The figures show a common-emitter amplifier, a common-collector amplifier, and a common-base amplifier. For each circuit, it is given that $R_{tb} = 1\, \text{k}\Omega$, $R_{te} = 100\, \Omega$, and $R_{tc} = 10\, \text{k}\Omega$. The transistors have the values $I_E = 1.5\, \text{mA}$, $V_T = 25\, \text{mV}$, $\beta = 99$, $r_x = 50\, \Omega$, and $r_0 = \infty$.

1. For each transistor, show that $g_m = 59.4\, \text{mS}$, $r_\pi = 1.667\, \text{k}\Omega$, $\alpha = 0.99$, and $r_e = 16.67\, \Omega$.

2. For the common-emitter amplifier of Figure (a):

   (a) Replace the BJT with the $\pi$ model. Label the controlled source in the collector $i'_c$, the base current $i'_c/\beta$, and the emitter current $i'_c/\alpha$. Let the voltage across $r_\pi$ be written $v_\pi = i'_c/g_m$. Write a loop equation around the base-emitter loop and solve for $i'_c$. Use the circuit to show that

   $$\frac{i'_c}{v_{tb}} = \frac{1}{R_{tb} + r_x + \frac{R_{te}}{\alpha}} = 7.785\, \text{mS}$$

   $$\frac{v_o}{v_{tb}} = \frac{-R_{tc}}{\frac{R_{tb} + r_x}{\beta} + \frac{R_{te}}{\alpha}} = -77.85$$

   Label the base current $i_b$ and the emitter current $(1 + \beta)i_b$. Write the loop equation and show that

   $$r_{in} = \frac{v_{tb}}{i_b} = R_{tb} + r_x + r_x + (1 + \beta)R_{te} = 12.72\, \text{k}\Omega$$

   Show that

   $$r_{out} = R_{tc} = 10\, \text{k}\Omega$$

   (b) Replace the BJT with the T model. Label the controlled source in the collector $i'_c$, the base current $i'_c/\beta$, and the emitter current $i'_c/\alpha$. Write a loop equation around the base-emitter loop and solve for $i'_c$. Use the circuit to show that

   $$\frac{i'_c}{v_{tb}} = \frac{1}{\frac{R_{tb} + r_x}{\beta} + \frac{r_e + R_{te}}{\alpha}} = 7.785\, \text{mS}$$
\[
\frac{v_o}{v_{tb}} = \frac{-R_{tc}}{R_{tb} + r_x + r_e + R_{te} \alpha} = -77.85
\]

Label the base current \(i_b\) and the emitter current \((1 + \beta)i_b\). Write the loop equation and show that

\[
\frac{v_{tb}}{i_b} = R_{tb} + r_x + (1 + \beta)(r_e + R_{te}) = 12.72 \text{ k\Omega}
\]

Show that

\[
r_{\text{out}} = R_{tc} = 10 \text{ k\Omega}
\]

(c) Show that the simplified T model and simplified \(\pi\) model give the same answers for \(v_o/v_{tb}\) and \(r_{\text{out}}\).

3. For the common-collector amplifier of Figure (b):

(a) Replace the BJT with the \(\pi\) model. Label the controlled source in the collector \(i'_c\), the base current \(i'_e/(1 + \beta)\), and the emitter current \(i'_e\). Write a loop equation around the base-emitter loop and solve for \(i'_e\). Use the circuit to show that

\[
\frac{i'_e}{v_{tb}} = \frac{1}{R_{tb} + r_x + r_{\pi} + (1 + \beta)R_{te}} = 7.864 \text{ mS}
\]

Label the base current \(i_b\) and the emitter current \((1 + \beta)i_b\). Write the loop equation and show that

\[
\frac{v_{tb}}{i_b} = R_{tb} + r_x + r_{\pi} + (1 + \beta)R_{te} = 12.72 \text{ k\Omega}
\]

The output resistance can be written \(r_{\text{out}} = R_{te}\|r_{ie}\), where \(r_{ie}\) is the resistance seen looking up into the emitter. This can be solved for as the ratio of the open-circuit output voltage with \(R_{te} = \infty\) to the short-circuit output current with \(R_{te} = 0\). Show that \(r_{ie}\) is given by

\[
r_{ie} = \frac{v_{o(oc)}}{i_{o(sc)}} = \frac{v_o}{v_o|_{R_{te}=\infty}} = \frac{R_{tb} + r_x + r_{\pi}}{1 + \beta} = 27.17 \text{ \Omega}
\]

and that \(r_{out}\) is

\[
r_{out} = R_{te}\|r_{ie} = 21.36 \text{ \Omega}
\]

