If $R_{ts} = 0$, the gains are the same. As $R_{ts}$ increases, the CG gain decreases. This if a high gain is desired, the CG stage should not be used with a source having a high $R_{ts}$.

The MOSFET Diff Amp with Body Effect

The differential amplifier or diff amp is a basic building block of analog electronics. It is an amplifier that has an output that is proportional to the difference between two input signals. The diff amp requires a dc current source to set its bias current. This can be realized with a MOSFET, a JFET, or a BJT. In some cases, a resistor is used.
We assume $M_1$ and $M_2$ are identical. For $V_{\bar{x}_1} = V_{\bar{x}_2} = 0$, $I_Q$ divides equally between $M_1$ and $M_2$ so that

$$I_{D1} = I_{D2} = \frac{1}{2} I_Q$$

The drain voltages are thus given by

$$V_{D1} = V_{D2} = V^+ - \frac{1}{2} I_Q R_D$$
For the ac signal analysis, we zero $V^+, V^-$, and $I_Q$ and replace the MOSFETs with their pi models. Because neither the drain or the source of $M_1$ or $M_2$ connect to ac ground, we omit $R_0$ for an approximate analysis.

\[ V_{o1} = -I_{d1}' R_D \]
\[ V_{o2} = -I_{d2}' R_D \]

\[ I_{d1}' = I_{dg1}' + I_{db1}' \]
\[ I_{d2}' = I_{dg2}' + I_{db2}' \]

\[ I_{d1}' = g_m v_{g1} + g_{mb} v_{b41} \]
\[ I_{d2}' = g_m v_{g2} + g_{mb} v_{b42} \]
\[ v_{gh1} = v_{i1} - v_A \]
\[ v_{gh2} = v_{i2} - v_A \]
\[ v_{b+i1} = -v_A \]
\[ v_{b+i2} = -v_A \]

\[ \Rightarrow \dot{i}_{d1} = g_m \cdot v_{i1} - (g_m + g_{mb}) \cdot v_A \]

\[ \dot{i}_{d2} = g_m \cdot v_{i2} - (g_m + g_{mb}) \cdot v_A \]

Now \[ \dot{i}_{d1} + \dot{i}_{d2} = 0 \]

\[ \Rightarrow g_m (v_{i1} + v_{i2}) - 2(g_m + g_{mb}) v_A = 0 \]

\[ \Rightarrow v_A = \frac{g_m}{g_m + g_{mb}} \cdot \frac{v_{i1} + v_{i2}}{2} \]

\[ \Rightarrow \dot{i}_{d1} = g_m \cdot v_{i1} - (g_m + g_{mb}) \cdot \frac{g_m}{g_m + g_{mb}} \cdot \frac{v_{i1} + v_{i2}}{2} \]

\[ = g_m \cdot \frac{v_{i1} - v_{i2}}{2} \]

Because \[ \dot{i}_{d1} + \dot{i}_{d2} = 0 \]

\[ \Rightarrow \dot{i}_{d2} = -g_m \cdot \frac{v_{i1} - v_{i2}}{2} \]
Thus for \( V_{01} \) and \( V_{02} \), we have

\[
V_{01} = - \frac{g_m R_D}{2} (V_{x1} - V_{x2})
\]

\[
V_{02} = + \frac{g_m R_D}{2} (V_{x1} - V_{x2})
\]

Note that the body effect has cancelled. Thus we could have omitted it from the pi model from the start.

Now that we see that the body effect cancels, let us use the T model without the body effect to calculate \( V_{01} \) and \( V_{02} \).
\[ \nu_{01} = -\kappa_{A1} R_D \quad \nu_{02} = -\kappa_{A2} R_D \]

\[ \kappa_{A1} = -\kappa_{A2} = \frac{\nu_{x1} - \nu_{x2}}{2R_A} \]

\[ \Rightarrow \nu_{01} = -\frac{R_D}{2R_A} (\nu_{x1} - \nu_{x2}) \]

\[ \nu_{02} = +\frac{R_D}{2R_A} (\nu_{x1} - \nu_{x2}) \]

Because \( R_A = \frac{1}{g_m} \), these are the same answers.

