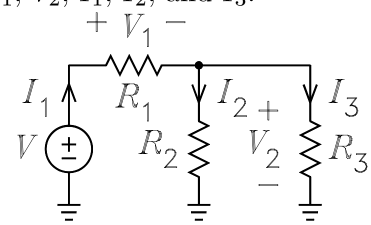
ECE3050 Homework Set 1

1. For $V = 18\,\text{V}$, $R_1 = 39\,\text{k}\Omega$, $R_2 = 43\,\text{k}\Omega$, and $R_3 = 11\,\text{k}\Omega$, use Ohm's Law, voltage division, and current division to solve for V_1, V_2, I_1, I_2, and I_3.

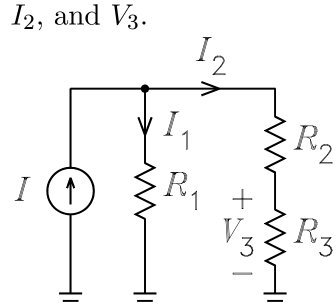


$$V_1 = 18\frac{39\,\text{k}\Omega}{39\,\text{k}\Omega + 43\,\text{k}\Omega \| 11\,\text{k}\Omega} = 14.7\,\text{V} \qquad V_2 = 18\frac{43\,\text{k}\Omega \| 11\,\text{k}\Omega}{39\,\text{k}\Omega + 43\,\text{k}\Omega \| 11\,\text{k}\Omega} = 3.30\,\text{V}$$

$$I_1 = \frac{18}{39\,\text{k}\Omega + 43\,\text{k}\Omega \| 11\,\text{k}\Omega} = 376.8\,\mu\text{A} \qquad I_2 = \frac{11\,\text{k}\Omega}{43\,\text{k}\Omega + 11\,\text{k}\Omega} I_1 = 76.8\,\mu\text{A}$$

$$I_3 = \frac{43\,\text{k}\Omega}{43\,\text{k}\Omega + 11\,\text{k}\Omega} I_1 = 300\,\mu\text{A}$$

2. For $I = 250\,\mu\text{A}$, $R_1 = 100\,\text{k}\Omega$, $R_2 = 68\,\text{k}\Omega$, and $R_3 = 82\,\text{k}\Omega$, use Ohm's Law, voltage division, and current division to solve for I_1, I_2, and V_3.

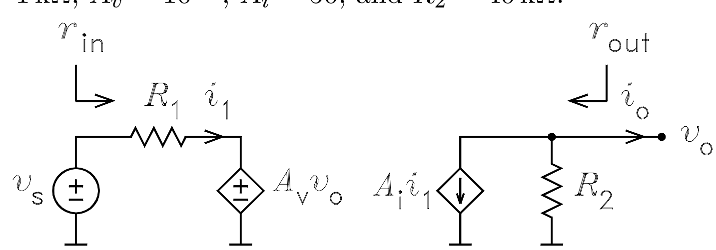


$$I_1 = 250\,\mu\text{A}\frac{68\,\text{k}\Omega + 82\,\text{k}\Omega}{100\,\text{k}\Omega + 68\,\text{k}\Omega + 82\,\text{k}\Omega} = 150\,\mu\text{A}$$

$$I_2 = 250\,\mu\text{A}\frac{100\,\text{k}\Omega}{100\,\text{k}\Omega + 68\,\text{k}\Omega + 82\,\text{k}\Omega} = 100\,\mu\text{A}$$

$$V_3 = 100\,\mu\text{A} \times 82\,\text{k}\Omega = 8.2\,\text{V}$$

3. It is given that $R_1 = 1\,\text{k}\Omega$, $A_v = 10^{-4}$, $A_i = 50$, and $R_2 = 40\,\text{k}\Omega$.



(a) With $i_o = 0$, use superposition to write the equations for i_1 and $v_{o(oc)}$. Solve the equations for $v_{o(oc)}$ as a function of v_s.

