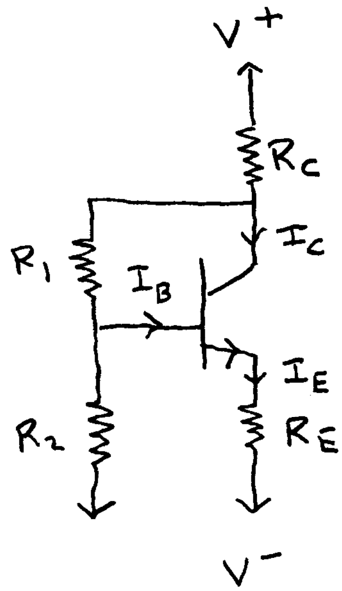
Example 3



$V^+ = +24\,V.$

$V^- = -24\,V.$

$R_c = 7.5\,K\Omega$

$R_1 = 2\,M\Omega$

$R_2 = 1.2\,M\Omega$

$R_E = 6.2\,K\Omega$

$\beta = 99$

$\alpha = 0.99$

$V_{BE} = 0.65\,V.$

$$V_{BB} = V^+ \frac{R_2}{R_c + R_1 + R_2} + V^- \frac{R_1 + R_c}{R_c + R_1 + R_2}$$

$$- I_c \frac{R_c}{R_c + R_1 + R_c} R_2$$

$$= -6.042 - 2806 I_c$$

$$R_{BB} = (R_1 + R_c) \| R_2 = 751.1\ K\Omega$$

$$V_{BB} - V_{EE} = -6.042 - 2806 I_c - (-24)$$

$$= 17.958 - 2806\, I_c$$

But $V_{BB} - V_{EE} = I_B R_{BB} + V_{BE} + I_E R_E$

$$= \frac{I_c}{\beta} R_{BB} + V_{BE} + \frac{I_c}{\alpha} R_E$$

$$= 13.85 \times 10^3 \, I_c + 0.65$$

$$\Rightarrow \quad 17.958 - 2806 \, I_c = 13.85 \times 10^3 \, I_c + 0.65$$

$$\Rightarrow \quad I_c = \frac{17.958 - 0.65}{2806 + 13.85 \times 10^3} = 1.039 \text{ mA}$$

Next we test for the active mode.

$$V_{CB} = (V_{CC} - I_c R_{CC}) - (V_{BE} + I_E R_E + V^-)$$

But $$V_{CC} = V^+ \frac{R_1 + R_2}{R_c + R_1 + R_2} + V^- \frac{R_c}{R_c + R_1 + R_2} - I_B \frac{R_2}{R_c + R_1 + R_2}$$

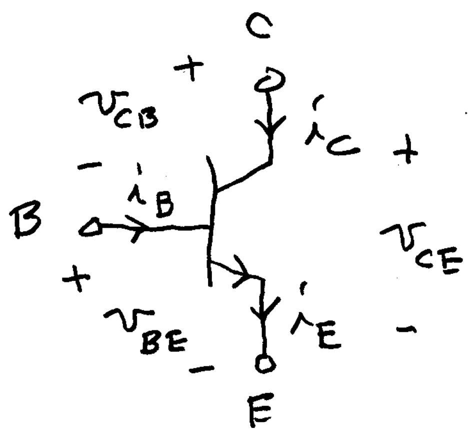
$$= 23.858 \text{ V.}$$

$$R_{CC} = R_c \parallel (R_1 + R_2) = 7482 \, \Omega$$

$$V_{CB} = (V_{CC} - I_c R_{CC}) - \left(V_{BE} + \frac{I_c}{\alpha} R_E + V^-\right)$$

$$= 32.92 \text{ V} \quad \Rightarrow \text{active mode}$$

The Small-Signal π Model of the BJT



We assume the active mode and neglect the leakage current terms

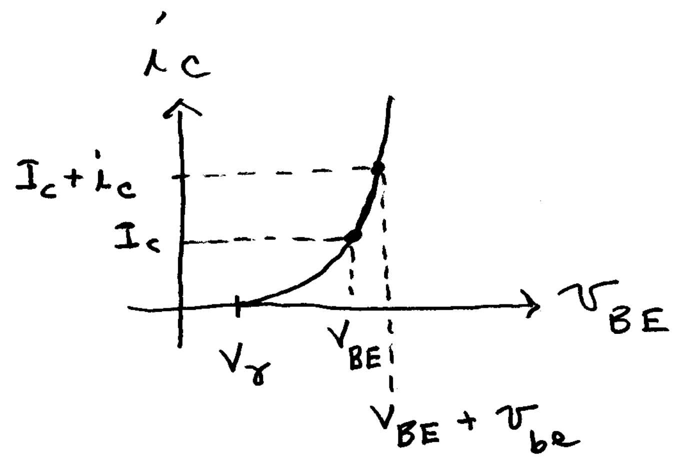
$$i_C = I_S \, e^{v_{BE}/V_T}$$

$$i_B = \frac{1}{\beta} \, i_C$$

$$I_S = I_{S0} \left(1 + \frac{v_{CE}}{V_A}\right)$$

$$\beta = \beta_0 \left(1 + \frac{v_{CE}}{V_A}\right)$$

First, we look at the transfer characteristics for $v_{CE} = \text{const.}$ $\Rightarrow I_S = \text{const.} \Rightarrow i_C$ is a function of v_{BE} only.



The Q point is (V_{BE}, I_C). We wish to relate the small-signal change in i_C to the small-signal change in v_{BE}. The slope of the curve at the Q point is used to do this.

$$I_C = I_S \, e^{V_{BE}/V_T}$$

$$\frac{dI_C}{dV_{BE}} = \frac{1}{V_T} I_C \, e^{V_{BE}/V_T} = \frac{I_C}{V_T}$$

We define the transconductance g_m by

$$g_m = \frac{dI_C}{dV_{BE}} = \frac{I_C}{V_T}$$

This must be equal to

$$g_m = \frac{\Delta i_C}{\Delta v_{BE}} = \frac{i_c}{v_{be}}$$

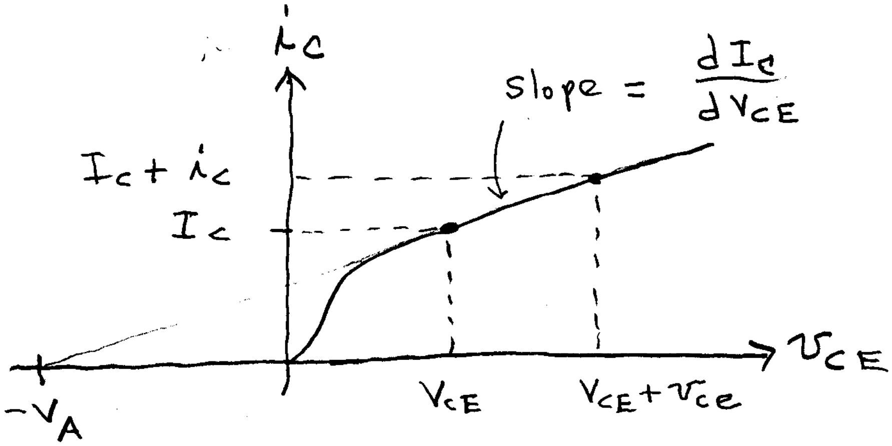
$$\Rightarrow \quad i_c = g_m v_{be}$$

This only holds for $v_{ce} = \text{const.}$

Now suppose we hold v_{be} const. and vary v_{CE}. We have already seen that if $v_{BE} = \text{const.}$, then $i_B = \text{const.}$

$$\Rightarrow \quad i_C = \beta i_B = \beta_o \left(1 + \frac{v_{CE}}{V_A}\right) i_B$$

