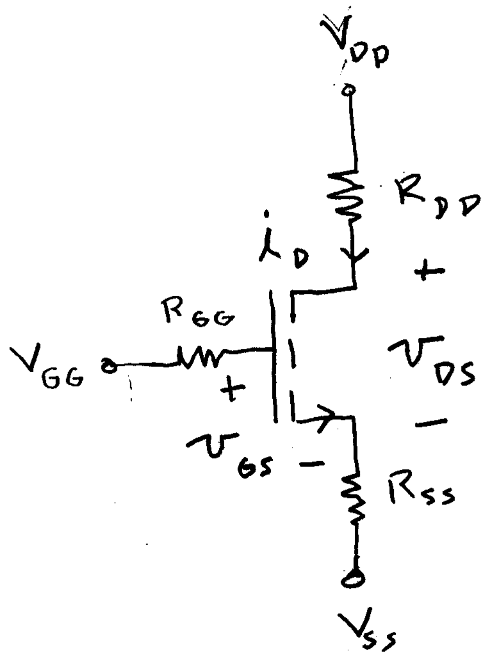
The MOSFET DC Bias Equation

Consider the circuit



We neglect the body effect to simplify the analysis

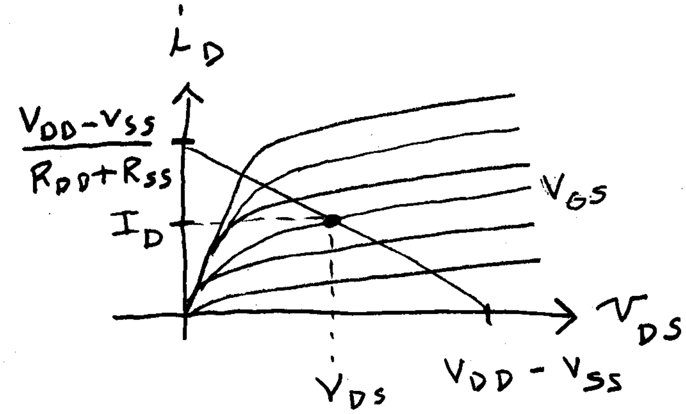
$$V_{DD} - V_{SS} = i_D R_{DD} + v_{DS} + i_D R_{SS}$$

$$\Rightarrow \quad i_D = \frac{V_{DD} - V_{SS}}{R_{DD} + R_{SS}} - \frac{v_{DS}}{R_{DD} + R_{SS}}$$

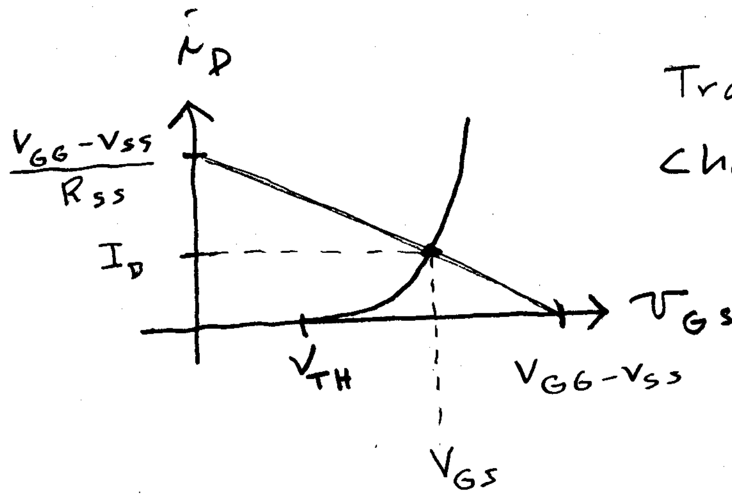
$$V_{GG} - V_{SS} = v_{GS} + i_D R_{SS}$$

$$\Rightarrow \quad i_D = \frac{V_{GG} - V_{SS}}{R_{SS}} - \frac{v_{GS}}{R_{SS}}$$

Graphical Solution



output characteristics



Transfer characteristic

You must plot both load lines to solve for I_D, V_{DS}, and V_{GS}.

Analytical Solution

$$i_D = K(v_{GS} - V_{TO})^2$$

$$V_{GG} - V_{SS} = v_{GS} + i_D R_{SS}$$

$$\Rightarrow v_{GS} = V_{GG} - V_{SS} - i_D R_{SS}$$

$$\Rightarrow i_D = K\left(V_{GG} - V_{SS} - i_D R_{SS} - V_{TO}\right)^2$$

Let $V_1 = V_{GG} - V_{SS} - V_{TO}$

$$\Rightarrow i_D = K(V_1 - i_D R_{SS})^2$$

$$\Rightarrow \frac{1}{\sqrt{K}} \sqrt{i_D} = V_1 - i_D R_{SS}$$

Let $i_D = x^2$

$$\Rightarrow \frac{1}{\sqrt{K}} x = V_1 - x^2 R_{SS}$$

$$\Rightarrow R_{SS} x^2 + \frac{1}{\sqrt{K}} x - V_1 = 0$$

$$\Rightarrow \quad x = \frac{\frac{-1}{\sqrt{K}} \pm \sqrt{\frac{1}{K} + 4R_{SS}V_1}}{2R_{SS}}$$