$$i_1 = \frac{v_s - A_v v_o}{R_1} \qquad v_{o(oc)} = -A_i i_1 R_2$$

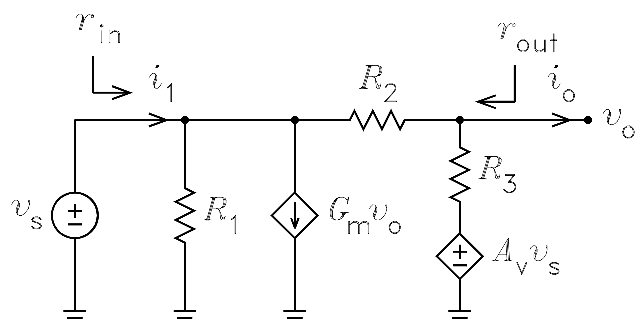
$$v_{o(oc)} = \frac{-A_i \dfrac{R_2}{R_1}}{1 - A_i A_v \dfrac{R_2}{R_1}} v_s = -2500 v_s$$

(b) With $v_o = 0$, use superposition to write the equations for i_1 and $i_{o(sc)}$. Solve the equations for $i_{o(sc)}$ as a function of v_s.

$$i_{o(sc)} = -\frac{A_i v_s}{R_1} = -0.05 v_s$$

(c) Use the solutions for $v_{o(oc)}$ and $i_{o(sc)}$ to show that $r_{out} = 50\,\text{k}\Omega$.
(d) Show that the Thévenin equivalent circuit seen looking into the output is a voltage source $v_{o(oc)} = -2500 v_s$ in series with a resistance $r_{out} = 50\,\text{k}\Omega$.
(e) Show that the Norton equivalent circuit seen looking into the output is a current source $i_{o(sc)} = -0.05 v_s$ in parallel with a resistance $r_{out} = 50\,\text{k}\Omega$.
(f) If a load resistor $R_L = 20\,\text{k}\Omega$ is connected from the output node to ground, use both the Thévenin and the Norton equivalent circuits to show that $v_o = -714.3 v_s$ for both.

4. It is given that $R_1 = 10\,\Omega$, $G_m = 0.5\,\text{S}$, $R_2 = 5\,\Omega$, $R_3 = 50\,\Omega$, and $A_v = 4$.



(a) With $i_o = 0$, use superposition to solve for $v_{o(oc)}$ as a function of v_s.

$$v_{o(oc)} = \frac{R_3}{R_2 + R_3} v_s + A_v v_s \frac{R_2}{R_2 + R_3} = \frac{R_3 + A_v R_2}{R_2 + R_3} v_s = 1.273 v_s$$

(b) With $v_o = 0$, use superposition to solve for $i_{o(sc)}$ as a function of v_s.

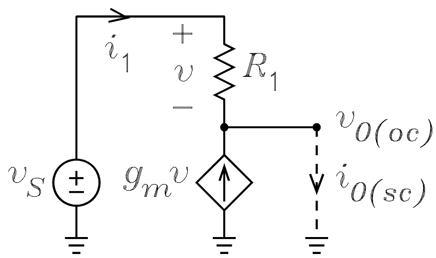
$$i_{o(sc)} = \frac{v_s}{R_2} + \frac{A_v v_s}{R_3} = v_s \left(\frac{1}{R_2} + \frac{A_v}{R_3} \right) = 0.28 v_s$$

(c) Solve for r_{out}.

$$r_{out} = \frac{v_{o(oc)}}{i_{o(sc)}} = 4.545\,\Omega$$

(d) Show that the Thévenin equivalent circuit seen looking into the output is a voltage source $v_{o(oc)} = 1.273 v_s$ in series with a resistance $r_{out} = 4.545\,\Omega$.
(e) Show that the Norton equivalent circuit seen looking into the output is a current source $i_{o(sc)} = 0.28 v_s$ in parallel with a resistance $r_{out} = 4.545\,\Omega$.
(f) If a load resistor $R_L = 5\,\Omega$ is connected from the output node to ground, show $v_o = 0.667 v_s$.