We plot the output characteristics with $i_B = \text{const.}$ as follows:



6/28/4 (4)

$$I_c = \beta_o \left(1 + \frac{V_{CE}}{V_A}\right) I_B$$

$$\frac{dI_c}{dV_{CE}} = \beta_o \frac{1}{V_A} I_B$$

But $\beta_o I_B = \dfrac{I_c}{1 + \frac{V_{CE}}{V_A}}$

$$\Rightarrow \frac{dI_c}{dV_{CE}} = \frac{1}{V_A} \frac{I_c}{1 + \frac{V_{CE}}{V_A}} = \frac{I_c}{V_A + V_{CE}}$$

Let us define the resistance r_o by

$$\frac{1}{r_o} = \frac{dI_c}{dV_{CE}} = \frac{I_c}{V_A + V_{CE}}$$

$$\Rightarrow r_o = \frac{V_A + V_{CE}}{I_c}$$

We can relate a small-signal change in i_c to a small-signal change in v_{CE} as follows

$$\frac{1}{r_o} = \frac{\Delta i_c}{\Delta v_{CE}} = \frac{i_c}{v_{ce}}$$

$$\Rightarrow i_c = \frac{v_{ce}}{r_o}$$

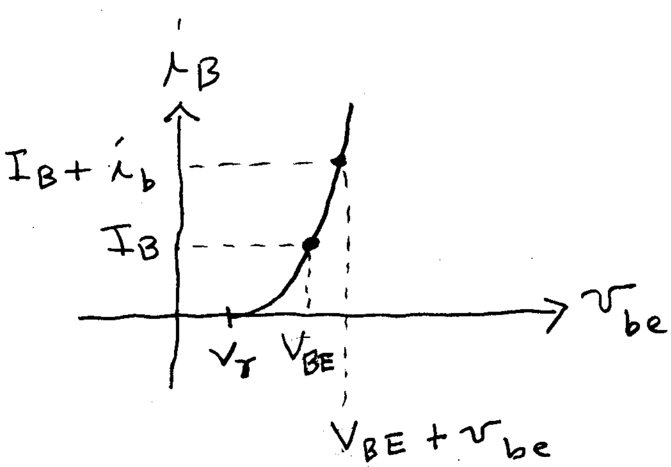
This only holds for v_{BE} = const. (or i_B = const.). Suppose both v_{BE} and v_{CE} vary. Let $v_{be} = \Delta v_{BE}$, $v_{ce} = \Delta v_{CE}$, and $i_c = \Delta i_C$. It follows that

$$i_c = g_m v_{be} + \frac{v_{ce}}{r_o}$$

Now, let us plot i_B versus v_{BE} for v_{CE} = const.

$$i_B = \frac{1}{\beta} i_C = \frac{I_S}{\beta} e^{v_{BE}/V_T} = \frac{I_{S0}}{\beta_0} e^{v_{BE}/V_T}$$

Note that the $1 + v_{CE}/V_A$ terms in I_S and β have cancelled.



The Q point is (V_{BE}, I_B). We wish to relate the small-signal change in i_B to the small-signal change in v_{BE}. The slope of the curve at the Q point is used to do this

$$I_B = \frac{I_{SO}}{\beta_o} e^{V_{BE}/V_T}$$

$$\frac{dI_B}{dV_{BE}} = \frac{1}{V_T} \frac{I_{SO}}{\beta_o} e^{V_{BE}/V_T}$$

$$= \frac{I_B}{V_T}$$

Let us define the resistance r_π by

$$\frac{1}{r_\pi} = \frac{dI_B}{dV_{BE}} = \frac{I_B}{V_T}$$

$$\Rightarrow \quad r_\pi = \frac{V_T}{I_B}$$

We can relate a small-signal change in i_B to a small-signal change in

v_{be} as follows

$$\frac{1}{r_\pi} = \frac{\Delta i_B}{\Delta v_{BE}} = \frac{i_b}{v_{be}}$$

$$\Rightarrow \quad i_b = \frac{v_{be}}{r_\pi}$$

Notice that i_b is not a function of v_{ce} because the Early effect cancels in the equation for i_B.

Summary:

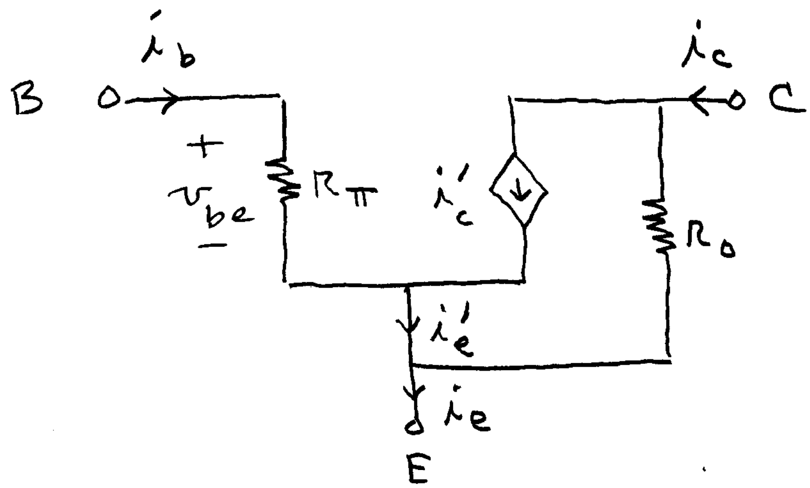
$$i_b = \frac{v_{be}}{r_\pi}$$

$$i_c = g_m v_{be} + \frac{v_{ce}}{r_o}$$

Let us define $i_c' = g_m v_{be}$ so that i_c can be written

$$i_c = i_c' + \frac{v_{ce}}{r_o}$$

The circuit which models these equations is



where $i_c' = g_m v_{be}$

The above circuit is called the hybrid-pi model of the BJT.