Must use the + sign for a positive x

$$\Rightarrow \quad x = \sqrt{\frac{1}{4KR_{SS}^2} + \frac{V_1}{R_{SS}}} - \frac{1}{2\sqrt{K}\,R_{SS}}$$

$$= \frac{1}{2\sqrt{K}\,R_{SS}}\left[\sqrt{1+4KV_1R_{SS}} - 1\right]$$

$$\Rightarrow \quad I_D = x^2$$

$$= \frac{1}{4KR_{SS}^2}\left[\sqrt{1+4KV_1R_{SS}} - 1\right]^2$$

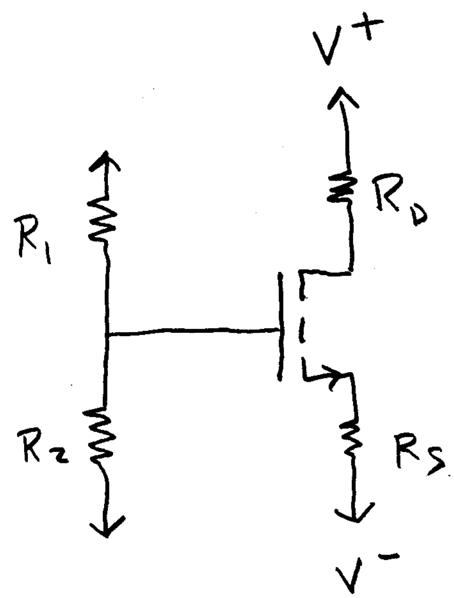
$$V_{DD} - V_{SS} = I_D R_{DD} + V_{DS} + I_D R_{SS}$$

$$\Rightarrow \quad V_{DS} = V_{DD} - V_{SS} - I_D(R_{DD} + R_{SS})$$

$$V_{GG} - V_{SS} = V_{GS} + I_D R_{SS}$$

$$\Rightarrow \quad V_{GS} = V_{GG} - V_{SS} - I_D R_{SS}$$

Example



$R_D = 5 \ k\Omega$

$V^+ = +24 \ V$

$V^- = -24 \ V$

$R_1 = 5 \ M\Omega$

$R_2 = 2 \ M\Omega$

$R_S = 3 \ k\Omega$

$k = 0.001 \ A/V^2$

$V_{TH} = 1.75 \ V.$

$$V_{GG} = V^+ \frac{R_2}{R_1+R_2} + V^- \frac{R_1}{R_1+R_2}$$

$$= -10.29 \ V.$$

$$R_{GG} = R_1 \| R_2 = 1.43 \ M\Omega$$

$V_{SS} = V^- = -24 \ V.$ $\quad R_{SS} = R_S = 3 \ k\Omega$

$V_{DD} = V^+ = +24 \ V$ $\quad R_{DD} = R_D = 5 \ k\Omega$

$$V_1 = V_{GG} - V_{SS} - V_{TH} = 11.96 \ V.$$

$$I_D = \frac{1}{4K R_S^2} \left[\sqrt{1 + 4K V_1 R_S} - 1 \right]^2$$

$$= 3.38 \quad mA$$

Test for saturation or active mode

$$V_{DS} > V_{GS} - V_{TH} \quad \text{must hold}.$$

$$V_{DS} = V_D - V_S = (V_{DD} - I_D R_D) - (V_{SS} + I_D R_{SS})$$

$$= 21.04$$

$$V_{GS} = V_{GG} - (V_S + I_D R_S)$$

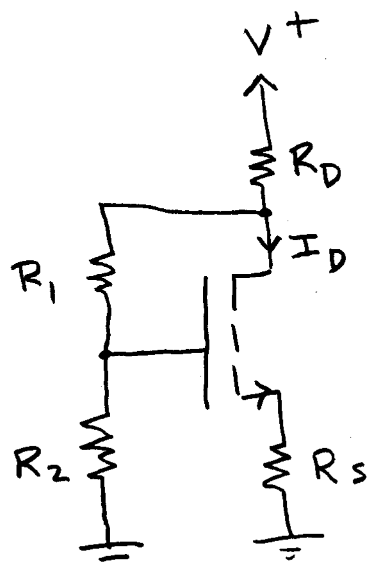
$$= 3.59 \quad V.$$

$$V_{GS} - V_{TH} = 1.84 \quad V, \quad \underline{\underline{\text{or}}} \quad \sqrt{\frac{I_D}{K}} = 1.84 \ V.$$

$$\therefore \quad V_{DS} > V_{GS} - V_{TH}$$

$\Rightarrow$ saturation mode

Example 2:



$V^+ = 24$ V

$R_D = 2.5\ K\Omega$

$R_S = 1\ K\Omega$

$R_1 = 1\ M\Omega$

$R_2 = 1\ M\Omega$

$K = 0.001\ A/V^2$

$V_{TH} = 1.5$ V.