5. It is given that $g_m = 0.025\,\text{S}$ and $R_1 = 40\,\text{k}\Omega$.



(a) With the output terminals open-circuited, use superposition of v_s and $g_m v$ to write equations for $v_{O(oc)}$ and v. Solve the two equations for $v_{O(oc)}$ as a function of v_S.

$$v_{O(oc)} = v_S + g_m v R_1 \qquad v = 0 - g_m v R_1 \Longrightarrow v = 0 \Longrightarrow v_{O(oc)} = v_s$$

(b) For the output terminals short-circuited, use superposition of v_S and $g_m v$ to write equations for $i_{O(sc)}$ and v. Solve the two equations for $i_{O(sc)}$ as a function of v_S.

$$i_{O(sc)} = \frac{v_S}{R_1} + g_m v \qquad v = v_S - 0 \Longrightarrow v = v_S \Longrightarrow i_{O(sc)} = v_S \left(\frac{1}{R_1} + g_m \right) = \frac{v_S}{39.96}$$

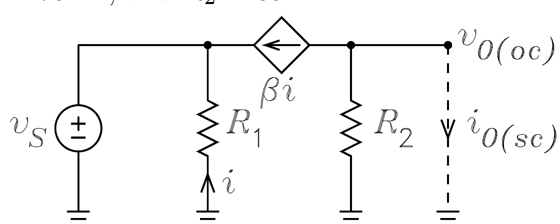
(c) Solve for the Thévenin equivalent circuit looking into the output terminals.

$$v_{O(oc)} = v_S \text{ in series with } r_{out} = 39.96\,\Omega$$

(d) Solve for the Norton equivalent circuit looking into the output terminals.

$$i_{O(sc)} = \frac{v_S}{39.96} \text{ in parallel with } r_{out} = 39.96\,\Omega$$

6. It is given that $\beta = 80$, $R_1 = 75\,\text{k}\Omega$, and $R_2 = 39\,\text{k}\Omega$.



(a) With the output terminals open-circuited, use superposition of v_S and βi to write equations for $v_{O(oc)}$ and i. Solve the two equations for $v_{O(oc)}$ as a function of v_S by eliminating i.

$$v_{O(oc)} = 0 - \beta i R_2 \qquad i = \frac{-v_S}{R_1} \Longrightarrow v_{O(oc)} = \frac{\beta R_2}{R_1} v_S = 41.6 v_S\,\text{V}$$

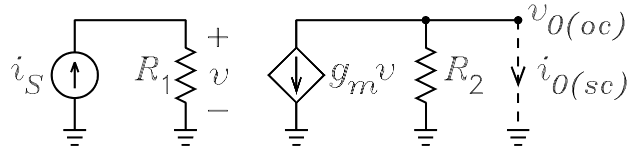
(b) With the output terminals short-circuited, use superposition of v_S and βi to write equations for $i_{O(sc)}$ and i. Solve the two equations for $i_{O(sc)}$ as a function of v_S by eliminating i.

$$i_{O(sc)} = 0 - \beta i \qquad i = \frac{-v_S}{R_1} \Longrightarrow i_{O(oc)} = \frac{\beta}{R_1} v_S = 1.067 v_S\,\text{mA}$$

(c) Solve for the Thévenin equivalent circuit looking into the output terminals. [$v_{O(oc)} = 41.6 v_S$ in series with $39\,\text{k}\Omega$]

(d) Solve for the Norton equivalent circuit looking into the output terminals. [$i_{O(sc)} = 1.067 \times 10^{-3} v_S$ in parallel with $39\,\text{k}\Omega$]

7. It is given that $g_m = 0.002\,\mathrm{S}$, $R_1 = 100\,\mathrm{k}\Omega$, and $R_2 = 1\,\mathrm{M}\Omega$.



(a) With the output terminals open-circuited, solve for v as a function of i_S and $v_{O(oc)}$ as a function of v. Solve the two equations for $v_{O(oc)}$ as a function of i_s by eliminating v.

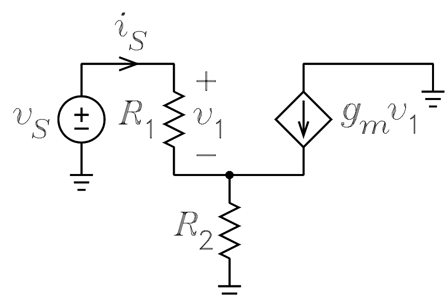
$$v_{O(oc)} = -g_m v R_2 \qquad v = i_S R_1 \Longrightarrow v_{O(oc)} = -g_m i_S R_2 R_1 = -2 \times 10^8 i_S$$