(b) Replace the BJT with the T model. Label the controlled source in the collector \(i'_c\), the base current \(i'_e/(1 + \beta)\), and the emitter current \(i'_e\). Write a loop equation around the base-emitter loop and solve for \(i'_e\). Use the circuit to show that

\[
\frac{i'_e}{v_{tb}} = \frac{1}{R_{tb} + r_x + r_{\pi} + (1 + \beta)R_{te}} = 7.864 \text{ mS}
\]
\[
\frac{v_o}{v_{tb}} = \frac{R_{te}}{R_{tb} + r_x + r_e + R_{te}} = 0.786
\]

Label the base current \(i_b\) and the emitter current \((1+\beta)i_b\). Write the loop equation and show that

\[
r_{in} = \frac{v_{tb}}{i_b} = R_{te} + r_x + (1 + \beta)(r_e + R_{te}) = 12.72 \, \text{k}\Omega
\]

The output resistance can be written \(r_{out} = R_{te} || r_{ie}\), where \(r_{ie}\) is the resistance seen looking up into the emitter. This can be solved for as the ratio of the open-circuit voltage with \(R_{te} = \infty\) to the short-circuit current with \(R_{te} = 0\). Show that \(r_{ie}\) is given by

\[
r_{ie} = \frac{v_{o(oc)}}{i_{o(sc)}} = \frac{v_o}{R_{te}}_{R_{te}=\infty} = \frac{R_{tb} + r_x}{1 + \beta} + r_e = 27.17 \, \Omega
\]

and that \(r_{out}\) is

\[
r_{out} = R_{te} || r_{ie} = 21.36 \, \Omega
\]

(c) Show that the simplified T model gives the same answers for \(v_o/v_{tb}\) and \(r_{out}\). Note that the simplified \(\pi\) model is not convenient because the \(v_o\) node does not appear in the circuit.

4. For the common-base amplifier of Figure (c):

\begin{enumerate}
\item[(a)] Replace the BJT with the \(\pi\) model. Label the controlled source in the collector \(\dot{i}_c\), the base current \(\dot{i}_b\), and the emitter current \(\dot{i}_e = \dot{i}_c/\alpha\). Let the voltage across \(r_\pi\) be written \(v_\pi = \dot{i}_c/g_m\). Write a loop equation around the base-emitter loop and solve for \(\dot{i}_c\) to show that

\[
\frac{\dot{i}_c}{v_{tb}} = \frac{-1}{\frac{R_{tb} + r_x + r_\pi}{\beta} + \frac{R_{te}}{\alpha}} = 7.785 \, \text{mS}
\]

\[
\frac{v_o}{v_{te}} = \frac{R_{te}}{R_{tb} + r_x + r_\pi + \frac{R_{te}}{\alpha}} = 77.85
\]

Label the base current \(\dot{i}_b/ (1+\beta)\) and the emitter current \(\dot{i}_e\). Write the loop equation and show that

\[
r_{in} = \frac{v_{te}}{-\dot{i}_e} = \frac{R_{te} + r_x + r_\pi}{1 + \beta} + R_{te} = 127.2 \, \Omega
\]

Show that

\[
r_{out} = R_{te} = 10 \, \text{k}\Omega
\]

\item[(b)] Replace the BJT with the T model. Label the controlled source in the collector \(\dot{i}_c\), the base current \(\dot{i}_b/\beta\), and the emitter current \(\dot{i}_e/\alpha\). Write a loop equation around the base-emitter loop and solve for \(\dot{i}_c\). Use the circuit to show that

\[
\frac{\dot{i}_c}{v_{tc}} = \frac{-1}{\frac{R_{tb} + r_x}{\beta} + \frac{r_e + R_{te}}{\alpha}} = 7.785 \, \text{mS}
\]
\end{enumerate}
\[
\frac{v_o}{v_{te}} = \frac{R_{tc}}{R_{tb} + r_x + r_e + R_{te}} = 77.85
\]

Label the base current \(i'_b/ (1 + \beta)\) and the emitter current \(i'_e\). Write the loop equation and show that

\[
\begin{align*}
    r_{in} &= \frac{v_{te}}{-i'_e} = \frac{R_{te} + r_x}{1 + \beta} + r_e + R_{te} = 127.2 \Omega \\
    \text{Show that} \quad r_{out} &= R_{tc} = 10 \, \text{k}\Omega
\end{align*}
\]

(c) Show that the simplified T model and the simplified π model give the same answers.

