Notational Conventions

DC Quantity – Upper case-letter, upper-case subscript
Small-Signal Quantity – Lower case-letter, lower-case subscript
Total Quantity – Lower case-letter, upper-case subscript
Phasor Quantity – Upper case-letter, lower-case subscript

\[ V_{BE}, I_D \]
\[ v_{be}, i_d \]
\[ v_{BE} = V_{BE} + v_{be}, i_D = I_D + i_d \]
\[ V_{be}, I_d \]

Independent Sources

Figure 1(a) shows the diagram of an independent voltage source. The voltage \( v \) is independent of the current \( i \) that flows through the source. Fig. 1(b) shows the diagram of an independent current source. The current \( i \) is independent of the voltage \( v \) across the source.

![Figure 1: (a) Independent voltage source. (b) Independent current source.](image)

Dependent Sources

**VCVS – Voltage Controlled Voltage Source**

Figure 2(a) shows the diagram of a voltage controlled voltage source. The output voltage is given by a voltage gain \( A_v \) multiplied by an input voltage \( v_1 \). Such a source in SPICE is called an E source.

![Figure 2: (a) Voltage controlled voltage source. (b) Voltage controlled current source. (c) Current controlled voltage source. (d) Current controlled current source.](image)

**CCVS – Voltage Controlled Current Source**

Figure 2(b) shows the diagram of a voltage controlled current source. The output current is given by a transconductance \( G_m \) multiplied by an input voltage \( v_1 \). Such a source in SPICE is called a G source.

**CCVS – Current Controlled Voltage Source**

Figure 2(c) shows the diagram of a current controlled voltage source. The output voltage is given by a transresistance \( R_m \) multiplied by an input current \( i_1 \). Such a source in SPICE is called an F source.

**CCCS – Current Controlled Current Source**

Figure 2(d) shows the diagram of a current controlled current source. The output current is given by a current gain \( A_i \) multiplied by an input current \( i_1 \). Such a source in SPICE is called an H source.
Passive Elements

Resistor
Figure 3(a) shows the diagram of a resistor. The voltage across it is given by

\[ v = iR \]

This relation is known as Ohm's law.

![Diagram of a resistor](image)

Figure 3: (a) Resistor. (b) Inductor. (c) Capacitor.

Inductor
Fig. 3(b) shows an inductor. The voltage across it is given by

\[ v = L \frac{di}{dt} \]

In the analysis of circuits having sinusoidal excitations, phasor analysis is usually used. In this case, the voltage across the inductor is given by

\[ V = LSsI \]

where \( V \) and \( I \) are phasors, \( s = j\omega \), and \( \omega \) is the radian frequency of the excitation. In the phasor domain, a multiplication by \( s \) is equivalent to a time derivative in the time domain. This is because the time domain excitation is assumed to be of the form \( \exp(st) \).

Capacitor
Fig. 3(c) shows a capacitor. The current through it is given by

\[ i = C \frac{dv}{dt} \]

For phasor representation of the signals, the current through the capacitor is given by

\[ I = CsV \]

Voltage Division and Current Division

Voltage Division
Figure 3(a) shows a two-resistor voltage divider. The voltages \( v_1 \) and \( v_2 \) are given by

\[
\begin{align*}
v_1 &= iSR_1 = \left( \frac{v_S}{R_1 + R_2} \right) R_1 = v_S \frac{R_1}{R_1 + R_2} \\
v_2 &= iSR_2 = \left( \frac{v_S}{R_1 + R_2} \right) R_2 = v_S \frac{R_2}{R_1 + R_2}
\end{align*}
\]
Current Division

Figure 3(b) shows a two-resistor current divider. The currents $i_1$ and $i_2$ are given by

\[ i_1 = \frac{v_S}{R_1} = \frac{i_S (R_1 \| R_2)}{R_1} = i_S \frac{R_2}{R_1 + R_2} \]

\[ i_2 = \frac{v_S}{R_2} = \frac{i_S (R_1 \| R_2)}{R_2} = i_S \frac{R_1}{R_1 + R_2} \]

Use of Superposition in Solving Circuits

This section illustrates the use of superposition in solving circuits containing dependent sources. The correct use of superposition requires the dependent sources to be treated as independent sources in writing equations. When a source is zeroed, its controlling variable is not zeroed.