By superposition of V^+ and I_D, we have

$$V_{GG} = V^+ \frac{R_2}{R_D + R_1 + R_2} - I_D \frac{R_D}{R_D + R_1 + R_2} R_2$$

$$= 7.491 - 1199\, I_D$$

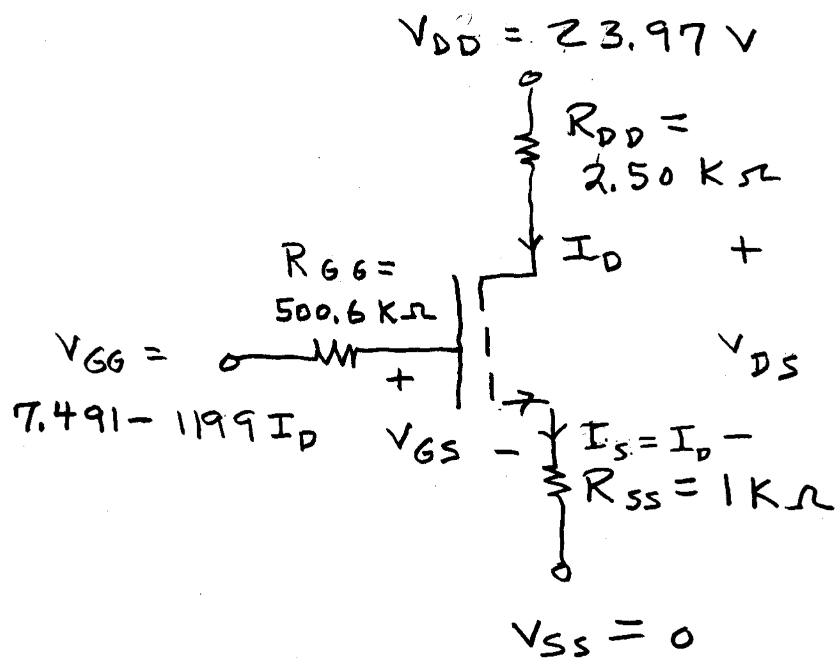
$$R_{GG} = R_2 \parallel (R_1 + R_D) = 500.6\ K\Omega$$

$$V_{SS} = 0 \qquad R_{SS} = R_S = 1K\Omega$$

$$V_{DD} = V^+ \frac{R_1 + R_2}{R_D + R_1 + R_2} = 23.97\ V$$

$$R_{DD} = R_D \parallel (R_1 + R_2) = 2.50\ K\Omega$$

The equivalent circuit becomes



$$V_{GG} - V_{SS} = V_{GS} + I_D R_{SS}$$

$$\Rightarrow \quad V_{GS} = V_{GG} - V_{SS} - I_D R_{SS}$$

$$I_D = K(V_{GS} - V_{TH})^2$$

$$= K\left(V_{GG} - V_{SS} - I_D R_{SS} - V_{TH}\right)$$

$$= 0.001\left(7.491 - 1199 I_D - 0 - 1000 I_D - 1.5\right)^2$$

$$= 0.001(5.991 - 2199 I_D)^2$$

Let $I_D = x^2$

$$\Rightarrow x^2 = 0.001\,(5.991 - 2199\,x^2)^2$$

$$\Rightarrow \frac{x}{\sqrt{0.001}} = 5.991 - 2199\,x^2$$

$$\Rightarrow 2199\,x^2 + \frac{x}{\sqrt{0.001}} - 5.991 = 0$$

$$\Rightarrow x = \frac{\frac{-1}{\sqrt{0.001}} \pm \sqrt{\frac{1}{0.001} + 4 \times 2199 \times 5.991}}{2 \times 2199}$$

Must use the + solution for a positive answer

$$\Rightarrow x = 454.99 \times 10^{-4}$$

$$\Rightarrow I_D = x^2 = 2.07 \text{ mA}$$

Now check for the saturation or active mode

$$V_D = V_{DD} - I_D R_{DD} = 10.02 \text{ V}$$

$$V_S = I_D R_{SS} + V_{SS} = 2.07 \text{ V.}$$

$$V_{DS} = V_D - V_S = 7.95 \text{ V.}$$

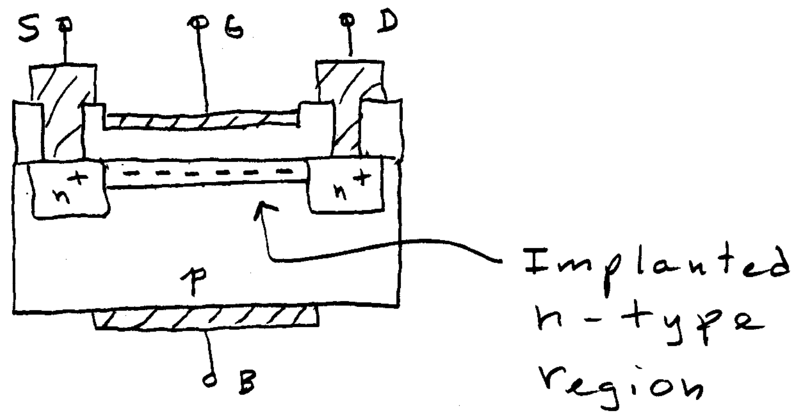
$$V_{GS} - V_{TH} = \sqrt{\frac{I_D}{K}} = 1.53 \text{ V.}$$

$$\therefore V_{DS} > V_{GS} - V_{TH} \Rightarrow \text{saturation mode}$$

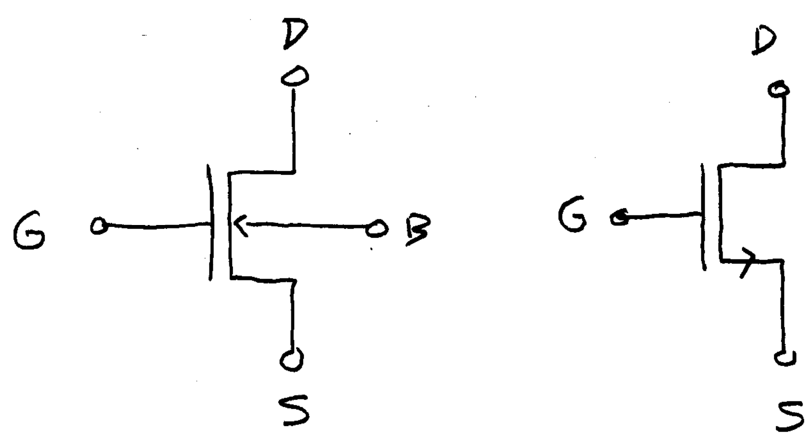
The Depletion Mode MOSFET

For the enhancement mode MOSFET, $V_{TH} > 0$ because the n channel does not form until v_{GS} is made positive. A built-in n channel can be formed by a process called ion implantation. Because the n channel exists for $v_{GS} = 0$, the threshold voltage is negative, i.e. $V_{TH} < 0$.