5. For the CE amplifier shown, let \(R_S = 1 \, \text{k}\Omega, R_1 = 300 \, \text{k}\Omega, R_2 = 150 \, \text{k}\Omega, R_3 = 50 \, \Omega, R_C = 4.3 \, \text{k}\Omega, R_E = 5.6 \, \text{k}\Omega, R_L = 20 \, \Omega, V^+ = 15 \, \text{V}, V^- = -15 \, \text{V}, V_{BE} = 0.65 \, \text{V}, \beta = 99, r_x = 20 \, \Omega, r_0 = 50 \, \text{k}\Omega, \text{and } V_T = 25 \, \text{mV}.

(a) Show that \(I_E = 1.417 \, \text{mA} \) and \(V_{CB} = 15.386 \, \text{V}\).

(b) Use the \(r_0\) approximations to show that \(v_o/v_s = -43.74\), \(r_{out} = 4.169 \, \text{k}\Omega\), and \(r_{in} = 6.315 \, \text{k}\Omega\).

(c) Show that the clipping levels are \(v^+_O = 4.964 \, \text{V} \) and \(v^-_O = -15.62 \, \text{V}\).

(d) Show that the emitter current which results in symmetrical clipping is \(I_E = 2.222 \, \text{mA}\).

(e) Show that the clipping levels for \(I_E = 2.222 \, \text{mA}\) are \(v^+_O = -v^-_O = 7.786 \, \text{V}\).

6. For the CC amplifier example shown, let \(R_s = 1 \, \text{k}\Omega, R_1 = 300 \, \text{k}\Omega, R_2 = 150 \, \text{k}\Omega, R_E = 5.6 \, \text{k}\Omega, R_L = 1 \, \text{k}\Omega, V^+ = 15 \, \text{V}, V^- = -15 \, \text{V}, V_{BE} = 0.65 \, \text{V}, \beta = 99, r_x = 20 \, \Omega, r_0 = 50 \, \text{k}\Omega, V_{CEsat} = 0.2 \, \text{V}, \text{and } V_T = 25 \, \text{mV}. \) In the ac signal circuit, combine \(r_0\) in parallel with \(R_E\) from the emitter to ground. This can be done because \(R_{tc} = 0\), which puts \(r_0\) in parallel with \(R_E\).

(a) Show that \(I_E = 1.417 \, \text{mA}, V_{CB} = 21.417 \, \text{V}\).

(b) Show that \(v_o/v_s = 0.958, r_{out} = 27.596 \, \text{\Omega}, \) and \(r_{in} = 46.01 \, \text{k}\Omega\).

(c) Show that the clipping levels are \(v^+_O = 3.31 \, \text{V} \) and \(v^-_O = -1.20 \, \text{V}\).

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(d) Show that the emitter current which results in symmetrical clipping is \( I_E = 2.661 \text{ mA} \).

(e) Show that the clipping levels for \( I_E = 2.661 \text{ mA} \) are \( v_O^+ = v_O^- = 2.257 \text{ V} \).

7. For the CB amplifier shown, let \( R_s = 50 \Omega, R_1 = 300 \text{k}\Omega, R_2 = 150 \text{k}\Omega, R_C = 4.3 \text{k}\Omega, R_E = 5.6 \text{k}\Omega, R_L = 20 \Omega, V^+ = 15 \text{ V}, V^- = -15 \text{ V}, V_{BE} = 0.65 \text{ V}, \beta = 99, r_x = 20 \Omega, r_0 = 50 \text{k}\Omega, \text{ and } V_T = 25 \text{ mV}.

(a) Show that \( I_E = 1.417 \text{ mA} \) and \( V_{CB} = 15.386 \text{ V} \).

(b) Use the \( r_0 \) approximations to show that \( v_o/v_s = 50.552, r_{\text{out}} = 4.202 \text{ k}\Omega, \) and \( r_{\text{in}} = 17.79 \Omega \).

