Transfer Functions For Chebyshev Filters

The dB Ripple

The dB ripple for a Chebyshev filter is the peak-to-peak passband ripple in the Bode magnitude plot of the filter response. The dB ripple determines the parameter ϵ as follows:

$$\text{dB ripple} = 10\log\left(1+\epsilon^2\right)$$

This can be solved for ϵ to obtain

$$\epsilon = \sqrt{10^{\text{dB}/10}-1}$$

The Parameter h

To write the Chebyshev transfer functions, the parameter h is required. Given the order n and the ripple parameter ϵ, h is defined by

$$h = \tanh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right)$$

Even-Order Chebyshev Filters

For an even-order Chebyshev low-pass filter of order n, the transfer function can be written in the product form

$$T_{\text{LP}}(s) = \prod_{i=1}^{n/2}\frac{1}{(s/a_i\omega_{\text{c}})^2 + (1/b_i)(s/a_i\omega_{\text{c}}) + 1}$$

The constants a_i and b_i are given by

$$a_i = \left(\frac{1}{1-h^2} - \sin^2\theta_i\right)^{1/2} \text{ for } 1 \le i \le n/2$$

$$b_i = \frac{1}{2}\left(1+\frac{1}{h^2\tan^2\theta_i}\right)^{1/2} \text{ for } 1 \le i \le n/2$$

where the θ_i are given by

$$\theta_i = \frac{2i-1}{n}\times 90^{\text{o}} \text{ for } 1 \le i \le n/2$$

Odd-Order Chebyshev Filters

For an odd-order Chebyshev low-pass filter of order n, the transfer function can be written in the form

$$T_{\mathrm{LP}}(s) = \frac{1}{s/a_{(n+1)/2}\omega_{\mathrm{c}} + 1} \times \prod_{i=1}^{(n-1)/2} \frac{1}{(s/a_i\omega_{\mathrm{c}})^2 + (1/b_i)(s/a_i\omega_{\mathrm{c}}) + 1}$$

The constants a_i and b_i are given by

$$a_{(n+1)/2} = \frac{h}{\sqrt{1-h^2}} \text{ for } i = (n+1)/2$$

$$a_i = \left(\frac{1}{1-h^2} - \sin^2\theta_i\right)^{1/2} \text{ for } 1 \le i \le (n-1)/2$$

$$b_i = \frac{1}{2}\left(1 + \frac{1}{h^2\tan^2\theta_i}\right)^{1/2} \text{ for } 1 \le i \le (n-1)/2$$

where the θ_i are given by

$$\theta_i = \frac{2i-1}{n} \times 90^{\mathrm{o}} \text{ for } 1 \le i \le (n-1)/2$$