The device is fabricated as follows:

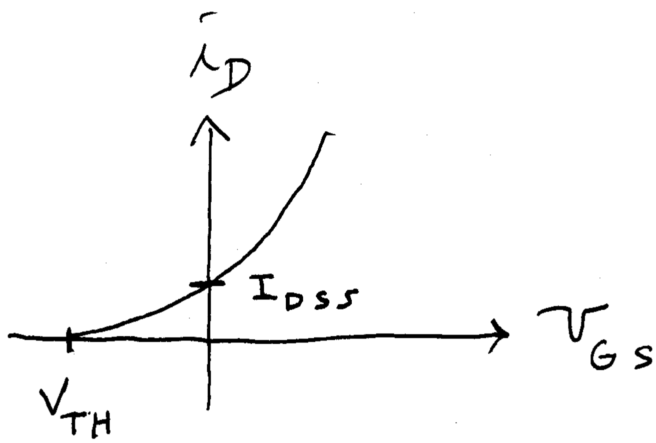


The circuit symbols are



The solid line indicates a conducting channel with $v_{GS} = 0$.

The drain current is given by the same equations as for the enhancement mode device except that V_{TH} is negative. Thus the i_D versus v_{GS} curve is of the form

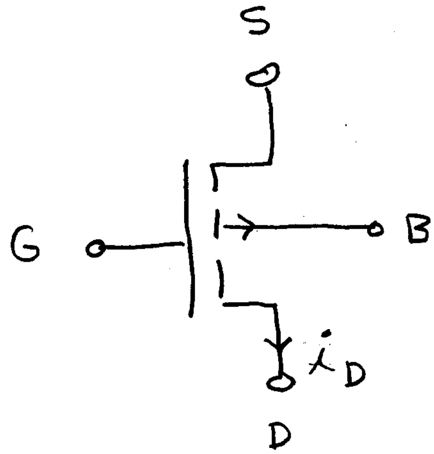


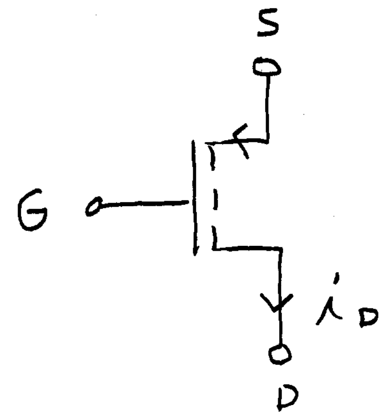
The value of i_D for $v_{GS} = 0$ is called I_{DSS}, the drain to source saturation current.

The P-Channel Devices

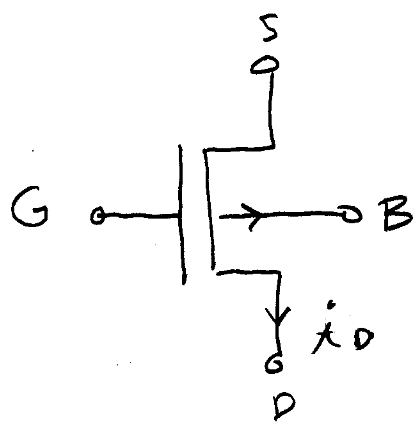
Flip over the symbols and change the directions of the arrows to obtain the following:

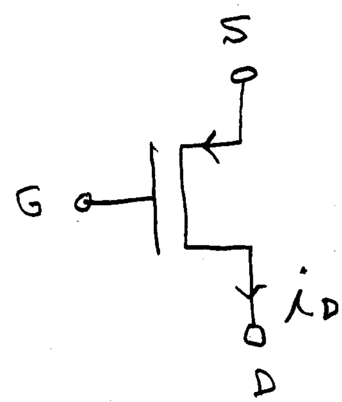
Enhancement Mode





Depletion Mode





In the current equations, interchange the subscripts for all voltages.

