1. The figure shows a CS amplifier with a current-mirror active load.

(a) If the $r_0$ approximations are used, show that the small-signal short-circuit output current is given by

$$i_o^{(sc)} = -\frac{v_i}{1 + \chi_1 r_{s1} + R_S} = \frac{g_{m1}}{1 + g_{m1} (1 + \chi_1) R_S} v_i$$

(b) Show that the small-signal output resistance is given by

$$r_{out} = r_{id1} r_{02} = \left[ r_{01} \left( 1 + \frac{R_S}{r_{s1}'} \right) + R_S \right] r_{02}$$

(c) Show that the small-signal open-circuit voltage is given by

$$v_o^{(oc)} = i_o^{(sc)} r_{out} = -\frac{g_{m1}}{1 + g_{m1} (1 + \chi_1) R_S} \left[ r_{01} \left( 1 + \frac{R_S}{r_{s1}'} \right) + R_S \right] r_{02} \times v_i$$

2. For the CS amplifier of Problem 1, each MOSFET has the parameters $K_0 = 0.002 \text{ A/V}^2$, $V_{TO} = 1.4 \text{ V}$, $\lambda = 0.02 \text{ V}^{-1}$, $\gamma = 1.5 \text{ V}^{1/2}$, and $\varphi = 0.6 \text{ V}$. It is given that $V^+ = 10 \text{ V}$, $V^- = -10 \text{ V}$, $I_{ref} = 1 \text{ mA}$, $R_S = 200 \Omega$.

(a) Use the equation $I_D = K_0 (1 + \lambda V_{SD}) (V_{SG} - V_{TO})^2$ to show that $V_{SG3} = 2.09 \text{ V}$. Note that $V_{SD3} = V_{SG3}$.

(b) Use the equation $I_D = K_0 (1 + \lambda V_{SD}) (V_{SG} - V_{TO})^2$ to show that $I_{D2} = 1.15 \text{ mA}$. Note that $V_{SG2} = V_{SG3}$.

(c) Show that $V_{DS1} = 9.77 \text{ V}$ and $V_{BS1} = -0.23 \text{ V}$. Note that the current through $R_S$ is equal to $I_{D2}$.

(d) Use the equation $\chi = 0.5\gamma/\sqrt{\varphi - V_{BS}}$ to show that $\chi_1 = 0.823$.

(e) Use the equation $K = K_0 (1 + \lambda V_{DS})$ to show that $K_1 = 4.78 \times 10^{-3} \text{ A/V}^2$.

(f) Use the equations $g_m = 2\sqrt{K T_D}$ and $r_0 = (V_{DS} + 1/\lambda) / I_D$ to show that $g_{m1} = 3.32 \times 10^{-3} \text{ S}$ and $r_{01} = 51.9 \text{ k}\Omega$.

(g) Use the equation $r_0 = (V_{SD} + 1/\lambda) / I_D$ to show that $r_{02} = 52.1 \text{ k}\Omega$. 

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(h) Show that the small-signal short-circuit output current is

\[ i_{o(sc)} = -\frac{v_i}{1 + \chi_1 r'_{s1} + R_S} = 1.50 \times 10^{-3} v_i \]

Assume \( r_{01} = \infty \) and use the simplified T model.

(i) Show that the small-signal output resistance and open-circuit output voltage are \( r_{out} = r_{id1} || r_{02} = 35.8 \text{k}\Omega \) and \( v_{o(oc)} = i_{o(sc)} \times r_{out} = -53.8 v_i \).

(j) If a load resistor \( R_L = 10 \text{k}\Omega \) is connected from output to ground, show that the output voltage changes by the factor \( R_L / (r_{out} + R_L) = 0.218 \) (or by \(-13.2 \text{ dB}\)) and the new voltage gain is \( v_o / v_i = -11.7 \).

(k) If \( R_S = 0 \), show that \( v_{o(oc)} \) increases to the value \( v_{o(oc)} = -86.28 v_i \). Show that the gain increases by \( 4.10 \text{ dB} \).