Often the output of the diff amp is taken differentially,

\[ \nu_0 = \nu_{01} - \nu_{02} \]

\[ = -g_m R_D (\nu_{x1} - \nu_{x2}) \]

or

\[ = -\frac{R_D}{R_A} (\nu_{x1} - \nu_{x2}) \]
The Bipolar Junction Transistor

The BJT is fabricated as two back-to-back p-n junctions. There are two types, the npn and the pnp. A diagram of each is as follows:

n-p-n (npn):

p-n-p (pnp):

The circuit symbols are

n-p-n (npn):

p-n-p (pnp):

The leads are called the collector (C), the base (B), and the emitter (E).
To understand its operation, consider the npn circuit

![NPN Circuit Diagram]

The battery $V_{CB}$ reverse biases the collector-to-base junction. The battery $V_{BE}$ forward biases the base-to-emitter junction. The two $n$ regions are doped very heavily with $n$-type impurities. The $p$ region is doped very lightly with a $p$-type impurity. Because of the way it is doped, the majority current carriers are free electrons.

The battery $V_{BE}$ causes free electrons to be injected or
emitted from the emitter region into the base region. The battery $V_{CB}$ sets up an electric field $E$ from the collector region into the base region. The base region is very narrow. This electric field attracts the free electrons injected into the base and pulls them into the collector. The fraction of electrons which are collected is denoted by $\alpha$. Thus we have

$$I_c = \alpha I_E$$

When $I_E = 0$, a small reverse saturation current flows across the reverse biased collector-to-base region. This current is denoted by $I_{CBO}$. The current flows from C to B with the E open circuited, i.e., with $I_E = 0$. Thus we add $I_{CBO}$ to the equation for $I_c$ to obtain
\[ I_c = \alpha I_E + I_{CBO} \]

The parameter \( \alpha \) is called the emitter-to-collector current gain. A typical value for \( \alpha \) is \( \alpha = 0.99 \).

Next, we relate \( I_c \) to \( I_B \).

By KCL, we have

\[ I_E = I_c + I_B \]

\[ \Rightarrow I_c = \alpha (I_c + I_B) + I_{CBO} \]

\[ \Rightarrow I_c = \frac{\alpha}{1 - \alpha} I_B + \frac{I_{CBO}}{1 - \alpha} \]

Let \( \beta \) and \( I_{CEO} \) be defined as follows:

\[ \beta = \frac{\alpha}{1 - \alpha} \quad I_{CEO} = \frac{I_{CBO}}{1 - \alpha} \]

Thus we can write
\[ I_C = \beta I_B + I_{CE0} \]

The parameter \( \beta \) is called the base-to-collector current gain. A typical value is \( \beta = 100 \). For \( d = 0.99 \), \( I_{CE0} \) is 100 times greater than \( I_{CBO} \).

Next, we look at the pnp device. Consider the circuit

\[ V_{EB} \] forward biases the emitter-to-base junction. \( V_{BC} \) reverse biases the base-to-collector junction. The p-type regions
are doped much heavier than the n-type region. Thus holes are the majority current carriers.

Holes that are emitted from the E to the B are collected by the E field across the base-to-collector junction and we have

\[ I_c = \alpha I_E + I_{CEO} \]

\[ = \beta I_B + I_{CEO} \]

where \[ \beta = \frac{\alpha}{1-\alpha} \]

\[ I_{CEO} = \frac{I_{CEO}}{1-\alpha} \]

The equations are identical to those for the npn except the voltage polarities and current directions are reversed.

Next, we wish to relate \( I_c \) to the B-E voltage.
Consider the circuit

Let $I_{Es}$ be the saturation current of the B to E diode. We can write

$$I_E = I_{Es} \left( e^{\frac{V_{BE}}{V_T}} - 1 \right)$$

Note that $\eta = 1$. This is because there are almost no recombinations due to the very lightly doped $p$-type base region.

For $I_c$, we can write
\[ I_c = \alpha I_E s \left( e^{V_{BE}/V_T} - 1 \right) + I_{cbo} \]

Let us define the BJT saturation current as

\[ I_s = \alpha I_E s \]

Thus we have

\[ I_c = I_s e^{V_{BE}/V_T} + (I_{cbo} - I_s) \]

For \( I_B \), we have

\[ I_B = \frac{I_c - I_{CEO}}{\beta} \]

\[ = \frac{1}{\beta} \left[ I_s e^{V_{BE}/V_T} + (I_{cbo} - I_s) - I_{CEO} \right] \]

