The figures show a common-emitter amplifier, a common-collector amplifier, and a common-base amplifier. For each circuit, it is given that $R_{tb} = 1 \, \text{k}\Omega$, $R_{te} = 100 \, \Omega$, and $R_{tc} = 10 \, \text{k}\Omega$. The transistors have the values $I_E = 1.5 \, \text{mA}$, $V_T = 25 \, \text{mV}$, $\beta = 99$, $r_x = 50 \, \Omega$, and $r_0 = \infty$.

1. For each transistor, show that $g_m = 59.4 \, \text{mS}$, $r_\pi = 1.667 \, \text{k}\Omega$, $\alpha = 0.99$, and $r_e = 16.67 \, \Omega$.

2. For the common-emitter amplifier of Figure (a):

(a) Replace the BJT with the $\pi$ model. Label the controlled source in the collector $i'_c$, the base current $i'_b/\beta$, and the emitter current $i'_e/\alpha$. Let the voltage across $r_\pi$ be written $v_\pi = i'_c/g_m$. Write a loop equation around the base-emitter loop and solve for $i'_c$. Use the circuit to show that

$$i'_c = \frac{v_{tb}}{R_{tb} + r_x + \frac{1}{g_m} + \frac{R_{te}}{\alpha}} = 7.785 \, \text{mS}$$

$$v_o = \frac{v_{tb}}{R_{tb} + r_x + \frac{1}{g_m} + \frac{R_{te}}{\alpha}} = -77.85$$

Label the base current $i_b$ and the emitter current $(1 + \beta) i_b$. Write the loop equation and show that

$$r_{in} = \frac{v_{tb}}{i_b} = R_{tb} + r_x + r_x + (1 + \beta) R_{te} = 12.72 \, \text{k}\Omega$$

Show that

$$r_{out} = R_{tc} = 10 \, \text{k}\Omega$$

(b) Replace the BJT with the $T$ model. Label the controlled source in the collector $i'_c$, the base current $i'_b/\beta$, and the emitter current $i'_e/\alpha$. Write a loop equation around the base-emitter loop and solve for $i'_c$. Use the circuit to show that

$$i'_c = \frac{v_{tb}}{R_{tb} + r_x + \frac{1}{r_e + R_{te}} + \frac{R_{te}}{\alpha}} = 7.785 \, \text{mS}$$
\[
\frac{v_o}{v_{tb}} = \frac{-R_{te}}{R_{tb} + r_x + \frac{r_e + R_{te}}{\alpha}} = -77.85
\]

Label the base current \(i_b\) and the emitter current \((1 + \beta) i_b\). Write the loop equation and show that

\[
\frac{v_{tb}}{i_b} = \frac{R_{tb} + r_x + (1 + \beta) (r_e + R_{te})}{\alpha + 1} = 12.72 \text{k}\Omega
\]

Show that

\[
r_{\text{out}} = R_{te} = 10 \text{k}\Omega
\]

(c) Show that the simplified T model and simplified \(\pi\) model give the same answers for \(v_o/v_{tb}\) and \(r_{\text{out}}\).

3. For the common-collector amplifier of Figure (b):

(a) Replace the BJT with the \(\pi\) model. Label the controlled source in the collector \(i'_c\), the base current \(i'_e/ (1 + \beta)\), and the emitter current \(i'_e\). Write a loop equation around the base-emitter loop and solve for \(i'_e\). Use the circuit to show that

\[
\frac{i'_e}{v_{tb}} = \frac{1}{R_{tb} + r_x + r_x + 1 + \beta} = 7.864 \text{mS}
\]

\[
\frac{v_o}{v_{tb}} = \frac{R_{te}}{R_{tb} + r_x + r_x + 1 + \beta + R_{te}} = 0.786
\]

Label the base current \(i_b\) and the emitter current \((1 + \beta) i_b\). Write the loop equation and show that

\[
\frac{v_{tb}}{i_b} = \frac{R_{tb} + r_x + (1 + \beta) R_{te}}{\alpha + 1} = 12.72 \text{k}\Omega
\]

