1. The figure shows the ac signal circuit of a BJT differential amplifier. It is given that $V_T = 0.025 \text{ V}$, $\beta = 99$, $r_x = 30 \Omega$, $I_E = 1.5 \text{ mA}$, $R_B = 1.5 \text{ k}\Omega$, $R_E = 50 \Omega$, $R_C = 12 \text{ k}\Omega$, and $R_Q = 10 \text{ k}\Omega$. To simplify the problem, assume that $r_0 = \infty$.

(a) Show that $r'_e = 31.967 \Omega$.

(b) Show that the Thévenin equivalent circuits seen looking out of the emitters in the ac signal circuit consist of the voltages and resistances $v_{te1} = 0.992v_s$, $v_{te2} = 0.992v_s$, and $R_{te1} = R_{te2} = 131.3 \Omega$.

(c) Use the emitter equivalent circuits on the simplified T model to show that $i_{0e1} = (v_s - 0.992v_s)/163.267$ and $i_{0e2} = (v_s - 0.992v_s)/163.267$.

(d) Use the collector equivalent circuits of the simplified T model to show that $v_{o1} = -72.764v_s + 72.173v_s$ and $v_{o2} = -72.764v_s + 72.173v_s$.

![Diagram of BJT differential amplifier](diagram.png)

2. An alternate solution to the differential amplifier of Problem 1 is to use common-mode and differential.

(a) For the common-mode input signals $v_{s1} = v_{s2} = v_{s(cm)}$, use the simplified T model to show that the common-mode gain to each output is given by

$$A_{cm} = \frac{v_{o1}}{v_{s(cm)}} = \frac{v_{o2}}{v_{s(cm)}} = -0.592$$

(b) For the differential input signals $v_{s1} = v_{s(d)}/2$ and $v_{s2} = -v_{s(d)}/2$, use the simplified T model to show that the differential gain is given by

$$A_d = \frac{v_{o1}}{v_{s(d)}} = -\frac{v_{o2}}{v_{s(d)}} = -72.468$$

(c) Solve for the common-mode rejection ratio at the $v_{o1}$ output to show that

$$CMRR = \left| \frac{v_{o1}/v_{s(d)}}{v_{o1}/v_{s(cm)}} \right| = \left| \frac{v_{o2}/v_{s(d)}}{v_{o2}/v_{s(cm)}} \right| = 122.501 = 41.763 \text{ dB}$$
(d) For \( v_{s(cm)} = (v_{s1} + v_{s2})/2 \) and \( v_{s(d)} = v_{s1} - v_{s2} \), show that
\[
\begin{align*}
v_{o1} &= \left( A_d + \frac{A_{cm}}{2} \right) v_{s1} - \left( A_d - \frac{A_{cm}}{2} \right) v_{s2} = -72.764v_{s1} + 72.173v_{s2} \\
v_{o2} &= -\left( A_d - \frac{A_{cm}}{2} \right) v_{s1} + \left( A_d + \frac{A_{cm}}{2} \right) v_{s2} = 72.173v_{s1} - 72.764v_{s2}
\end{align*}
\]
(e) Show that
\[
v_{o(d)} = v_{o1} - v_{o2} = -144.937(v_{s1} - v_{s2})
\]
(a) With \( v_{i1} = v_{i2} = 0 \), show that \( I_{E1} = I_{E2} = 1.258 \text{ mA} \), \( r_0 = \infty \), \( r_e = 19.876 \Omega \), \( r'_e = 21.076 \Omega \), and \( r_{ib} = 15.08 \text{ k}\Omega \).

(b) Show that \( R_{te} = 80.946 \Omega \) for each BJT, \( v_{te1} = 0.997v_{i2} \), and \( v_{te2} = 0.997v_{i1} \).

(c) Show that \( i'_c/v_{ib} = -i'_c/v_{te} = 9.704 \times 10^{-3} \Omega \) for each BJT.

(d) Show that \( v_{o1}/v_{i1} = v_{o2}/v_{i2} = -72.778 \) and \( v_{o2}/v_{i1} = v_{o1}/v_{i2} = +72.593 \).

(e) If the output is taken from either the collector of \( Q_1 \) or the collector of \( Q_2 \), show that \( CMRR = 392.071 \) or \( 51.876 \text{ dB} \).

(f) For \( v_{i1} = v_{i(d)}/2 \) and \( v_{i2} = -v_{i(d)}/2 \), show that the differential input resistance between the two inputs in series is \( r_{id} = 10.2 \text{ k}\Omega \).

(g) For \( v_{i1} = v_{i2} = v_{i(cm)} \), show that the common-mode input resistance to the two inputs in parallel is \( r_{icm} = 4.005 \text{ M}\Omega \).