(b) With the output terminals short-circuited, solve for v as a function of i_S and $i_{O(sc)}$ as a function of v. Solve the two equations for $i_{O(sc)}$ as a function of i_S by eliminating v.

$$i_{O(sc)} = -g_m v \qquad v = i_S R_1 \Longrightarrow i_{O(sc)} = -g_m i_S R_1 = -200 i_S$$

(c) Show that the Thévenin equivalent circuit looking into the output terminals is a voltage source $v_{O(oc)} = -2 \times 10^8 i_S$ in series with a resistance $r_{out} = 1\,\mathrm{M}\Omega$]
(d) Show that the Norton equivalent circuit looking into the output terminals is a current source $i_{O(sc)} = -200 i_S$ in parallel with a resistance $r_{out} = 1\,\mathrm{M}\Omega$.

8. It is given that $R_1 = 3\,\mathrm{k}\Omega$, $R_2 = 2\,\mathrm{k}\Omega$, and $g_m = 0.1$. This problem illustrates two solutions for the input resistance to the circuit. In one solution, the source is a voltage source. In the other it is a current source.



(a) Use superposition of v_S and $g_m v_1$ to solve for i_S.

$$i_S = \frac{v_S}{R_1 + R_2} - g_m v_1 \frac{R_2}{R_1 + R_2}$$

(b) Use superposition of v_S and $g_m v_1$ to solve for v_1.

$$v_1 = v_S \frac{R_1}{R_1 + R_2} - g_m v_1 R_1 \| R_2 \Longrightarrow v_1 = v_S \frac{R_1}{R_1 + R_2} \times \frac{1}{1 + g_m R_1 \| R_2}$$

(c) Solve the two equations for i_S as a function of v_S by eliminating v_1.

$$\begin{aligned} i_S &= \frac{v_S}{R_1 + R_2}\left(1 - \frac{g_m R_2}{R_1 + R_2}\,\frac{R_1}{1 + g_m R_1 \| R_2}\right) = \frac{v_S}{R_1 + R_2}\left(1 - \frac{g_m R_1 \| R_2}{1 + g_m R_1 \| R_2}\right) \\ &= \frac{v_S}{R_1 + R_2}\,\frac{1}{1 + g_m R_1 \| R_2} = \frac{v_S}{605\,\mathrm{k}\Omega} \end{aligned}$$

(d) Solve for the input resistance to the circuit.

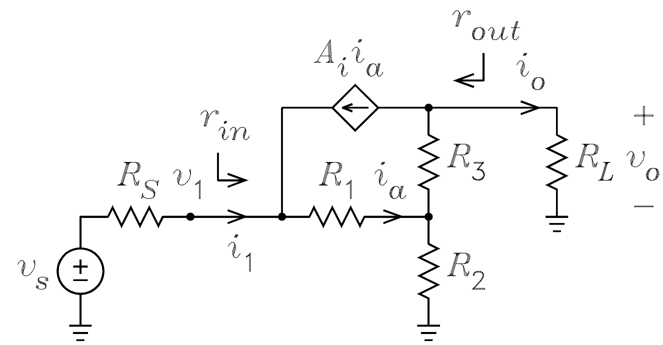
$$r_{in} = \frac{v_S}{i_S} = 605\,\mathrm{k}\Omega$$

(e) Replace v_S with an independent current source i_S. Repeat the problem to solve for r_{in}. Which solution is simpler?

$$v_S = i_S(R_1 + R_2) + g_m v_1 R_2 \qquad v_1 = i_S R_1 \Longrightarrow v_S = i_S(R_1 + R_2 + g_m R_1 R_2)$$

$$r_{in} = \frac{v_S}{i_S} = R_1 + R_2 + g_m R_1 R_2 = 605\,\mathrm{k\Omega}$$

9. The figure shows an amplifier equivalent circuit. It is given that $A_i = 0.99$, $R_S = 1\,\mathrm{k\Omega}$, $R_1 = 25\,\Omega$, $R_2 = 100\,\Omega$, and $R_3 = 30\,\mathrm{k\Omega}$.