We can relate g_m and r_π as follows:

$$g_m = \frac{I_C}{V_T} = \frac{\beta I_B}{V_T} = \frac{\beta}{r_\pi}$$

$$\Rightarrow \quad \beta = g_m r_\pi$$

We can relate the currents as follows:

$$i_c' = g_m v_{be} = g_m i_b r_\pi = \beta i_b$$

$$\Rightarrow i_c' = \beta i_b$$

$$i_e' = i_b + i_c' = \frac{i_c'}{\beta} + i_c' = \frac{1+\beta}{\beta} i_c'$$

But $\alpha = \dfrac{\beta}{1+\beta} \Rightarrow i_e' = \dfrac{i_c'}{\alpha}$

$$\Rightarrow i_c' = \alpha i_e'$$

Thus we have 3 equations for i_c':

$$i_c' = g_m v_{be} = \beta i_b = \alpha i_e'$$

Either of these can be used in analyzing a BJT circuit.

Next, we convert the pi model to the T model.

$$v_{be} = i_b r_\pi = \frac{i_c'}{\beta} r_\pi = \frac{\alpha i_e'}{\beta} r_\pi$$

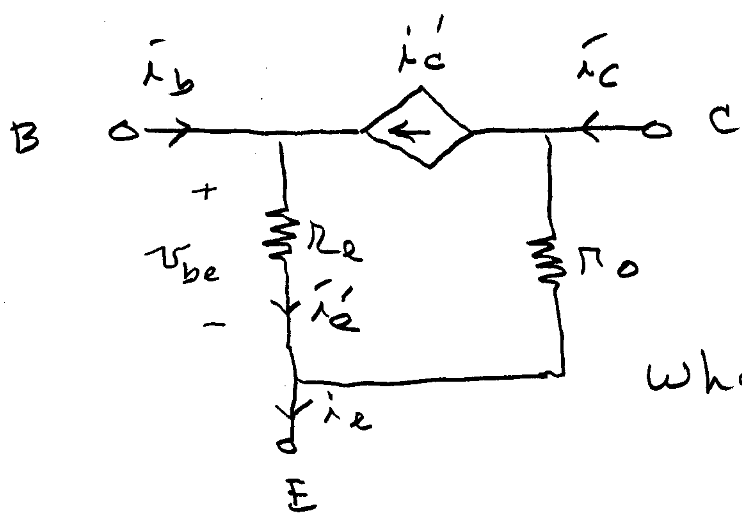
Let us define the resistance r_e by

$$r_e = \frac{\alpha}{\beta} r_\pi = \frac{\alpha}{\beta} \frac{V_T}{I_B} = \frac{\alpha V_T}{I_C} = \frac{V_T}{I_E}$$

$$\Rightarrow \quad v_{be} = i_e' \, r_e$$

$$i_c' = \alpha i_e' + \frac{v_{ce}}{r_o}$$

The circuit which models these equations is

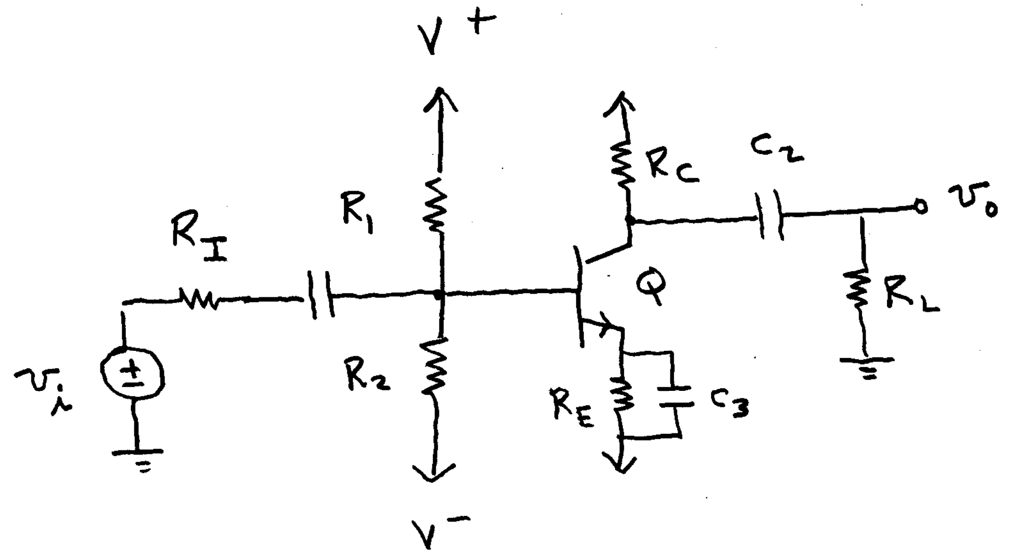


where $i_c' = \alpha i_e'$

This is called the T model. In this circuit, i_c' can be calculated from one of the 3 equations

$$i_c' = g_m v_{be} = \beta i_b = \alpha i_e'$$

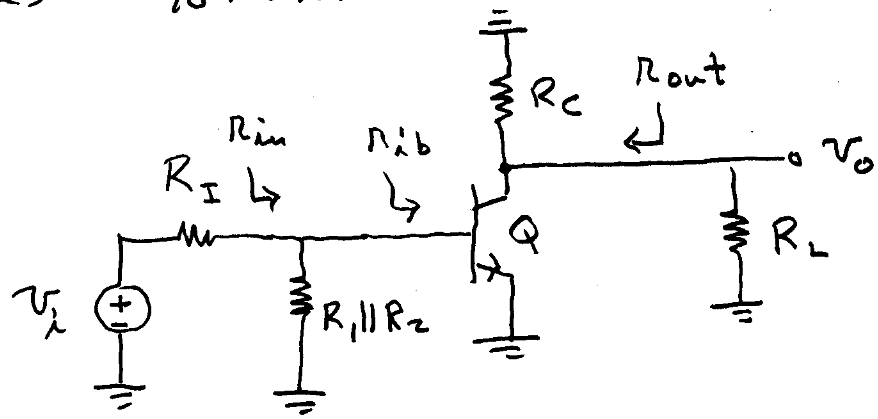
The Common Emitter Amplifier



The Q point values are calculated with C_1, C_2, and C_3 open circuited. We assume this solution is known.

For a sinusoidal signal, the complex impedance of a capacitor is given by $Z_c = 1/j\omega C$. If ωC is sufficiently large, $|Z_c|$ can be made small enough to be considered an ac short circuit. We assume each C can be considered an ac short circuit.

The ac signal equivalent circuit is as follows:

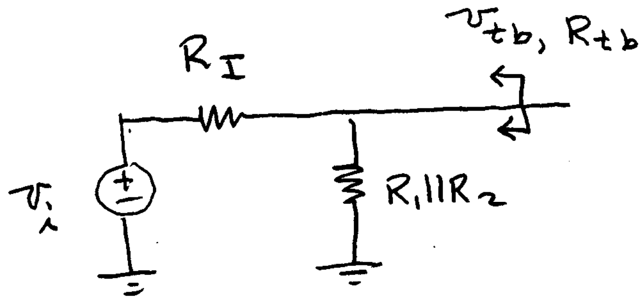


The input resistance is given by

$$r_{in} = R_1 \| R_2 \| r_{ib}$$

where r_{ib} is the small-signal resistance looking into the base. From the pi model, this is $r_{ib} = r_\pi = V_T / I_B$. Note that $r_{ib} \neq r_\pi$ if the emitter is not grounded.