But \( I_{cbo} - I_{CEO} = I_{cbo} - \frac{I_{cbo}}{1 - \alpha} \)

\[ = I_{cbo} \left( 1 - \frac{1}{1 - \alpha} \right) \]

\[ = -\beta I_{cbo} \]
Thus we can write

\[ I_B = \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}} - \left( I_{CBO} + \frac{I_S}{\beta} \right) \]

The Modes of operation

Consider the circuit

We can write

\[ I_B = \frac{V_{BB} - V_{BE}}{R_{BB}} \]

\[ I_C = \beta I_B + I_{CEO} \]
\[ V_{CE} = V_{CC} - I_c R_{cc} \]

To see how \( V_{CE} \) varies with \( I_B \), let us assume \( I_{CEO} \) is small so that we can write

\[ I_c = \beta I_B \]

\[ \Rightarrow V_{CE} = V_{CC} - \beta I_B R_{cc} \]

As \( I_B \) is increased, \( V_{CE} \) decreases, but it cannot go negative because the B-C junction becomes forward biased. When this happens, \( V_{CE} \) levels off at a value labeled.
$V_{CE(sat)}$. This is given by

$$V_{CE(sat)} = V_{BE(sat)} - V_{BC(sat)}$$

The transistor is said to be saturated. Typically, for most BJT's

$$0 < V_{CE(sat)} < 0.2 \text{ V}$$

The value of $I_B$ at which saturation occurs is obtained from

$$V_{CE(sat)} = V_{cc} - \beta I_{B1} R_c$$

$$\Rightarrow I_{B1} = \frac{V_{cc} - V_{CE(sat)}}{\beta R_c}$$

For $I_B > I_{B1}$, the BJT is saturated. For $0 < I_B < I_{B1}$, the BJT is said to be in its linear or active mode.
The base current is given by

\[ I_B = \frac{V_{BB} - V_{BE}}{R_{BB}} \]

For \( V_{BB} = 0 \), \( I_B \) is zero. As \( V_{BB} \) is increased, \( I_B \) will begin to flow when the B-E junction turns on. We denote the value of \( V_{BE} \) at which the BJT turns on by \( V_f \). This is also called the cut-in voltage. Typically \( V_f \approx 0.5 \text{V} \) to \( 0.6 \text{V} \).

As \( V_{BB} \) is increased, \( I_B \) begins to flow for \( V_{BB} > V_f \). The BJT enters the active region. In this region \( V_{BE} \) (active) = \( 0.6 \text{V} \) to \( 0.7 \text{V} \). For \( I_B > I_{Bi} \), the BJT is saturated. In this case, both junctions are forward biased.
Regions of operation

1. Active - $V_{BE} > 0$ and $V_{CB} > 0$
2. Saturation - $V_{BE} > 0$ and $V_{CB} < 0$
3. Cutoff - $V_{BE} < 0$ and $V_{CB} > 0$
4. Reverse Active - $V_{BC} > 0$ and $V_{BE} < 0$

**Summary Graph**

The Early Effect

When $V_{CE}$ changes, the width of the base region changes. This is called base width modulation or the Early effect. It causes both $I_S$ and $\beta$ to vary with $V_{CE}$. 
They are given by

\[ I_s = I_{so} \left( 1 + \frac{V_{CE}}{V_A} \right) \]

\[ \beta = \beta_0 \left( 1 + \frac{V_{CE}}{V_A} \right) \]

where \( V_A \) is the Early Voltage, \( I_{so} \) is the value of \( I_s \) with \( V_{CE} = 0 \), and \( \beta_0 \) is the value of \( \beta \) with \( V_{CE} = 0 \).

General Current Equations for the BJT

We will assume that the device is in its active mode and that the leakage currents can be neglected.
\[ \dot{I}_c = I_s e^{\frac{V_{BE}}{V_T}} \]

\[ \dot{I}_B = \frac{\dot{I}_C}{\beta} \]

\[ \dot{I}_E = \dot{I}_C + \dot{I}_B \]

\[ I_s = I_{so} \left( 1 + \frac{V_{CE}}{V_A} \right) \]

\[ \beta = \beta_0 \left( 1 + \frac{V_{CE}}{V_A} \right) \]

The transfer characteristic is a plot of \( \dot{I}_C \) versus \( V_{BE} \) with \( V_{CE} = \text{constant} \). If \( V_{CE} = \text{constant} \), then \( I_s = \text{constant} \).

\[ \dot{I}_C = I_s e^{\frac{V_{BE}}{V_T}} \]

\[ \text{increases rapidly with } V_{BE} \]

\[ \text{V}_s \quad \rightarrow \quad V_{BE} \]
The output characteristics are plots of $i_C$ versus $V_{CE}$ for various values of $i_B$.

$$i_C = \beta i_B = \beta_0 \left(1 + \frac{V_{CE}}{V_A}\right) i_B$$

In the saturation region $V_{CB} < 0$ so that $V_{CE} = V_{CB} + V_{BE} < V_{BE}$. That is $V_{CE} < V_{BE}$.