Example 1 For the circuit in Fig. 5(a), it is given that $R_1 = 20 \, \text{k}\Omega$, $R_2 = 1 \, \text{k}\Omega$, and $A_i = 50$. Solve for the Thévenin and Norton equivalent circuits seen looking into the output terminals.

\[ v_{OC} = v_S \frac{R_2}{R_1 + R_2} + A_i i_1 \frac{R_2}{R_1 + R_2} \]

Next, we use superposition to solve for $i_1$.

\[ i_1 = \frac{v_S}{R_1 + R_2} - A_i i_1 \frac{R_2}{R_1 + R_2} \]

This can be solved for $i_1$ to obtain

\[ i_1 = \frac{v_S}{R_1 + R_2} \frac{1}{1 + A_i \frac{R_2}{R_1 + R_2}} = \frac{v_S}{R_1 + (1 + A_i) R_2} \]
We substitute the solution for \(i_1\) into the expression for \(v_{OC}\) to obtain

\[
v_{OC} = \frac{v_S}{R_1 + R_2} + A_i \frac{(R_1 \parallel R_2)}{R_1 + (1 + A_i) R_2} \frac{v_S}{R_1 + (1 + A_i) R_2} \\
= \frac{v_S R_2}{R_1 + R_2} \left[ R_2 + \frac{A_i R_1 R_2}{R_1 + (1 + A_i) R_2} \right] \\
= \frac{v_S}{20 \, \Omega} \left[ 1 \, \Omega + \frac{5 \times 20 \, \Omega \times 1 \, \Omega}{20 \, \Omega + (1 + 50) 1 \, \Omega} \right] \\
= \frac{v_S}{21} \left( 1 + \frac{5 \times 20}{20 + 51} \right) \\
= 0.718 v_S
\]

The Thévenin equivalent circuit is shown in Fig. 5(b).

By superposition, the short-circuit output current is given by

\[
i_{SC} = \frac{v_S}{R_1} + A_i i_1
\]

where \(i_1\) is given by

\[
i_1 = \frac{v_S}{R_1}
\]

We substitute the expression for \(i_1\) into the expression for \(i_{SC}\) to obtain

\[
i_{SC} = \frac{v_S}{R_1} + A_i \frac{v_S}{R_1} = v_S \frac{1 + A_i}{R_1} = v_S \frac{1 + 50}{1 \, \Omega} = 2.55 \times 10^{-3} v_S
\]

The output resistance is given by

\[
R_{out} = \frac{v_{OC}}{i_{SC}} = \frac{0.718 v_S}{2.55 \times 10^{-3} v_S} = \frac{0.718}{2.55 \times 10^{-3}} = 282 \, \Omega
\]

The Norton equivalent circuit is shown in Fig. 5(c).

**Example 2** For the circuit in Fig. 6(a), it is given that \(R_1 = 3 \, k\Omega\), \(R_2 = 2 \, k\Omega\), and \(G_m = 0.1 \, S\). Solve for the input resistance to the circuit.

**Solution.** The input resistance is given by the ratio of the source voltage to the source current. By superposition, we can write

\[
v_S = i_S (R_1 + R_2) + G_m v_1 R_2
\]

where \(v_1\) is given by

\[
v_1 = i_S R_1
\]

Substitute the expression for \(v_1\) into the expression for \(v_S\) to obtain

\[
v_S = i_S (R_1 + R_2) + G_m i_S R_1 R_2 = i_S (R_1 + R_2 + G_m R_1 R_2)
\]
Thus the input resistance is given by

\[ R_{in} = \frac{v_S}{i_S} = R_1 + R_2 + G_mR_1R_2 = 3 \, \text{k}\Omega + 2 \, \text{k}\Omega + 0.1 \times 3 \, \text{k}\Omega \times 2 \, \text{k}\Omega = 605 \, \text{k}\Omega \]

The equivalent circuit seen looking into the input is shown in Fig. 6(b).

**Example 3** Solve for the open circuit output voltage and the short circuit output current for the circuit in Fig. 7(a).

**Figure 7: Circuit for Example 3**

**Solution.** The circuit contains a floating current source. To make superposition simpler to apply, this source can be broken into two series sources as shown in Fig. 7(b). The node between the sources is shown connected to ground. Although no current flows from this node to ground when both sources are active, a current does flow when either is zeroed. However, by superposition, the sum of these currents must be zero. Because the currents flow into the ground node, the voltages or current in the circuit are not affected. By superposition, the open circuit output voltage is given by

\[ v_{OC} = (i_S + g_m v_1) \frac{R_3}{R_1 + R_2 + R_3} \frac{1 - g_m v_1}{R_1 R_2} \]

The voltage \( v_1 \) is given by

\[ v_1 = (i_S + g_m v_1) R_1 \frac{R_3}{R_1 + R_2 + R_3} \]

When this is used in the equation for \( v_{OC} \), we obtain

\[ v_{OC} = i_S \frac{R_3}{R_1 + R_2 + R_3} \left[ R_1 - g_m \frac{R_3}{R_1 R_2} \frac{R_1 + (R_2 + R_3)}{R_1 R_2} \right] \]

By superposition, the short circuit output current is given by

\[ i_{SC} = (i_S + g_m v_1) \frac{R_1}{R_1 + R_2} - g_m v_1 = i_S \frac{R_1}{R_1 + R_2} - g_m v_1 \frac{R_2}{R_1 + R_2} \]

The voltage \( v_1 \) is given by

\[ v_1 = (i_S + g_m v_1) \frac{R_1}{R_2} \]
This can be solved for $v_1$ to obtain

$$v_1 = i_S \frac{R_1 \| R_2}{1 - g_m (R_1 \| R_2)}$$

When this is used in the equation for $i_{SC}$, we obtain

$$i_{SC} = i_S \frac{1}{R_1 + R_2} \left[ R_1 - g_m R_2 \frac{(R_1 \| R_2)}{1 - g_m (R_1 \| R_2)} \right]$$

**Example 4** For the circuit of Fig. 8(a), it is given that $i_S = 5 \text{ mA}$, $R_1 = 4 \text{ k}\Omega$, $R_2 = 2 \text{ k}\Omega$, $R_3 = 3 \text{ k}\Omega$, and $A_i = 3$. Solve for the Norton equivalent circuit seen looking into the output terminals and the voltage at the output if a load resistor of value $R_L = 6 \text{ k}\Omega$ is connected from the output to ground.