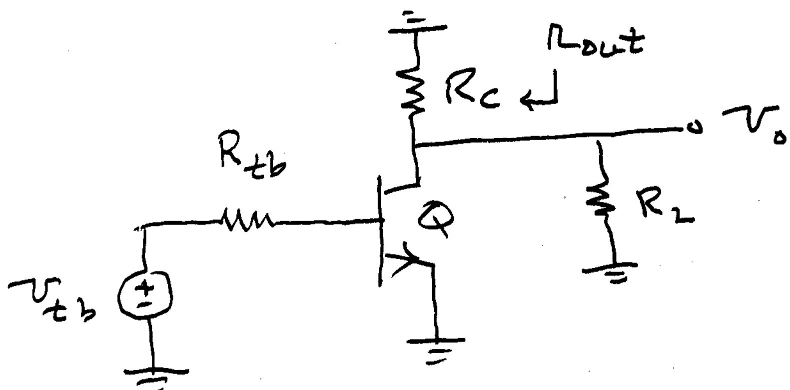
To solve for the output voltage, we make a Thevenin equivalent circuit looking out of the base



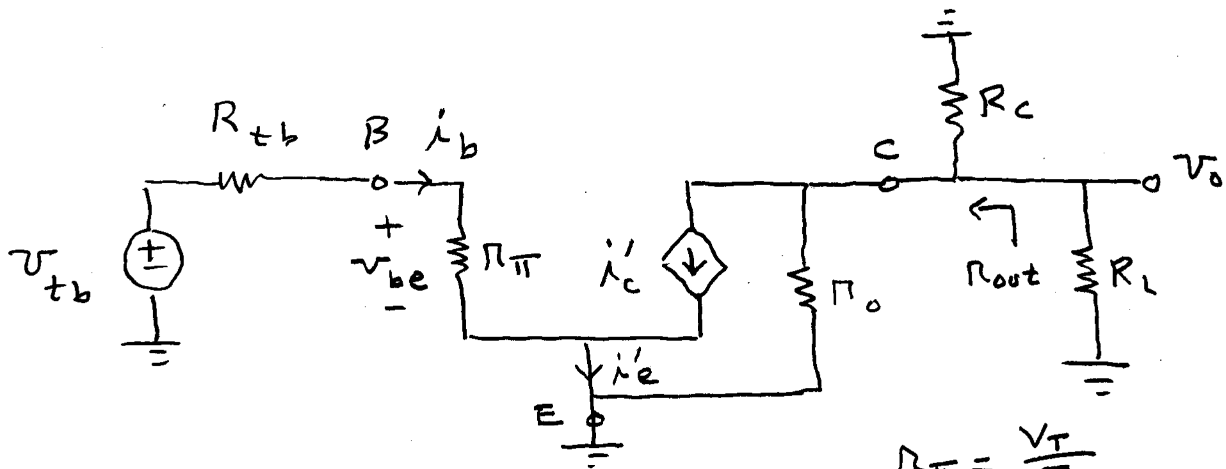
$$v_{tb} = v_i \frac{R_1 \| R_2}{R_I + R_1 \| R_2}$$

$$R_{tb} = R_I \| R_1 \| R_2$$

The new circuit is



To solve for v_o and R_{out}, we replace Q with either the pi or the T model. Let us first use the pi model.



$$r_\pi = \frac{V_T}{I_B}$$

$$r_o = \frac{V_A + V_{CE}}{I_C}$$

$$v_o = -i_c' \left(r_o \| R_c \| R_L\right)$$

$$i_c' = \beta i_b = \beta \frac{v_{tb}}{R_{tb} + r_\pi}$$

$$\Rightarrow \quad v_o = -\beta \frac{r_o \| R_c \| R_L}{R_{tb} + r_\pi} v_{tb}$$

$$= -\beta \frac{r_o \| R_c \| R_L}{R_{tb} + r_\pi} \frac{R_1 \| R_2}{R_I + R_1 \| R_2} v_i$$

Alternate solution ($g_m = I_c / V_T$)

$$i_c' = g_m v_{be} = g_m v_{tb} \frac{r_\pi}{R_{tb} + r_\pi}$$

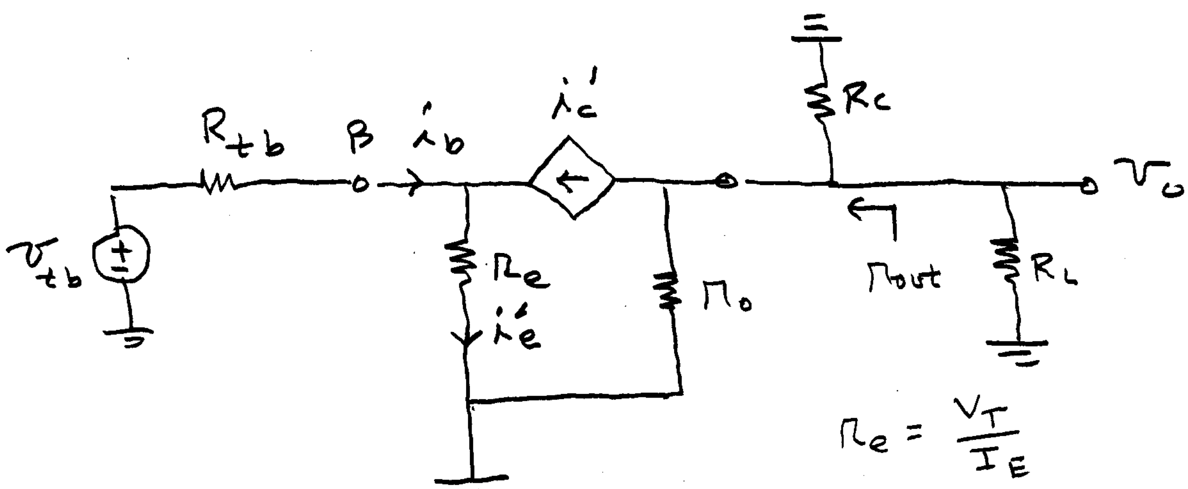
But we remember that $g_m r_\pi = \beta$

$$\Rightarrow \quad i_c' = \beta \frac{v_{tb}}{R_{tb} + r_\pi}$$

This is the same solution obtained above. The output resistance is given by

$$r_{out} = r_o \parallel R_C$$

Next, we repeat the solution with the T model.