(a) With $R_L = \infty$, use superposition of v_s and $A_i i_\alpha$ to show that $v_{o(oc)}$ and i_α are given by

$$\begin{aligned} v_{o(oc)} &= v_s \frac{R_2}{R_S + R_1 + R_2} - A_i i_\alpha \left(R_3 + \frac{R_1}{R_S + R_1 + R_2} R_2 \right) \\ &= \frac{v_s}{11.25} - 29702.2 i_\alpha \end{aligned}$$

$$\begin{aligned} i_\alpha &= \frac{v_s}{R_S + R_1 + R_2} + A_i i_\alpha \frac{R_S + R_2}{R_S + R_1 + R_2} \\ &= \frac{v_s}{1125} + \frac{i_\alpha}{1.03306} \end{aligned}$$

(b) Solve the equations to show that $v_{o(oc)} = -824.97 v_s$.

(c) With $R_L = 0$, use superposition of v_s and $A_i i_\alpha$ to show that $i_{o(sc)}$ and i_α are given by

$$\begin{aligned} i_{o(sc)} &= \frac{v_s}{R_S + R_1 + R_2 \| R_3} \frac{R_2}{R_2 + R_3} - A_i i_\alpha \left(1 - \frac{R_S}{R_S + R_1 + R_2 \| R_3} \frac{R_2}{R_2 + R_3} \right) \\ &= \frac{v_s}{338525} - \frac{i_\alpha}{1.01309} \end{aligned}$$

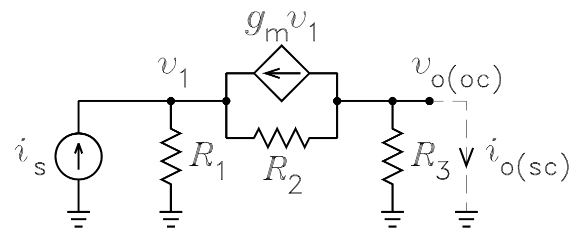
$$\begin{aligned} i_\alpha &= \frac{v_s}{R_S + R_1 + R_2 \| R_3} + A_i i_\alpha \frac{R_S}{R_S + R_1 + R_2 \| R_3} \\ &= \frac{v_s}{1124.67} - \frac{i_\alpha}{1.13603} \end{aligned}$$

(d) Solve the equations to show that $i_{o(sc)} = -v_s/136.486$.

(e) Solve for $r_{out} = v_{o(oc)}/i_{o(sc)}$. [$r_{out} = 112.597\,\mathrm{k\Omega}$]

(f) With $R_L = 10\,\mathrm{k\Omega}$, solve for v_o. [$v_o = -67.2913 v_s$]

10. For the circuit shown, it is given that $i_s = 4\,\text{mA}$, $R_1 = 2\,\text{k}\Omega$, $R_2 = 1\,\text{k}\Omega$, $R_3 = 9\,\text{k}\Omega$, and $g_m = 0.005\,\text{S}$.



(a) Use superposition to solve for $v_{o(oc)}$. Express your answer in symbolic form.

$$v_1 = i_s\left[R_1 \| (R_2 + R_3)\right] + g_m v_1 \frac{R_2}{R_1 + R_2 + R_3} R_1$$
$$= i_s \frac{R_1 \| (R_2 + R_3)}{1 - g_m R_1 \| (R_2 + R_3) + g_m \dfrac{R_3}{R_1 + R_2 + R_3} R_1}$$
$$v_{o(oc)} = i_s \frac{R_1}{R_1 + R_2 + R_3} R_3 - g_m v_1 \frac{R_2}{R_1 + R_2 + R_3} R_3$$

(b) Use superposition to solve for $i_{o(sc)}$. Express your answer in symbolic form.

$$v_1 = (i_s + g_m v_1) R_1 \| R_2 = i_s \frac{R_1 \| R_2}{1 - g_m R_1 \| R_2}$$
$$i_{o(sc)} = i_s \frac{R_1}{R_1 + R_2} - g_m v_1 \frac{R_2}{R_1 + R_2}$$

(c) Solve for the Thévenin equivalent circuit seen looking into the output terminal. Express your answer in numerical form.

$$v_{th} = v_{o(oc)} = -144\,\text{V} \qquad R_{th} = \frac{v_{o(oc)}}{i_{o(sc)}} = -3150\,\Omega$$