Linear or Triode Region

$$i_D = 2K\left(v_{SG} - V_{TH} - \frac{v_{SD}}{2}\right) v_{SD}$$

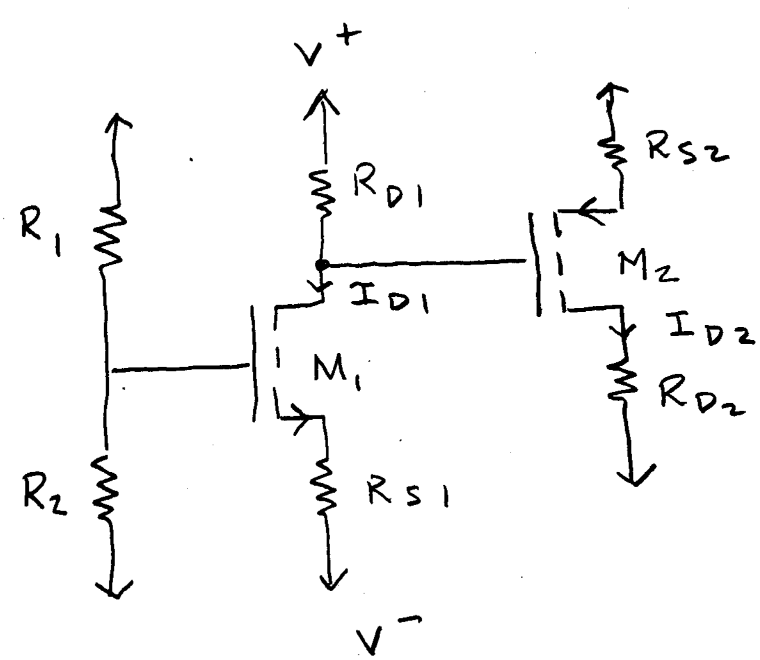
Saturation Region

$$i_D = K\left(v_{SG} - v_{TH}\right)^2$$

$$K = \frac{K'}{2}\frac{W}{L}\left(1 + \lambda v_{SD}\right)$$

$$K' = \mu C_{ox}$$

Bias Example



$V^+ = +24$ V. $\quad V^- = -24$ V.

$R_1 = 5\ M\Omega \quad R_2 = 1\ M\Omega$

$R_{D1} = 10\ K\Omega \quad R_{S1} = 3\ K\Omega$

$R_{D2} = 10\ K\Omega \quad R_{S2} = 1\ K\Omega$

$K = 0.001$ V.

$V_{TH1} = V_{TH2} = 1.75$ V

From a previous example, we have

$$I_{D1} = 1.655 \text{ mA}$$

For M_2, we have

$$V_{GG2} = V^+ - I_{D1} R_{D1}$$

$$= 19.04 \text{ V.}$$

$$R_{GG2} = R_{D1} = 3\ K\Omega$$

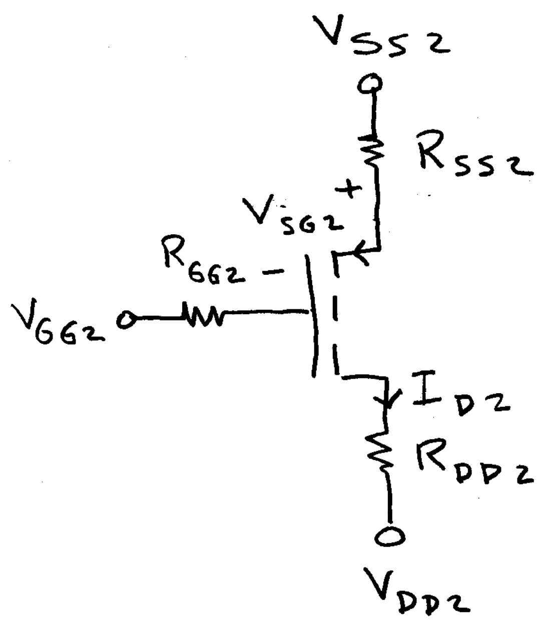
$$V_{DD2} = V^- = -24 \text{ V.}$$

$$R_{DD2} = R_{D2} =$$

$$V_{SS2} = V^{+} = +24\ V,$$

$$R_{SS2} = R_{S2} = 10\ K\Omega$$

The equivalent circuit for M_2 is



The bias equation is (omit the subscript 2)

$$V_{SS} - V_{GG} = I_D R_{SS} + V_{SG}$$

$$\Rightarrow \quad V_{SG} = V_{SS} - V_{GG} - I_D R_{SS}$$

Also

$$I_D = K\left(V_{SG} - V_{TO}\right)^2$$

$$\Rightarrow \quad I_D = K\left(V_{SS} - V_{GG} - I_D R_{SS} - V_{TO}\right)^2$$

$$= K\left(V_{SS} - V_{GG} - V_{TO} - I_D R_{SS}\right)^2$$

Let $\quad I_D = x^2$

$$V_{SS} - V_{GG} - V_{TO} = V_1$$

$$\Rightarrow \quad x^2 = K\left(V_1 - x^2 R_{SS}\right)^2$$

$$\Rightarrow \quad x^2 R_{SS} + \frac{x}{\sqrt{K}} - V_1 = 0$$

$$x = \frac{\frac{-1}{\sqrt{K}} \pm \sqrt{\frac{1}{K} + 4 R_{SS} V_1}}{2 R_{SS}} \qquad \text{choose } +$$