$$r_e = \frac{V_T}{I_E}$$

$$r_o = \frac{V_A + V_{CE}}{I_C}$$

$$v_{tb} = i_b R_{tb} + i_e' r_e$$

$$= \frac{i_e'}{1+\beta} R_{tb} + i_e' r_e$$

$$\Rightarrow i_e' = \frac{v_{tb}}{\frac{R_{tb}}{1+\beta} + r_e}$$

$$v_o = - i_c' \, (r_o \parallel R_C \parallel R_L)$$

But $i_c' = \alpha i_e'$

$$\Rightarrow v_o = -\alpha i_e' (r_o \| R_c \| R_L)$$

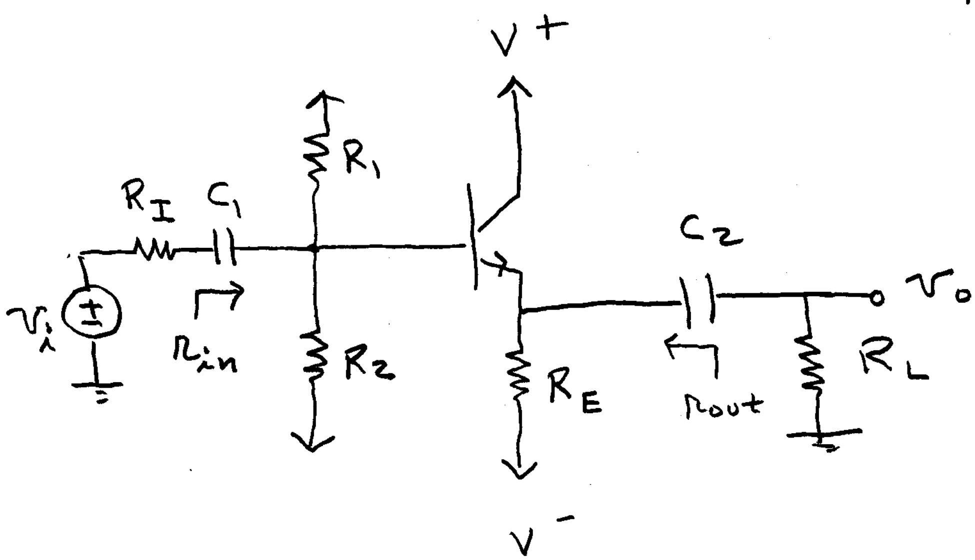
$$= -\alpha \frac{v_{tb}}{\frac{R_{tb}}{1+\beta} + r_e} (r_o \| R_c \| R_L)$$

$$= -\alpha \frac{r_o \| R_c \| R_L}{\frac{R_{tb}}{1+\beta} + r_e} \frac{R_1 \| R_2}{R_I + R_1 \| R_2} v_i$$

$$r_{out} = r_o \| R_c$$

The two solutions are equivalent.

The Common Collector Amplifier

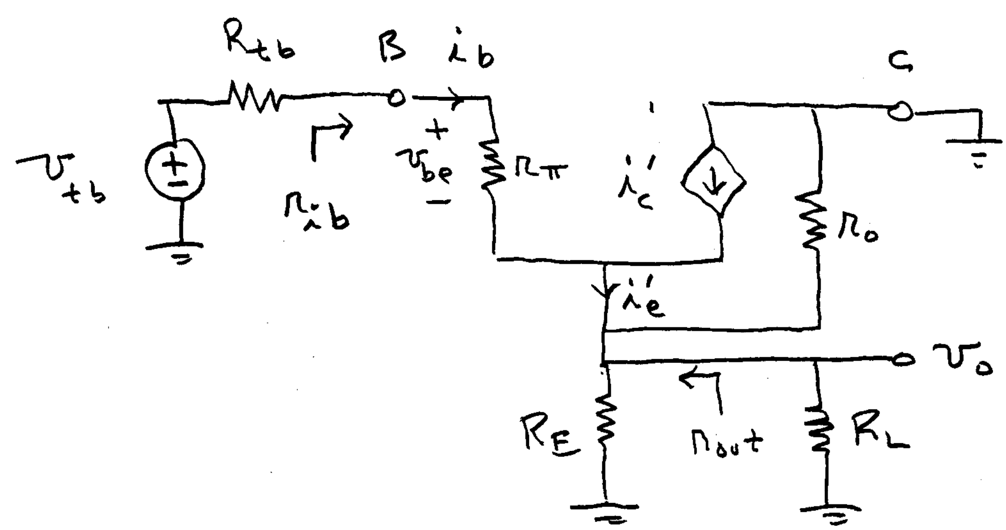


We assume the Q point solution. In the ac signal circuit, make a Thévenin equivalent looking out of the base as in the CE amplifier.

$$v_{tb} = v_i \frac{R_1 \| R_2}{R_I + R_1 \| R_2}$$

$$R_{tb} = R_I \| R_1 \| R_2$$

The pi model circuit is



$$v_o = i_e' \left(r_o \| R_E \| R_L \right)$$

$$v_{tb} = i_b' \left(R_{tb} + r_\pi \right) + i_e' \left(r_o \| R_E \| R_L \right)$$

$$= \frac{i_e'}{1+\beta} \left(R_{tb} + r_\pi \right) + i_e' \left(r_o \| R_E \| R_L \right)$$

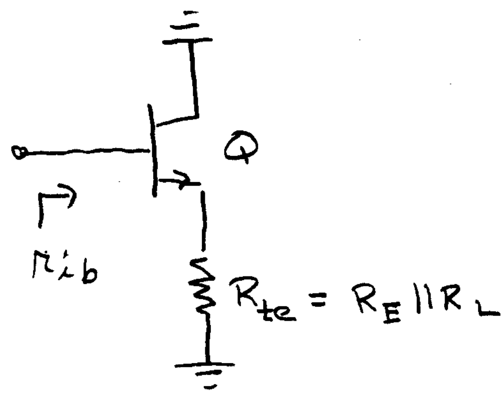
$$\Rightarrow i_e' = \frac{v_{tb}}{\frac{R_{tb}+r_\pi}{1+\beta} + r_o \| R_E \| R_L}$$

$$\Rightarrow v_o = \frac{r_o \| R_E \| R_L}{\frac{R_{tb}+r_\pi}{1+\beta} + r_o \| R_E \| R_L} \cdot v_{tb}$$

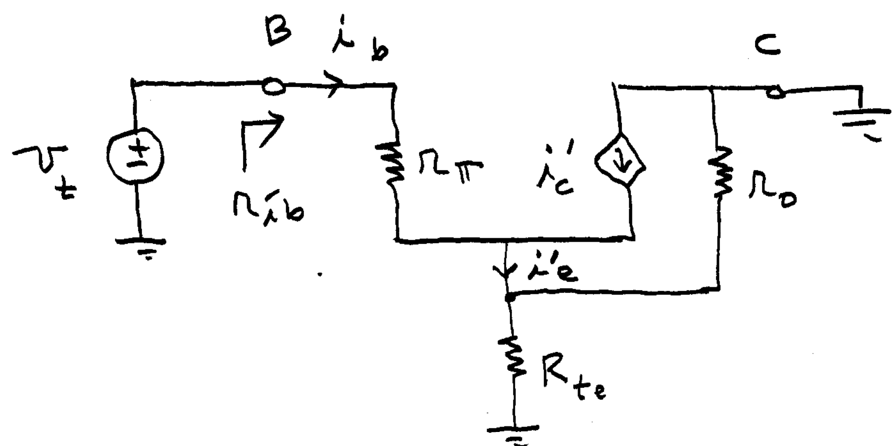
$$= \frac{r_o \| R_E \| R_L}{\frac{R_{tb}+r_\pi}{1+\beta} + r_o \| R_E \| R_L} \; \frac{R_1 \| R_2}{R_I + R_1 \| R_2} \, v_i$$

Note that $\frac{v_o}{v_i} \leq 1$

To solve for r_{in}, we must solve for r_{ib}.