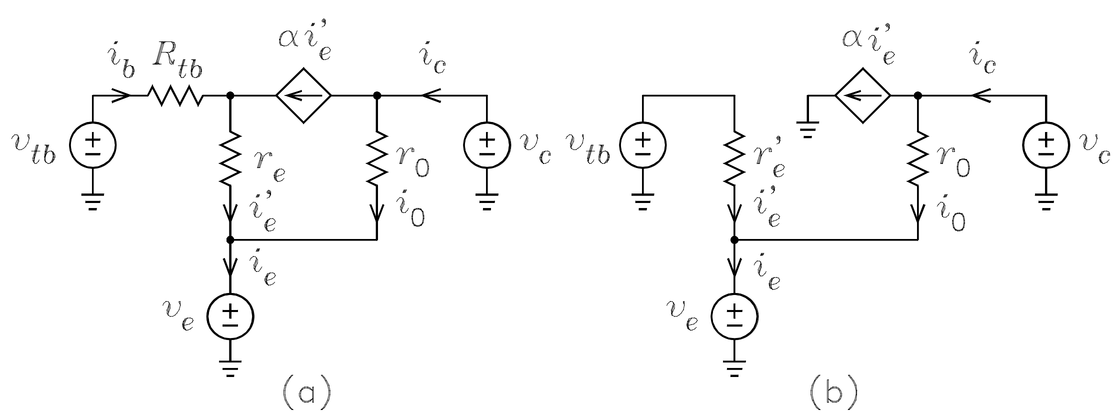
(d) Solve for the Norton equivalent circuit seen looking into the output terminal. Express your answer in numerical form.

$$i_{nor} = i_{o(sc)} = 4.571\,\text{mA} \qquad R_{nor} = \frac{v_{o(oc)}}{i_{o(sc)}} = -3150\,\Omega$$

(e) If a load resistor $R_l = 9\,\text{k}\Omega$ is connected to the output, what is the load voltage?

$$v_L = v_{th} \frac{R_L}{R_{th} + R_L} \overset{or}{=} i_{nor} R_{nor} \| R_L = 57.6\,\text{V}$$

11.



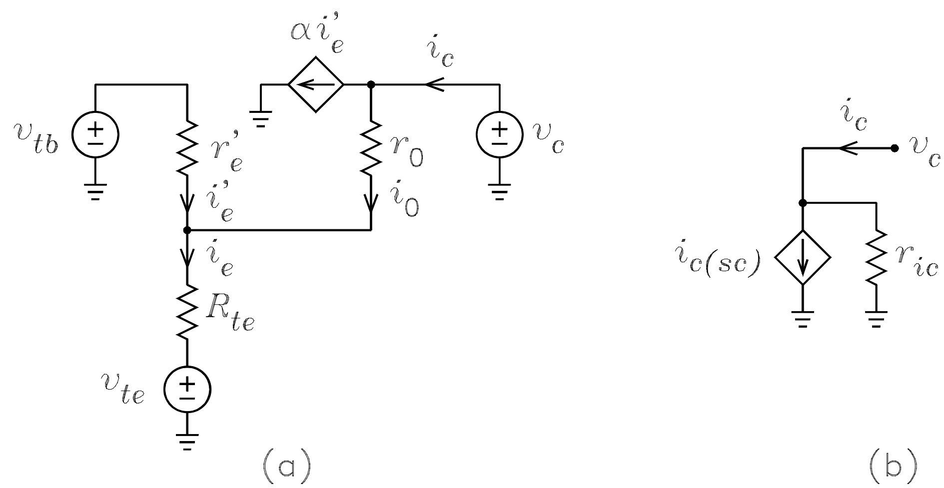
(a) For the circuit of figure (a), show that

$$i_b = (1-\alpha)\, i'_e \qquad i'_e = \frac{v_{tb} - v_e}{(1-\alpha) R_{tb} + r_e} \qquad i_0 = \frac{v_c - v_e}{r_0} \qquad i_c = \alpha i'_e + i_0 \qquad i_e = i'_e + i_0$$

(b) For the circuit of figure (b), show that i'_e, i_e, and i_c are the same as those in figure (a) provided r'_e is given by

$$r'_e = (1-\alpha) R_{tb} + r_e$$

12.