$$= \frac{\sqrt{1 + 4 K R_{SS} V_1} - 1}{2\sqrt{K}\, R_{SS}}$$

$$I_{D2} = x^2 = \frac{1}{4KR_{SS}^2}\left[\sqrt{1+4KR_{SS}V_1} - 1\right]^2$$

$$V_1 = V_{SS} - V_{GG} - V_{TH}$$

$$= 24 - 19.01 - 1.75$$

$$= 3.24 \text{ V}$$

$$\Rightarrow I_{D2} = 1.69 \text{ mA}$$

$$V_{SG2} - V_{TH2} = \sqrt{\frac{I_{D2}}{K}} = 1.30 \text{ V.}$$

$$V_{SD2} = V_{S2} - V_{D2}$$

$$= (V_{SS2} - I_{D2}R_{SS2}) - (V_{DD2} + I_{D2}R_{D2})$$

$$= V_{SS2} - V_{DD2} - I_{D2}(R_{SS2} + R_{D2})$$

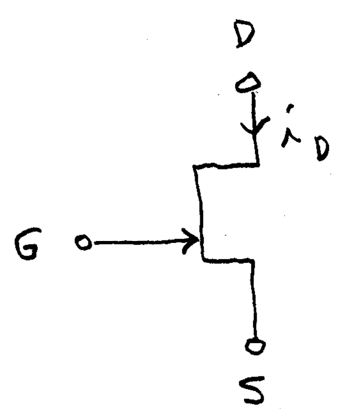
$$= 29.41 \text{ V.}$$

$$\Rightarrow V_{SD2} > V_{SG2} - V_{TH2} \Rightarrow \text{saturation}$$

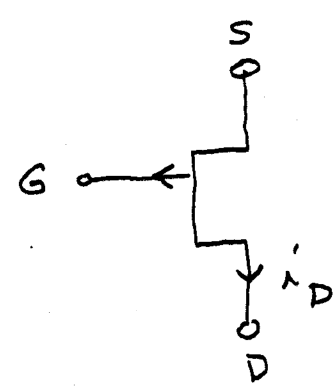
The Junction Field Effect Transistor

The JFET differs from the MOSFET in that the gate is separated from the channel by a pn junction rather than an insulating layer or dielectric. The devices are depletion mode devices.

Circuit Symbols



N-channel



P-channel

For no gate current to flow, we must have

$v_{GS} \leq 0$ for the n-channel
$v_{SG} \leq 0$ for the p-channel

This reverse biases the gate to channel pn junction.

For the n-channel device

$$i_D = \beta\,(v_{GS} - V_{T0})^2$$

$$\beta = \beta_0\,(1 + \lambda v_{DS})$$

β = transconductance parameter

V_{T0} = threshold voltage

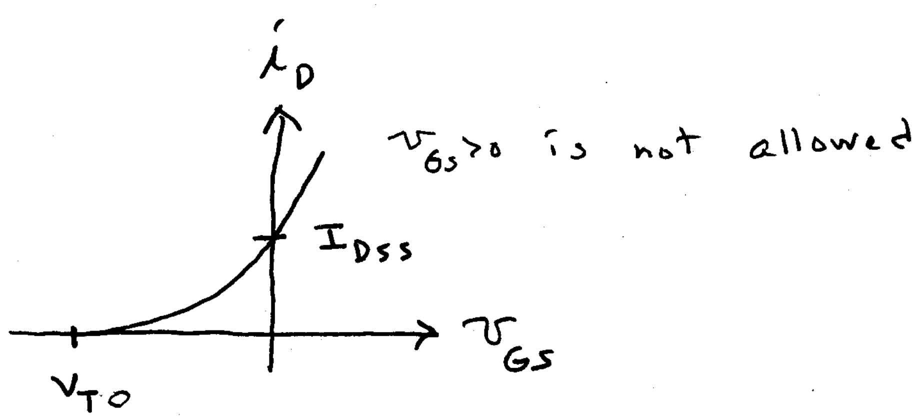
For the p-channel device

$$i_D = \beta\,(v_{SG} - V_{T0})^2$$

$$\beta = \beta_o (1 + \lambda v_{SD})$$

For both devices V_{TO} is negative.

Transfer Characteristics
n-channel device



$$I_{DSS} = \beta (0 - V_{TO})^2 = \beta V_{TO}^2$$

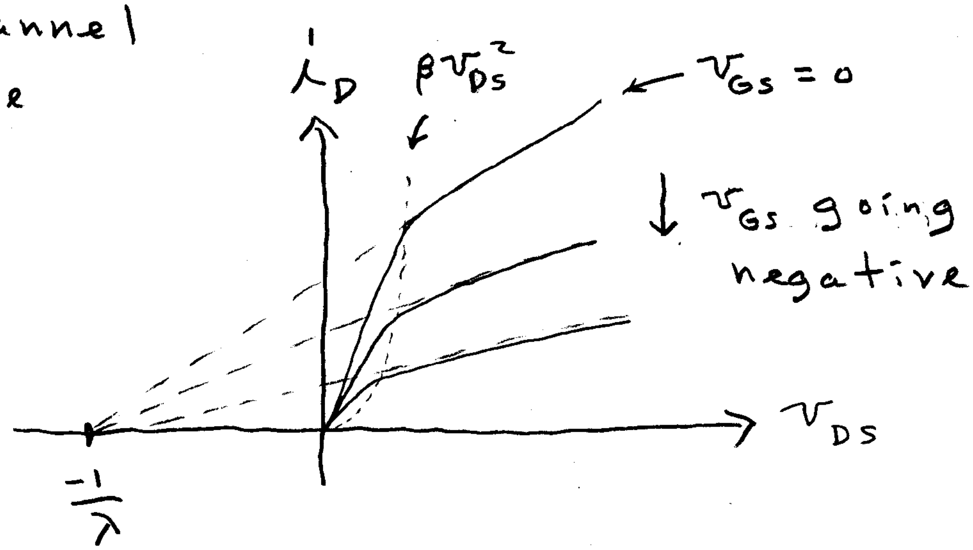
I_{DSS} is called the drain-to-source saturation current.

For the p-channel device, replace v_{GS} with v_{SG}.