![Figure 8: Circuit for Example 4.](image)

**Solution.** By superposition, the open-circuit output voltage is given by

$$v_{OC} = i_S \frac{R_1}{R_1 + R_2 + R_3} R_3 - A_i i_2 \left[ (R_1 + R_2) \| R_3 \right]$$

The current $i_2$ is given by

$$i_2 = i_S \frac{R_1}{R_1 + R_2 + R_3} + A_i i_2 \frac{R_3}{R_1 + R_2 + R_3}$$

Solution yields

$$i_2 = i_S \frac{R_1}{R_1 + R_2 + R_3} \frac{1}{1 - \frac{A_i R_3}{R_1 + R_2 + R_3}} = i_S \frac{R_1}{R_1 + R_2 + (1 - A_i) R_3}$$

When this equation is used in the equation for $v_{OC}$, we obtain

$$v_{OC} = i_S \left[ \frac{R_1 R_3}{R_1 + R_2 + R_3} - A_i \left[ (R_1 + R_2) \| R_3 \right] \right] \frac{R_1}{R_1 + R_2 + (1 - A_i) R_3}$$

For the element values given, it follows that $v_{OC} = -\infty$. This means that the output resistance of the circuit is $\infty$, i.e. the circuit looks like a current source to any load.

By superposition, the short-circuit output current is given by

$$i_{SC} = i_S \frac{R_1}{R_1 + R_2} - A_i i_2$$

The current $i_2$ is given by

$$i_2 = i_S \frac{R_1}{R_1 + R_2}$$

When this is used in the equation for $i_{SC}$, we obtain

$$i_{SC} = i_S \left[ \frac{R_1}{R_1 + R_2} - A_i \frac{R_1}{R_1 + R_2} \right] = i_S \frac{(1 - A_i) R_1}{R_1 + R_2} = -\frac{4}{3} i_S$$

Fig. 8(b) shows the circuit with $R_L$ connected to its output. The output voltage is given by

$$v_O = -\frac{4}{3} \times 5 \text{ mA} \times 6 \text{ k}\Omega = -40 \text{ V}$$
Amplifier Representations

Figure 9 shows the diagram of an amplifier. The source is represented by a Thévenin equivalent circuit. In general, the input resistance $R_{in}$ is a function of the load resistance $R_L$ and the output resistance $R_{out}$ is a function of the source resistance $R_S$.

![Amplifier model diagram]

Figure 9: Amplifier model.

The output circuit of the amplifier can be represented by either a Thévenin equivalent circuit or a Norton equivalent circuit using one of the four dependent sources described above. The four equivalent circuits are summarized below.

**VCVS Model**

Figure 10(a) shows the amplifier model with the output represented by a voltage-controlled voltage source. The output voltage is given by

$$v_O = A_v v_I \frac{R_L}{R_{out} + R_L} = A_v \left( v_S \frac{R_{in}}{R_S + R_{in}} \right) \frac{R_L}{R_{out} + R_L}$$

![VCVS model diagram](a)

Figure 10: (a) Voltage controlled voltage source amplifier. (b) Voltage controlled current source amplifier.

**VCCS Model**

Figure 10(b) shows the amplifier model with the output represented by a voltage-controlled current source. The output voltage is given by

$$v_O = G_m v_I \left( R_{out} \parallel R_L \right) = G_m \left( v_S \frac{R_{in}}{R_S + R_{in}} \right) \left( R_{out} \parallel R_L \right)$$

![VCCS model diagram](b)

**CCVS Model**

Figure 11(a) shows the amplifier model with the output represented by a current-controlled voltage source. The output voltage is given by

$$v_O = R_m i_S \frac{R_L}{R_{out} + R_L} = R_m \left( v_S \frac{R_{in}}{R_S + R_{in}} \right) \frac{R_L}{R_{out} + R_L}$$

![CCVS model diagram](a)
Figure 11: (a) Current controlled voltage source amplifier. (b) Current controlled current amplifier.

**CCCS Model**

Figure 11(b) shows the amplifier model with the output represented by a current-controlled current source. The output voltage is given by

\[ v_O = A_i i_S \frac{R_L}{R_{out} + R_L} = A_i \left( \frac{v_S}{R_S + R_m} \right) \frac{R_{out} R_L}{R_{out} + R_L} \]