The input characteristics are a plot of $i_B$ versus $V_{BE}$ for $V_{CE} = \text{constant}$. 
\[ I_B = \frac{I_C}{\beta} = \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}} = I_{So} \frac{V_{BE}}{V_T} \]

Note that the Early effect cancels out so that \( I_B \) is not a function of \( V_{CE} \).

The BJT Bias Equation

Assume the external circuits are represented by Thévenin equivalent circuits. For the npn device, the general dc bias circuit is
The loop equation for the B to E loop is

\[ V_{BB} - V_{EE} = I_B R_{BB} + V_{BE} + I_E R_{EE} \]

To solve for \( I_c \), let \( I_B = I_c / \beta \) and \( I_E = I_c / \alpha \)

\[ \Rightarrow V_{BB} - V_{EE} = I_c \frac{R_{BB}}{\beta} + V_{BE} + I_c \frac{R_{EE}}{\alpha} \]

\[ \Rightarrow I_c = \frac{V_{BB} - V_{EE} - V_{BE}}{\frac{R_{BB}}{\beta} + \frac{R_{EE}}{\alpha}} \]

Neglect leakage currents so that \( I_c = \beta I_B = \alpha I_E \)
Often it is desired to solve for $I_E$. To do this, we write $I_B$ as follows:

$$I_B = I_E - I_C = I_E - \alpha I_E = (1 - \alpha) I_E$$

But $1 - \alpha = 1 - \frac{\beta}{1+\beta} = \frac{1}{1+\beta}$

$$\Rightarrow I_B = \frac{I_E}{1+\beta}$$

$$\Rightarrow V_{BB} - V_{EE} = I_E \frac{R_{BB}}{1+\beta} + V_{BE} + I_E R_{EE}$$

$$\Rightarrow I_E = \frac{V_{BB} - V_{EE} - V_{BE}}{\frac{R_{BB}}{1+\beta} + R_{EE}}$$

**Example 1**

- $V^+ = 12$ V.
- $R_B = 2 \text{ M} \Omega$
- $R_C = 10 \text{ k} \Omega$
- $R_E = 1 \text{ k} \Omega$
- $\beta = 99$
- $V_{BE} = 0.65$ V
\[ I_E = \frac{V^+ - V_{BE}}{R_B + \frac{R_E}{1+\beta}} = 0.5405 \text{ mA} \]

Next, we test for the active mode. This requires \( V_{CB} > 0 \).

\[ V_{CB} = V_C - V_B = (V^+ - I_C R_C) - (V_{BE} + I_E R_E) = (V^+ - \alpha I_E R_C) - (V_{BE} + I_E R_E) = 5.459 \text{ V} \]

Thus the BJT is in the active mode.

Example 2

\[ V^+ = +12 \text{ V} \]
\[ V^- = -12 \text{ V} \]
\[ \beta = 99 \]
\[ V_{BE} = 0.65 \text{ V} \]
\[ R_1 = 300 \text{ k}\Omega \]
\[ R_2 = 20 \text{ k}\Omega \]
\[ R_c = 5.6 \text{ k}\Omega \]
\[ R_E = 2 \text{ k}\Omega \]
\[ V_{BB} = V^+ \frac{R_2}{R_1 + R_2} + V^- \frac{R_1}{R_1 + R_2} = -10.5 \, V \]

\[ R_{BB} = R_1 || R_2 = 18.75 \, K\Omega \]

\[ I_C = \frac{V_{BB} - V^- - V_{BE}}{\frac{R_{BB}}{\beta} + \frac{R_E}{\alpha}} = 0.3847 \, mA \]

Next, we test for the active mode.

\[ V_{CB} = (V^+ - I_C R_C) - (V_{BE} + \frac{I_C}{\alpha} R_E + V^-) \]

\[ = 14.88 \, V. \]

Because \( V_{CB} > 0 \), the BJT is in the active mode.