Replace Q with the pi model and add a test source at the base.



$$r_{ib} = \frac{v_t}{i_b}$$

$$v_t = i_b r_\pi + i_e' (r_o \| R_{te})$$

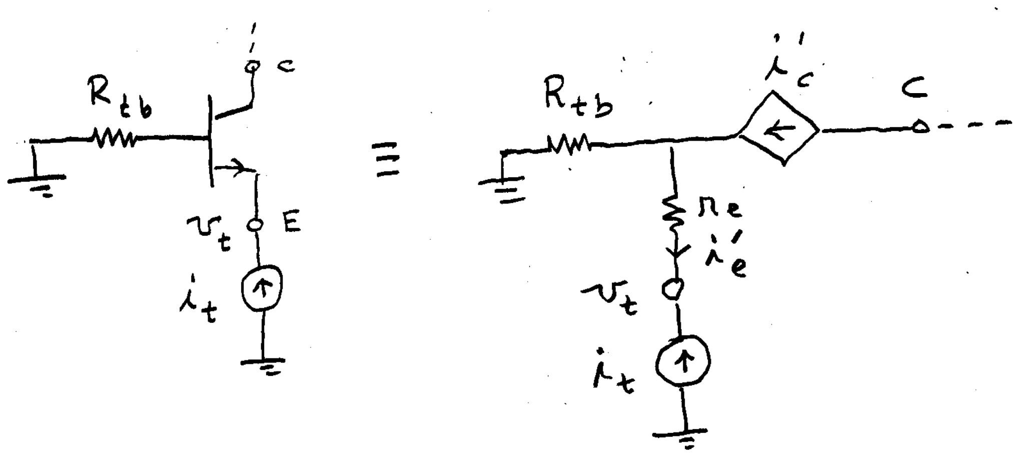
$$= i_b r_\pi + (1+\beta) i_b (r_o \| R_{te})$$

$$\Rightarrow \quad r_{ib} = \frac{v_t}{i_b} = r_\pi + (1+\beta)(r_o \| R_{te})$$

The input resistance to the original circuit is

$$r_{in} = R_1 \| R_2 \| r_{ib}$$

To solve for r_{out}, we need the resistance seen looking up into the i_e' branch. We can use the T model and a test current source to solve for this. Set $v_i = 0 \Rightarrow v_{tb} = 0$.



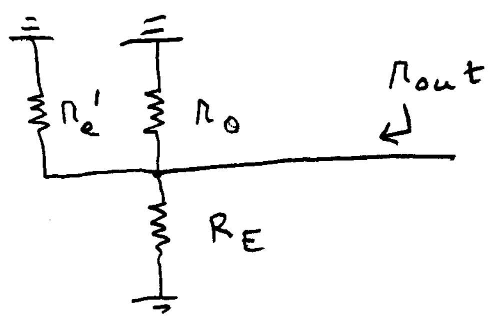
$$v_t = i_t r_e + (i_t + i_c') R_{tb}$$

But $i_c' = \alpha i_e' = -\alpha i_t$

$$\Rightarrow \quad v_t = i_t r_e + i_t (1-\alpha) R_{tb}$$

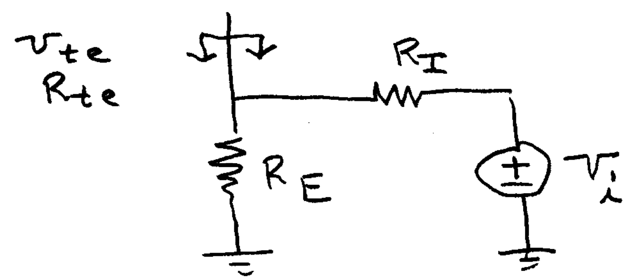
Let $r_e' = \dfrac{v_t}{i_t} = r_e + (1-\alpha) R_{tb} = r_e + \dfrac{R_{tb}}{1+\beta}$

The circuit for r_{out} is



$$r_{out} = r_e' \parallel r_o \parallel R_E$$

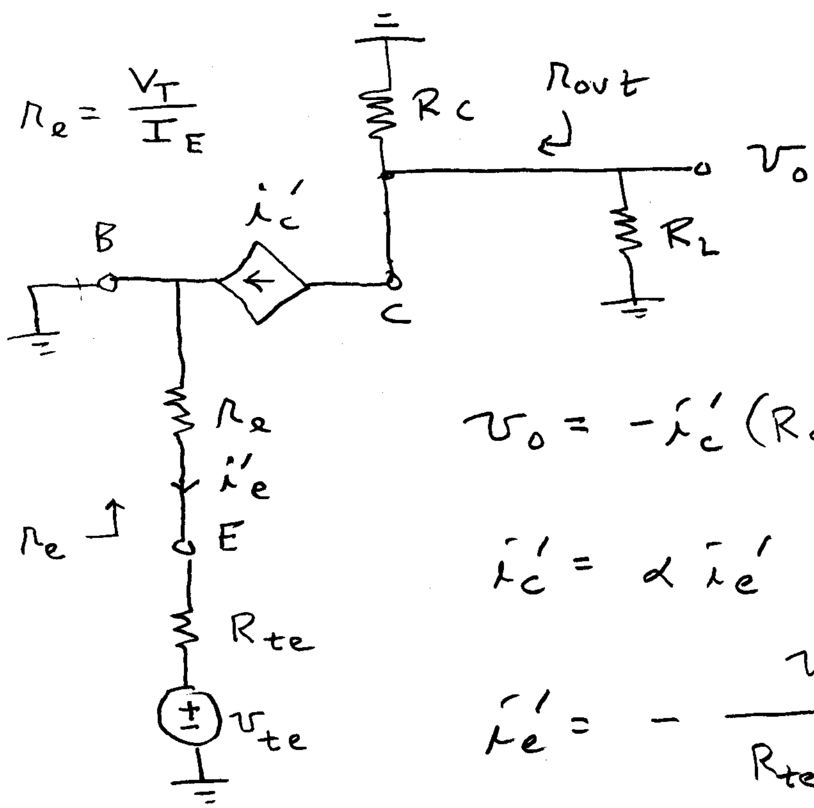
We assume the Q point solution is known. For the ac analysis, we make a Thévenin equivalent looking out of the emitter.



$$v_{te} = v_i \frac{R_E}{R_I + R_E} \qquad R_{te} = R_E \| R_I$$

In the ac circuit, neither end of r_o is grounded, so we omit it to obtain an approximate solution.