(a) For the circuit of figure (a), use superposition of v_c, $\alpha i'_e$, v_{tb}, and v_{te} to show that

$$i_c = \frac{v_c}{r_0 + r'_e \| R_{te}} + \alpha i'_e - \frac{v_{tb}}{r'_e + R_{te} \| r_0} \frac{R_{te}}{R_{te} + r_0} - \frac{v_{te}}{R_{te} + r'_e \| r_0} \frac{r'_e}{r'_e + r_0}$$

$$i'_e = \frac{v_{tb}}{r'_e + R_{te} \| r_0} - \frac{v_{te}}{R_{te} + r'_e \| r_0} \frac{r_0}{r_0 + r'_e} - \frac{v_c}{r_0 + r'_e \| R_{te}} \frac{R_{te}}{R_{te} + r_0}$$

(b) Substitute the i'_e into the i_c equation to show that

$$\begin{aligned} i_c &= \frac{v_{tb}}{r'_e + R_{te} \| r_0} \left(\alpha - \frac{R_{te}}{R_{te} + r_0} \right) - \frac{v_{te}}{R_{te} + r'_e \| r_0} \frac{\alpha r_0 + r'_e}{r_0 + r'_e} \\ &\quad + \frac{v_c}{r_0 + r'_e \| R_{te}} \left(1 - \frac{\alpha R_{te}}{r'_e + R_{te}} \right) \end{aligned}$$

(c) Show that the circuit of figure (b) gives the same value for i_c provided

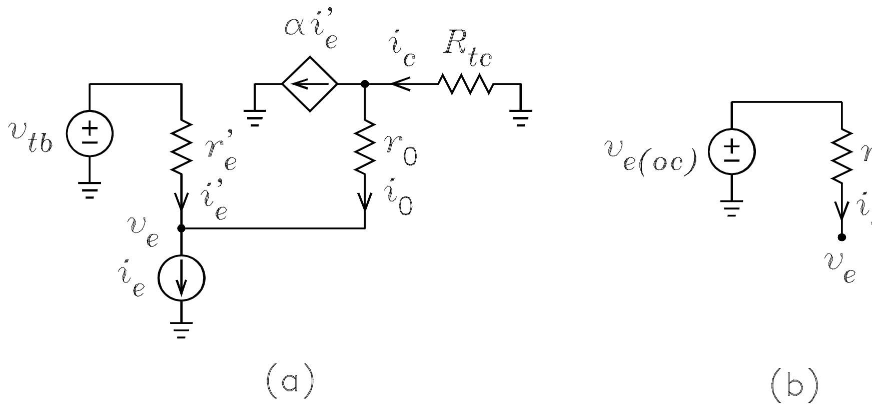
$$i_c = G_{mb} v_{tb} - G_{me} v_{te} + \frac{v_c}{r_{ic}} \qquad G_{mb} = \frac{1}{r'_e + R_{te} \| r_0} \left(\alpha - \frac{R_{te}}{R_{te} + r_0} \right)$$

$$G_{me} = \frac{1}{R_{te} + r'_e \| r_0} \frac{\alpha r_0 + r'_e}{r_0 + r'_e} \qquad r_{ic} = \frac{r_0 + r'_e \| R_{te}}{1 - \dfrac{\alpha R_{te}}{r'_e + R_{te}}}$$

(d) If β is sufficiently large, show that $G_{mb} = G_{me} = G_m$, where G_m is given by

$$G_m = \frac{1}{r'_e + R_{te}}$$

13.



(a) For the circuit of figure (a), use superposition of v_{tb}, i_e, and $\alpha i'_e$, to show that

$$v_e = v_{tb}\frac{r_0 + R_{tc}}{r'_e + r_0 + R_{tc}} - i_e\left[r'_e \| \left(r_0 + R_{tc}\right)\right] - \alpha i'_e \frac{R_{tc} r'_e}{R_{tc} + r_0 + r'_e}$$

$$i'_e = \frac{v_{tb} - v_e}{r'_e}$$

(b) Eliminate i'_e between the equations to show that

$$v_e\left(1 - \frac{\alpha R_{tc}}{R_{tc} + r_0 + r'_e}\right) = v_{tb}\frac{r_0 + (1-\alpha) R_{tc}}{r_0 + r'_e + R_{tc}} - i_e\left[r'_e \| \left(r_0 + R_{tc}\right)\right]$$