3. The figure shows a CG amplifier with a current-mirror active load. The voltage \( V_G \) is a dc bias voltage. Each MOSFET has the parameters \( g_m = 2.5 \times 10^{-3} \text{S} \), \( r_0 = 40 \text{k}\Omega \), and \( \chi = 0.5 \).

It is given that \( R_S = 200 \text{\Omega} \). It can be assumed that the dc value of the output voltage is zero.

(a) Solve for the Norton short-circuit output current. Assume \( r_{01} = \infty \) and use the simplified T model. Answer: \( i_{o(sc)} = v_i / (r'_{s1} + R_S) = 2.14 \times 10^{-3} v_i \).

(b) Solve for the Thévenin equivalent circuit seen looking into the \( v_o \) node, i.e. solve for \( r_{out} \) and \( v_{o(oc)} \). Answers: \( r_{out} = r_{id1} || r_{02} = 25.5 \text{k}\Omega \) and \( v_{o(oc)} = i_{o(sc)} \times r_{out} = 54.6 v_i \).

(c) By what factor does \( v_o \) change if a load resistor \( R_L = 10 \text{k}\Omega \) is connected from output to ground? What is the new voltage gain? Answer: \( R_L / (r_{out} + R_L) = 0.282 \) or by \(-11 \text{ dB}\) and \( v_o / v_i = 15.4 \).

(d) Show that the input resistance is \( r_{in} = r'_{s1} = 267 \Omega \).

4. The figure shows a CD amplifier with a current-mirror active load. The voltage \( V_G \) is a dc bias voltage. Each MOSFET has the parameters \( g_m = 2.5 \times 10^{-3} \text{S} \), \( r_0 = 40 \text{k}\Omega \), and \( \chi = 0.5 \).
(a) Solve for the Norton short-circuit output current. Use the simplified T model. Answer: 
\[ i_{o(sc)} = \frac{v_i}{r_{s1}} = g_m v_i = 2.5 \times 10^{-3} v_i. \] Note that the body effect cancels when \( v_o = 0 \).

(b) Solve for the Thévenin equivalent circuit seen looking into the \( v_o \) node, i.e. solve for \( r_{out} \) and \( v_{o(oc)} \). Answers: 
\[ r_{out} = r_{s1} || r_{01} || r_{02} = 263 \Omega \] and 
\[ v_{o(oc)} = i_{o(sc)} \times r_{out} = 0.658 v_i. \]

(c) By what factor does \( v_o \) change if a load resistor \( R_L = 10 \text{k}\Omega \) is connected from output to ground? What is the new voltage gain? Answers: 
\[ \frac{R_L}{r_{out} + R_L} = 0.974 \] or by 
\[ -0.226 \text{ dB} \] and 
\[ \frac{v_o}{v_i} = 0.641. \]

5. The figure shows a cascode amplifier. \( M_1 \) is operated as a CS amplifier with a small-signal voltage \( v_s \) and a dc bias voltage \( V_{B1} \) applied to its gate. \( M_2 \) is operated as a CG amplifier with a dc bias voltage \( V_{B2} \) applied to its gate. \( M_3 \) and \( M_4 \) form a current mirror with an input dc current \( I_{REF} \). For each MOSFET, it is given that \( g_m = 0.005 \text{ S}, \ g_{mb} = 0.0025 \text{ S}, \) and \( r_0 = 50 \text{k} \Omega \).

(a) To simplify the solution for \( i_{o(sc)} \), assume \( r_{01} = r_{02} = \infty \). Show that \( i_{o(sc)}/v_s = i'_{d2} = \frac{I_{REF}}{V_{B1}} = 0.005 \text{ S}. \)

(b) With \( r_{01} = r_{02} = 50 \text{k} \Omega \), show that the output resistance is 
\[ r_{out} = r_{03} || \left[ r_{02} \left( 1 + \frac{r_{01}}{r_{s2}} \right) + r_{02} \right] = 49.87 \text{k} \Omega \]

(c) Show that 
\[ \frac{v_{o(oc)}}{v_s} = - \left[ \frac{i_{o(sc)}}{v_s} \right] \times r_{out} \left| r_{03} = -249.3. \]