Either the pi or the T model can be used. We will use the T model.



$$v_o = -i_c'(R_C \parallel R_L)$$

$$i_c' = \alpha\, i_e'$$

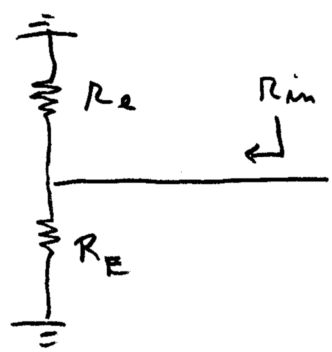
$$i_e' = -\frac{v_{te}}{R_{te} + r_e}$$

$$\Rightarrow \quad v_o = +\alpha \frac{R_C \parallel R_L}{R_{te} + r_e} v_{te}$$

$$= +\alpha \frac{R_C \parallel R_L}{R_{te} + r_e} \frac{R_E}{R_I + R_E} v_i$$

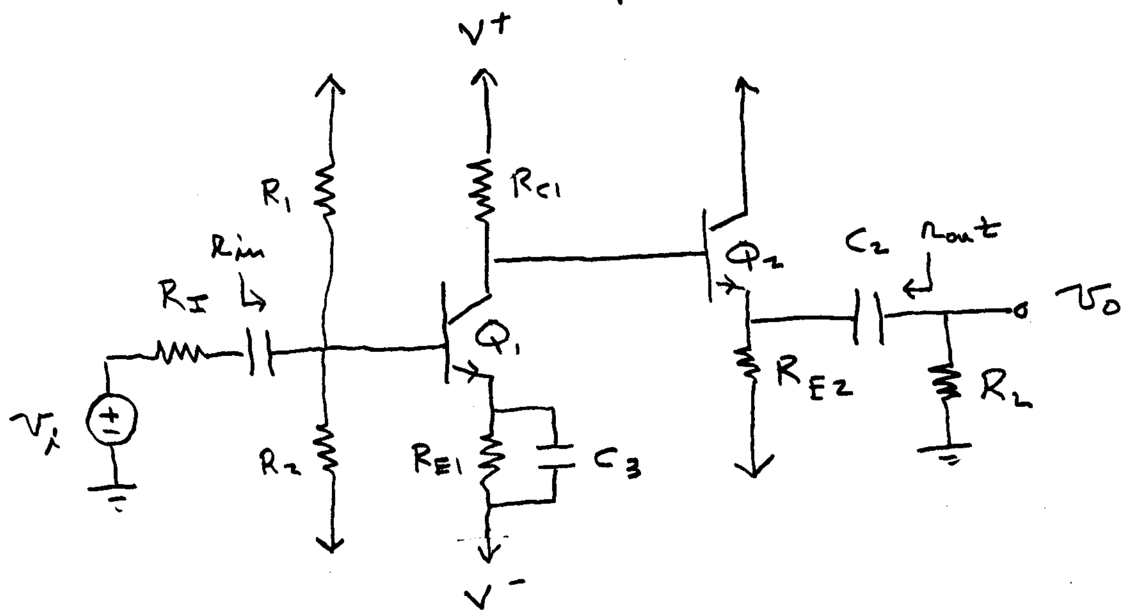
$$r_{out} = R_C \parallel R_L$$

The resistance seen looking up into the emitter node is r_e. Thus the equivalent circuit for R_{in} is

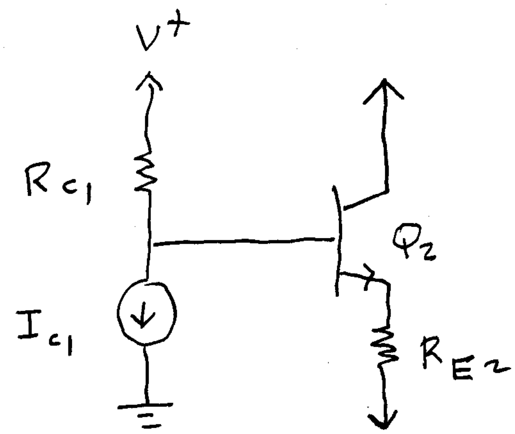


$R_{in} = r_e \parallel R_E \qquad r_e = \dfrac{V_T}{I_E}$

The CE - CC Amplifier

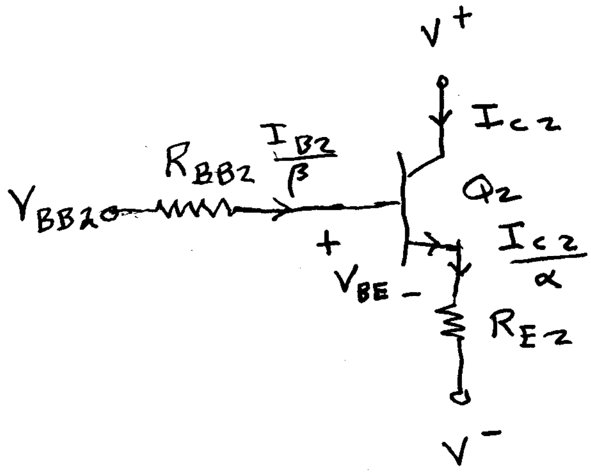


The dc bias solution for Q_1 is the same as for the CE amp. The dc bias circuit for Q_2 is



$$V_{BB2} = V^+ - I_{C1} R_{C1} \qquad R_{BB2} = R_{C1}$$

Thus we can redraw the circuit to obtain



$$V_{BB2} - V^- = \frac{I_{C2}}{\beta} R_{BB2} + V_{BE} + \frac{I_{C2}}{\alpha} R_{E2}$$

$$\Rightarrow \quad I_{C2} = \frac{V_{BB2} - V^- - V_{BE}}{\frac{R_{BB2}}{\beta} + \frac{R_{E2}}{\alpha}}$$

$$= \frac{V^+ - I_{C1} R_{C1} - V^- - V_{BE}}{\frac{R_{BB2}}{\beta} + \frac{R_{E2}}{\alpha}}$$

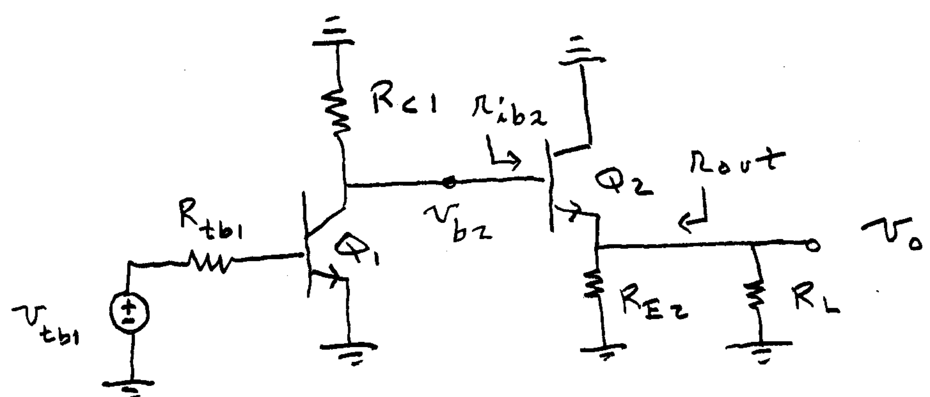
For Q_2 to be in the active mode, V_{CB2} must be positive.