The output resistance can be written \(r_{\text{out}} = R_{te} || r_{ie}\), where \(r_{ie}\) is the resistance seen looking up into the emitter. This can be solved for as the ratio of the open-circuit output voltage with \(R_{te} = \infty\) to the short-circuit output current with \(R_{te} = 0\). Show that \(r_{ie}\) is given by

\[
r_{ie} = \frac{v_{o(oc)}}{i_{o(sc)}} = \frac{v_o}{i_{o(sc)}} = \frac{v_o}{\frac{R_{te}}{R_{te}} \bigg|_{R_{te} = 0}} = \frac{R_{tb} + r_x + r_x}{1 + \beta} = 27.17 \Omega
\]

and that \(r_{\text{out}}\) is

\[
r_{\text{out}} = R_{te} || r_{ie} = 21.36 \Omega
\]

(b) Replace the BJT with the T model. Label the controlled source in the collector \(i'_c\), the base current \(i'_e/ (1 + \beta)\), and the emitter current \(i'_e\). Write a loop equation around the base-emitter loop and solve for \(i'_e\). Use the circuit to show that

\[
\frac{i'_e}{v_{tb}} = \frac{1}{R_{tb} + r_x + r_x + 1 + \beta} = 7.864 \text{mS}
\]

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\[
\frac{v_o}{v_{tb}} = \frac{R_{te}}{R_{tb} + r_x + r_e + R_{te}} = 0.786
\]

Label the base current \(i_b\) and the emitter current \((1 + \beta)i_b\). Write the loop equation and show that

\[
\frac{v_{tb}}{i_b} = R_{te} + r_x + (1 + \beta)(r_e + R_{te}) = 12.72 \, \text{k}\Omega
\]

The output resistance can be written \(r_{out} = R_{te} \parallel r_{ie}\), where \(r_{ie}\) is the resistance seen looking up into the emitter. This can be solved for as the ratio of the open-circuit voltage with \(R_{te} = \infty\) to the short-circuit current with \(R_{te} = 0\). Show that \(r_{ie}\) is given by

\[
r_{ie} = \frac{v_o}{i_{o(oc)}} = \frac{v_o}{R_{te}} = \frac{v_o}{1 + \beta} = 27.17 \, \Omega
\]

and that \(r_{out}\) is

\[
r_{out} = R_{te} \parallel r_{ie} = 21.36 \, \Omega
\]

(c) Show that the simplified T model gives the same answers for \(v_o/v_{tb}\) and \(r_{out}\). Note that the simplified π model is not convenient because the \(v_o\) node does not appear in the circuit.

4. For the common-base amplifier of Figure (c):

(a) Replace the BJT with the π model. Label the controlled source in the collector \(i'_c\), the base current \(i'_b\), and the emitter current \(i'_e = i'_c/\alpha\). Let the voltage across \(r_\pi\) be written \(v_\pi = i'_c/g_m\). Write a loop equation around the base-emitter loop and solve for \(i'_c\) to show that

\[
\frac{i'_c}{v_{tb}} = \frac{-1}{R_{tb} + r_x + r_\pi} + \frac{R_{te}}{\alpha} = 7.785 \, \text{mS}
\]

\[
\frac{v_o}{v_{te}} = \frac{R_{te}}{R_{tb} + r_x + r_\pi + R_{te}} = 77.85
\]

Label the base current \(i'_b/(1 + \beta)\) and the emitter current \(i'_e\). Write the loop equation and show that

\[
\frac{v_{te}}{-i'_e} = \frac{R_{te} + r_x + r_\pi}{1 + \beta} + R_{te} = 127.2 \, \Omega
\]

Show that

\[
r_{out} = R_{te} = 10 \, \text{k}\Omega
\]

(b) Replace the BJT with the T model. Label the controlled source in the collector \(i'_c\), the base current \(i'_b/\beta\), and the emitter current \(i'_e/\alpha\). Write a loop equation around the base-emitter loop and solve for \(i'_e\). Use the circuit to show that

\[
\frac{i'_c}{v_{tc}} = \frac{-1}{R_{tb} + r_x} + \frac{r_e + R_{te}}{\alpha} = 7.785 \, \text{mS}
\]
\[
\frac{v_o}{v_{te}} = \frac{R_{te}}{\frac{R_{tb} + r_x}{\beta} + \frac{r_e + R_{te}}{\alpha}} = 77.85
\]

Label the base current \(i'_b/ (1 + \beta)\) and the emitter current \(i'_e\). Write the loop equation and show that
\[
r_{in} = \frac{v_{te}}{-i'_e} = \frac{R_{te} + r_x}{1 + \beta} + r_e + R_{te} = 127.2 \, \Omega
\]

Show that
\[
r_{out} = R_{te} = 10 \, k\Omega
\]

(c) Show that the simplified T model and the simplified π model give the same answers.