8. The figure shows the ac signal circuit of a cascade common-emitter amplifier. For each BJT, it is given that \( I_E = 1.5 \text{ mA}, \alpha = 0.99, \beta = 99, r_x = 20 \Omega, r_0 = \infty, \) and \( V_T = 0.025 \text{ V} \). The circuit element values are \( R_S = 1 \text{k}\Omega, R_{E1} = R_{E2} = 47 \Omega, R_{C1} = R_{C2} = 10 \text{k}\Omega. \)
(a) Looking out of the base of $Q_2$, use the Norton collector circuit of $Q_1$ to show that 

$$v_{tb2} = -i_{c1}R_{c1} = -134.03v_s \quad R_{tb2} = R_{C1} = 10 \text{ k}\Omega$$

(b) Use the Norton collector circuit for $Q_2$ to show that 

$$v_o = 8097v_s \quad r_{out} = R_{C2} = 10 \text{ k}\Omega$$

(c) Show that 

$$r_{in} = r_{ib1} = 6.387 \text{ k}\Omega$$

(d) If a resistor $R_L = 1 \text{ k}\Omega$ is connected from output to ground, show that the new gain is reduced by a factor $R_L / (r_{out} + R_L)$ to the value $v_o/v_s = 736.1$ and that the dB decrease in gain is 20.83 dB.

9. The ac signal circuit of a common-base amplifier driving a common-collector amplifier is shown. For each BJT, it is given that $I_E = 1.5 \text{ mA}$, $\alpha = 0.99$, $\beta = 99$, $r_x = 20 \Omega$, $r_0 = \infty$, and $V_T = 0.025 \text{ V}$. The circuit element values are $R_S = 50 \Omega$, $R_C = 10 \text{ k}\Omega$, $R_E = 1 \text{ k}\Omega$.

(a) Looking out of the base of $Q_2$ use the Norton collector circuit of $Q_1$ to show that 

$$v_{tb2} = 148.06v_s \quad R_{tb2} = 10 \text{ k}\Omega$$
(b) Use the simplified T model for $Q_2$ to show that

$$v_o = 132.56v_s \quad r_{out} = r_{ie2}\| R_E = 104.638\Omega$$

(c) Use the simplified T model for $Q_1$ to show that

$$r_{in} = r_{ie1} = 16.867\Omega$$

(d) If a resistor $R_L = 1\, k\Omega$ is connected from output to ground, show that the new gain is reduced by the factor $R_L/(r_{out} + R_L)$ to the value $v_o/v_s = 120.01$ and the dB decrease in gain is 0.864 dB. Note that the gain does not change nearly as much as it did in problem 8. Explain.

10. The figure shows a cascade common-collector amplifier, also called a Darlington connection. For each BJT, it is given that $\alpha = 0.99$, $\beta = 99$, $r_x = 20\, \Omega$, $r_0 = \infty$, and $V_T = 0.025\, V$. The emitter current in $Q_2$ is $I_{E2} = 10\, mA$. The circuit element values are $R_S = 10\, k\Omega$ and $R_E = 100\, \Omega$.

![Circuit Diagram]

(a) Looking out of the base of $Q_2$, use the simplified T model for $Q_1$ to show that

$$v_{tb2} = v_s \quad R_{tb2} = r_{ie1} = 350.2\, \Omega$$

(b) Use the simplified T model for $Q_2$ to show that

$$v_o = 0.942v_s \quad r_{out} = r_{ie2}\| R_E = 5.84\, \Omega$$

(c) Show that

$$r_{in} = r_{ib1} = 1.052\, M\Omega$$

(d) If $Q_1$ and $Q_2$ are removed and $R_E$ is connected to the right node of $R_S$, show that $v_o/v_s = 9.901 \times 10^{-3}$, a decrease of 39.56 dB.

11. The figure shows a common-collector stage driving a common-base stage. For each BJT, it is given that $I_E = 1.5\, mA$, $\alpha = 0.99$, $\beta = 99$, $r_x = 20\, \Omega$, $r_0 = \infty$, and $V_T = 0.025\, V$. The circuit element values are $R_S = 1\, k\Omega$, $R_E = 100\, \Omega$, $R_C = 10\, k\Omega$. 

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(a) Use the simplified T model for $Q_1$ to show that

\[ v_{te2} = v_s \quad R_{te2} = R_E + r_{ie1} = 126.87 \, \Omega \]

(b) Use the Norton collector circuit of $Q_2$ to show that

\[ v_o = 68.88v_s \quad r_{out} = R_C = 10 \, k\Omega \]

(c) Show that

\[ r_{in} = r_{ib1} = 13.37 \, k\Omega \]