(c) Show that the above equation simplifies to

$$v_e = v_{tb}\frac{r_0 + (1-\alpha) R_{tc}}{r_0 + r'_e + (1-\alpha) R_{tc}} - i_e\frac{r'_e\left(r_0 + R_{tc}\right)}{r'_e + r_0 + (1-\alpha) R_{tc}}$$

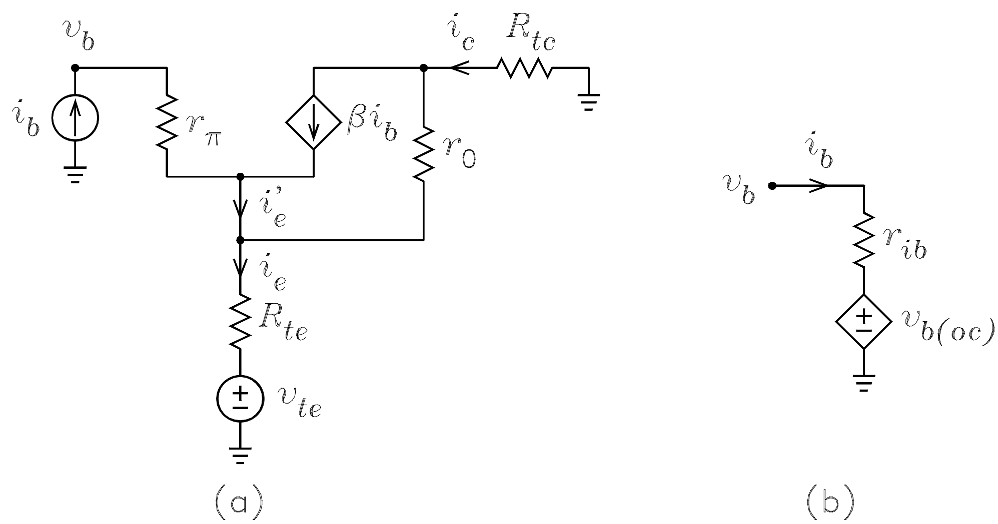
(d) Show that the circuit of figure (b) gives the same value for v_2 provided

$$v_{e(oc)} = v_{tb}\frac{r_0 + (1-\alpha) R_{tc}}{r_0 + r'_e + (1-\alpha) R_{tc}} \qquad r_{ie} = \frac{r'_e\left(r_0 + R_{tc}\right)}{r_0 + r'_e + (1-\alpha) R_{tc}}$$

(e) If r_0 is sufficiently large, show that the above answers reduce to

$$v_{e(oc)} = v_{tb} \qquad r_{ie} = r'_e$$

14.



(a) For the circuit of figure (a), use superposition of v_{te}, i_b, and βi_b to show that

$$v_b = v_{te}\frac{r_0 + R_{tc}}{R_{te} + r_0 + R_{tc}} + i_b\left[r_\pi + R_{te}\|\left(r_0 + R_{tc}\right)\right] + \beta i_b \frac{r_0 R_{te}}{r_0 + R_{te} + R_{tc}}$$

(b) Show that the equation for v_b simplifies to

$$v_b = v_{te}\frac{r_0 + R_{tc}}{r_0 + R_{te} + R_{tc}} + i_b\left[r_\pi + R_{te}\frac{(1+\beta)\,r_0 + R_{tc}}{r_0 + R_{te} + R_{tc}}\right]$$

(c) Show that the circuit of figure (b) gives the same value for v_b provided

$$v_{b(oc)} = v_{te}\frac{r_0 + R_{tc}}{r_0 + R_{te} + R_{tc}} \qquad r_{ib} = r_\pi + R_{te}\frac{(1+\beta)\,r_0 + R_{tc}}{r_0 + R_{te} + R_{tc}}$$

(d) If r_0 is sufficiently large, show that the above answers reduce to

$$v_{b(oc)} = v_{te} \qquad r_{ib} = r_\pi + (1+\beta)\,R_{te}$$