6/9/4 (5)

Output Characteristics

n-channel Device



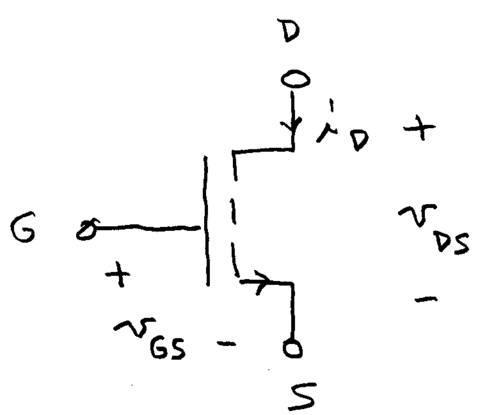
For the device to be in the saturation region, it must be operated to the right of the βv_{DS}^2 parabola. In this region

$$v_{DS} > v_{GS} - V_{TO}$$

For the p-channel device, replace v_{GS} with v_{SG} and v_{DS} with v_{SD}.

To obtain a JFET equation from a MOSFET equation, simply replace K with β in the equation. It must be remembered that the gate-to-channel junction must be reverse biased.

The Small-Signal π Model of the MOSFET



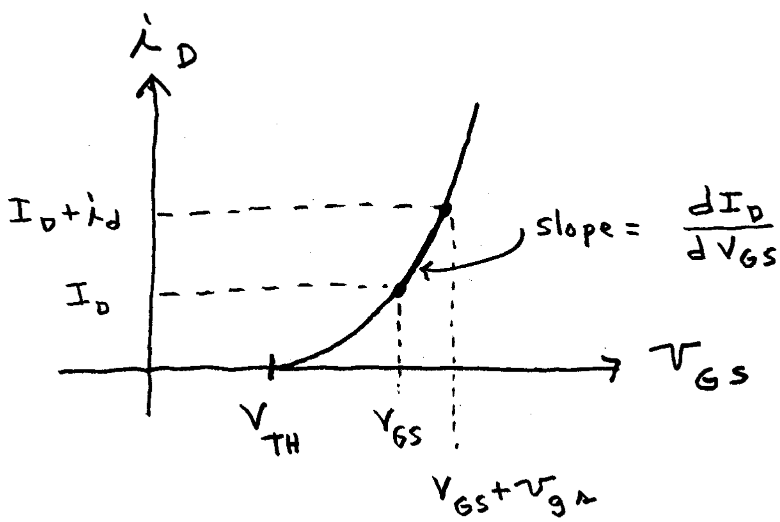
We assume that the body is connected to the source for this derivation.

$$i_D = K\,(v_{GS} - V_{TH})^2$$

$$K = \frac{K'}{2}\,\frac{W}{L}\,(1 + \lambda v_{DS})$$

$$K' = \mu C_{ox}$$

First, we look at the transfer characteristics for v_{DS} = const.



The Q point is (V_{GS}, I_D). We wish to relate the small-signal change in i_D to the small-signal change in v_{GS}. The slope of the curve at the Q point is used to do this.

$$I_D = K(V_{GS} - V_{TH})^2$$

$$\frac{dI_D}{dV_{GS}} = 2K(V_{GS} - V_{TH})$$

But $(V_{GS} - V_{TH}) = \sqrt{\frac{I_D}{K}}$

$$\Rightarrow \frac{dI_Q}{dV_{GS}} = 2K\sqrt{\frac{I_D}{K}} = 2\sqrt{KI_D}$$

We define the transconductance g_m by

$$g_m = \frac{dI_Q}{dV_{GS}} = 2\sqrt{KI_D}$$

This must be equal to

$$g_m = \frac{\Delta i_D}{\Delta v_{GS}} = \frac{i_d}{v_{gs}}$$

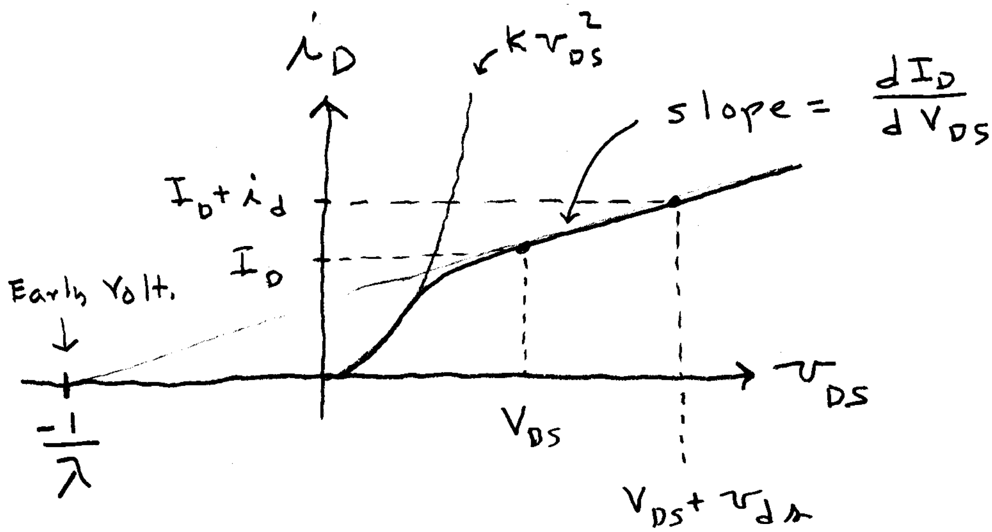
$$\Rightarrow i_d = g_m v_{gs}$$

This only holds for v_{DS} = constant.