$$V_{CB2} = V_{C2} - V_{B2}$$
$$= V^+ - (V_{BE} + I_{E2} R_{E2} + V^-)$$
$$= V^+ - (V_{BE} + \frac{I_{C2}}{\alpha} R_{E2} + V^-)$$

The ac signal circuit is obtained by shorting all capacitors and setting $V^+ = 0$ and $V^- = 0$. As with the CE amplifier, let

$$v_{tb1} = v_i \frac{R_1 \| R_2}{R_I + R_1 \| R_2} \qquad R_{tb1} = R_I \| R_1 \| R_2$$

The ac signal circuit is



Note that r_{ib2} is the load resistor R_{L1} for Q_1. From our common-collector amplifier analysis, it is given by

$$r_{ib2} = r_{\pi 2} + (1+\beta_2)(r_{o2} \| \underbrace{R_{E2} \| R_L}_{R_{te2}})$$

Thus we can use our CE amplifier analysis to write

$$v_{b2} = -\beta_1 \frac{r_{o1} \| R_{C1} \| r_{ib2}}{R_{tb1} + r_{\pi 1}} v_{tb1}$$

Because there is no resistor between the v_{b2} node and the base of Q_2, it follows that

$$v_{tb2} = v_{b2} \qquad R_{tb2} = 0$$

Thus we can use our CC amplifier analysis to write

$$v_o = \frac{r_{o2} \| R_{E2} \| R_L}{\frac{r_\pi}{1+\beta} + r_{o2} \| R_{E2} \| R_L} v_{b2}$$

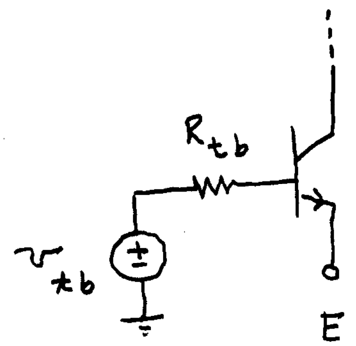
The overall voltage gain is given by

$$\frac{v_o}{v_i} = \frac{v_{tb1}}{v_i} * \frac{v_{b2}}{v_{tb1}} * \frac{v_o}{v_{b2}}$$

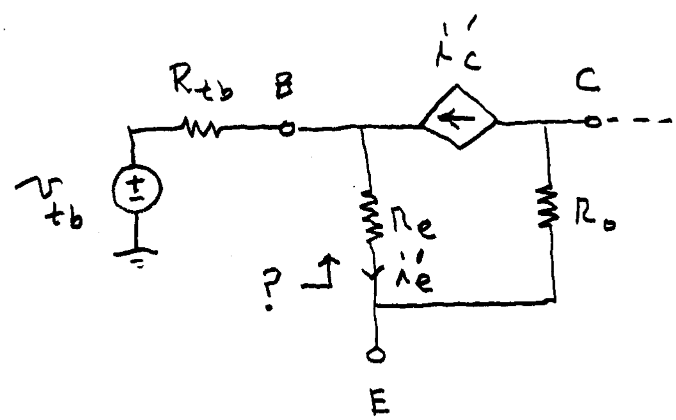
where the 3 gain terms are found from the above analysis.

The Thévenin Equivalent Circuit Seen Looking Into The i_e' Branch of the BJT Small-Signal Model

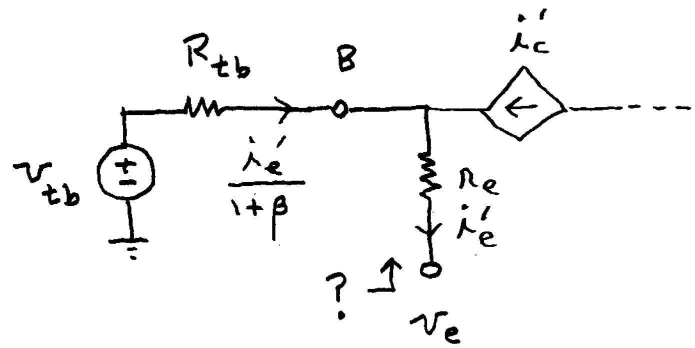
Consider the ac signal circuit



The T model is



We wish to solve for the Thévenin equivalent circuit seen looking up into the i_e' branch. The part of the circuit which we need for this is as follows



We can write

$$v_{tb} - v_e = \frac{i_e'}{1+\beta} R_{tb} + i_e' r_e$$

$$= i_e' \left(\frac{R_{tb}}{1+\beta} + r_e \right)$$

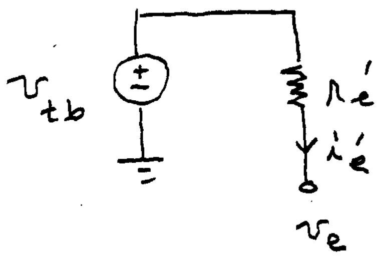
Let us define r_e' as follows

$$r_e' = \frac{R_{tb}}{1+\beta} + r_e$$

$$\Rightarrow \quad v_{tb} - v_e = i_e' r_e'$$

$$\Rightarrow \quad v_e = v_{tb} - i_e' r_e'$$

The circuit which this equation applies to is

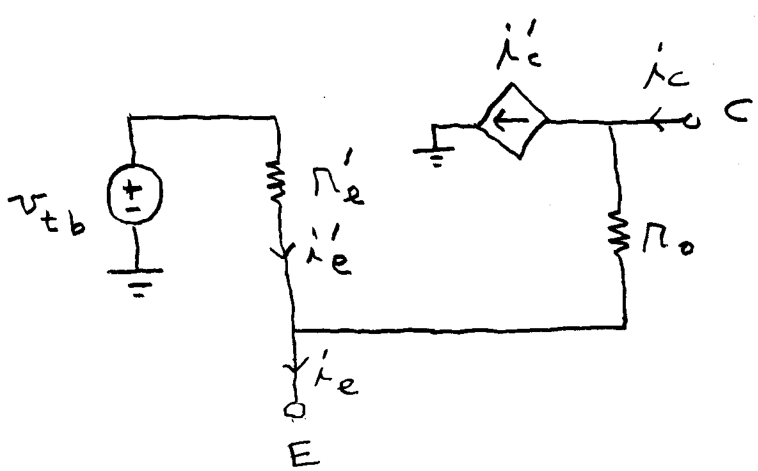


$$v_e' = v_{tb} - i_e' r_e'$$

$$r_e' = \frac{R_{tb}}{1+\beta} + r_e$$

This is the Thévenin equivalent circuit seen looking up into the i_e' branch.

We can now draw a simplified form of the T model as follows:



Note that the base node B has been absorbed. We use this model to solve the BJT differential amplifier.