1. The figure shows the ac signal circuit of a BJT differential amplifier. It is given that $V_T = 0.025\, \text{V}$, $\beta = 99$, $r_x = 30\, \Omega$, $I_E = 1.5\, \text{mA}$, $R_B = 1.5\, \text{k}\Omega$, $R_E = 50\, \Omega$, $R_C = 12\, \text{k}\Omega$, and $R_Q = 10\, \text{k}\Omega$. To simplify the problem, assume that $r_0 = \infty$.

(a) Show that $r'_e = 31.967\, \Omega$.

(b) Show that the Thévenin equivalent circuits seen looking out of the emitters in the ac signal circuit consist of the voltages and resistances $v_{te1} = 0.992\, v_{s1}$, $v_{te2} = 0.992\, v_{s2}$, and $R_{te1} = R_{te2} = 131.3\, \Omega$.

(c) Use the emitter equivalent circuits on the simplified T model to show that $i_{0e1} = \frac{(v_{s1} - 0.992\, v_{s2})}{163.267}$ and $i_{0e2} = \frac{(v_{s2} - 0.992\, v_{s1})}{163.267}$.

(d) Use the collector equivalent circuits of the simplified T model to show that $v_{o1} = -72.764\, v_{s1} + 72.173\, v_{s2}$ and $v_{o2} = -72.764\, v_{s2} + 72.173\, v_{s1}$.

2. An alternate solution to the differential amplifier of Problem 1 is to use common-mode and differential.

(a) For the common-mode input signals $v_{s1} = v_{s2} = v_{s(cm)}$, use the simplified T model to show that the common-mode gain to each output is given by

$$A_{cm} = \frac{v_{o1}}{v_{s(cm)}} = \frac{v_{o2}}{v_{s(cm)}} = -0.592$$

(b) For the differential input signals $v_{s1} = v_{s(d)}/2$ and $v_{s2} = -v_{s(d)}/2$, use the simplified T model to show that the differential gain is given by

$$A_d = \frac{v_{o1}}{v_{s(d)}} = -\frac{v_{o2}}{v_{s(d)}} = -72.468$$

(c) Solve for the common-mode rejection ratio at the $v_{o1}$ output to show that

$$CMRR = \frac{|v_{o1}/v_{s(d)}|}{|v_{o1}/v_{s(cm)}|} = \frac{|v_{o2}/v_{s(d)}|}{|v_{o2}/v_{s(cm)}|} = 122.501 = 41.763\, \text{dB}$$
(d) For \( v_{s(cm)} = (v_{s1} + v_{s2}) / 2 \) and \( v_{s(d)} = v_{s1} - v_{s2} \), show that

\[
\begin{align*}
    v_{o1} &= \left( A_d + \frac{A_{cm}}{2} \right) v_{s1} - \left( A_d - \frac{A_{cm}}{2} \right) v_{s2} = -72.764v_{s1} + 72.173v_{s2} \\
    v_{o2} &= -\left( A_d - \frac{A_{cm}}{2} \right) v_{s1} + \left( A_d + \frac{A_{cm}}{2} \right) v_{s2} = 72.173v_{s1} - 72.764v_{s2}
\end{align*}
\]

(e) Show that

\[ v_{o(d)} = v_{o1} - v_{o2} = -144.937(v_{s1} - v_{s2}) \]

3. A differential amplifier has a differential gain of \(-100\) and a common-mode gain of \(-0.01\). The input voltages are given by \( v_{i1} = 0.1 \sin \omega_1 t - 0.01 \sin \omega_2 t \) and \( v_{i2} = 0.1 \sin \omega_1 t + 0.01 \sin \omega_2 t \).

(a) Show that the differential and common-mode input voltages are given by

\[
\begin{align*}
    v_{id} &= -0.02 \sin \omega_2 t \\
    v_{icm} &= 0.1 \sin \omega_1 t
\end{align*}
\]

(b) If the output is taken from the collector of \( Q_1 \), show that

\[
    v_{o1} = -0.001 \sin \omega_1 t + 2 \sin \omega_2 t
\]

(c) If the \( \omega_1 \) term is an unwanted interference signal and the \( \omega_2 \) term is a desired signal, show that the input signal-to-noise ratio is \( SNR_{in} = -14 \) dB, which is pretty low. Show that the output signal-to-noise ratio is \( SNR_{out} = 66 \) dB.

(d) Show that the improvement in the signal-to-noise ratio between the input and the output is \( 80 \) dB. Note: This problem illustrates how a common-mode noise voltage on a signal pair can be reduced by a large amount with a differential amplifier input stage.

(e) What is the CMRR? Answer: \( 80 \) dB.

4. For the diff amp shown, it is given that \( I_Q' = 1.8 \) mA, \( R_Q = 20 \) k\( \Omega \), \( R_C = 7500 \) \( \Omega \), \( R_E = 30 \) \( \Omega \), \( R_B = 100 \) \( \Omega \), \( r_x = 20 \) \( \Omega \), \( V^+ = +15 \) V, \( V^- = -15 \) V, \( V_{BE} = 0.65 \) V, \( \beta = 149 \), \( V_A = \infty \), and \( V_T = 25 \) mV.
(a) With \( v_{i1} = v_{i2} = 0 \), show that \( I_{E1} = I_{E2} = 1.258 \text{ mA} \), \( r_0 = \infty \), \( r_e = 19.876 \Omega \), \( r_e' = 21.076 \Omega \), and \( r_{ib} = 15.08 \text{ k}\Omega \).

(b) Show that \( R_{te} = 80.946 \Omega \) for each BJT, \( v_{te1} = 0.997 v_{i2} \), and \( v_{te2} = 0.997 v_{i1} \).

(c) Show that \( i_e'/v_{ib} = -i_e'/v_{te} = 9.704 \times 10^{-3} \Omega \) for each BJT.

(d) Show that \( v_{o1}/v_{i1} = v_{o2}/v_{i2} = -72.778 \) and \( v_{o2}/v_{i1} = v_{o1}/v_{i2} = +72.593 \).

(e) If the output is taken from either the collector of \( Q_1 \) or the collector of \( Q_2 \), show that \( CMRR = 392.071 \) or \( 51.876 \text{ dB} \).

(f) For \( v_{i1} = v_{i(d)}/2 \) and \( v_{i2} = -v_{i(d)}/2 \), show that the differential input resistance between the two inputs in series is \( r_{i(d)} = 10.2 \text{ k}\Omega \).

(g) For \( v_{i1} = v_{i2} = v_{i(cm)} \), show that the common-mode input resistance to the two inputs in parallel is \( r_{i(cm)} = 4.005 \text{ M}\Omega \).