Now, suppose we hold v_{GS} constant and vary v_{DS}.

$$i_D = \frac{K'}{2}\frac{W}{L}(1+\lambda v_{DS})(v_{GS}-V_{TH})^2$$

We plot the output characteristics for $v_{GS} = \text{const}$ as follows:



$$I_D = \frac{K'}{2}\frac{W}{L}(1+\lambda V_{DS})(V_{GS}-V_{TH})^2$$

$$\frac{dI_D}{dV_{DS}} = \frac{K'}{2}\frac{W}{L}\lambda(V_{GS}-V_{TH})^2$$

But $\frac{K'}{2}\frac{W}{L}(V_{GS}-V_{TH})^2 = \frac{I_D}{1+\lambda V_{DS}}$

$$\Rightarrow \frac{dI_D}{dV_{DS}} = \frac{\lambda I_D}{1+\lambda V_{DS}} = \frac{I_D}{\underbrace{\frac{1}{\lambda}}_{\text{Early Volt.}} + V_{DS}}$$

Let us define the resistance r_o by

$$\frac{1}{r_o} = \frac{dI_D}{dV_{DS}} = \frac{I_D}{\frac{1}{\lambda} + V_{DS}}$$

We can relate a small-signal change in i_D to a small-signal change in v_{DS} as follows

$$\frac{1}{r_o} = \frac{\Delta i_D}{\Delta v_{DS}} = \frac{i_d}{v_{ds}}$$

$$\Rightarrow \quad i_d = \frac{v_{ds}}{r_o}$$

This only holds for v_{GS} = constant. Suppose both v_{GS} and v_{DS} vary. Let $v_{gs} = \Delta v_{GS}$, $v_{ds} = \Delta v_{DS}$, and $i_d = \Delta i_D$. It follows that

$$i_d = g_m v_{gs} + \frac{v_{ds}}{r_o}$$

Because no gate current flows, we have $\Delta i_G = i_g = 0$

Thus we have the two equations

$$i_g = 0$$

$$i_d = g_m v_{gs} + \frac{v_{ds}}{r_o}$$

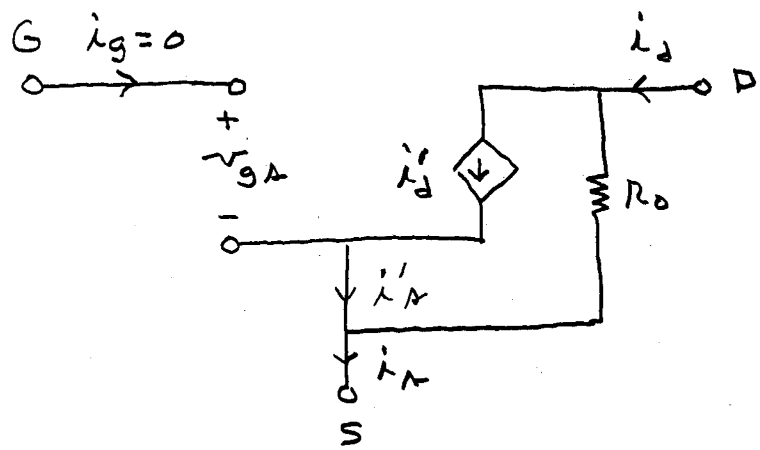
where $g_m = 2\sqrt{K I_D}$

$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D}$$

Note that $\Delta i_S = i_s = i_d + i_g$ must hold. Thus we have

$$i_s = i_d$$

Thus we can draw the following small signal circuit model



where $i_d' = g_m v_{gs}$

This is called the π model. It cannot be used to predict Q point values.

Because $i_s' = i_d' = g_m v_{gs}$, we have

$$v_{gs} = \frac{1}{g_m} i_s'$$

Let us define $R_s = \frac{1}{g_m}$

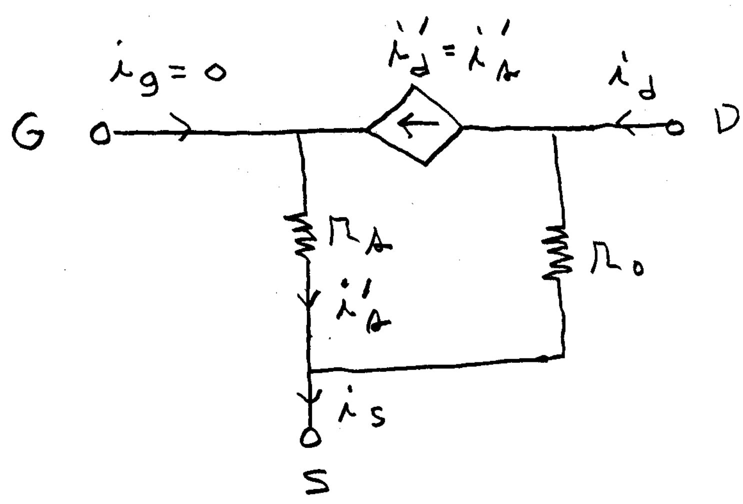
Thus we have the alternate set of equations

$$i_g = 0$$

$$i_d = i_s' + \frac{v_{ds}}{r_o}$$

$$i_s' = \frac{v_{gs}}{r_s}$$

The circuit which models these equations is



This is called the T model. Either model can be used to calculate the currents.