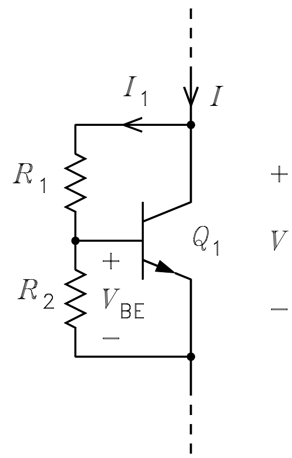
ECE3050 – Assignment 16

1. The figure shows a V_{BE} multiplier.



   (a) By writing a single node equation at the base node of Q_1, show that

$$I_1 = I_B + \frac{V_{BE}}{R_2} \Longrightarrow V = V_{BE}\left(1 + \alpha\frac{R_1}{R_2}\right) + \frac{I}{1+\beta}R_1$$

   Hint: Consider the emitter node to be the datum or zero voltage reference node and use

$$I_B = \frac{I_C}{\beta} = \frac{1}{\beta}(I - I_1) = \frac{1}{\beta}\left(I - \frac{V - V_{BE}}{R_1}\right)$$

   (b) If $\beta \to \infty$, show that the answer for part (a) is the same as the one derived in class.

   (c) For a given I and I_1, show that

$$R_1 = \frac{V - V_{BE}}{I_1} \qquad \text{and} \qquad R_2 = \frac{V_{BE}}{I_1/\alpha - I/\beta}$$

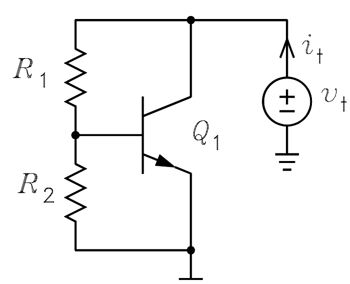
   (d) For $V_T = 25\,\text{mV}$, $I_S = 7.5 \times 10^{-15}\,\text{A}$, $\beta = 99$, and $I = 5\,\text{mA}$, solve for R_1 and R_2 if $V = 2.4\,\text{V}$ and $I_1 = 0.1I$. Answers: $R_1 = 3.44\,\text{k}\Omega$, $R_2 = 1.49\,\text{k}\Omega$.

   (e) Calculate the new values of R_1 and R_2 if the base current in the BJT is neglected. Answers: $R_1 = 3.44\,\text{k}\Omega$, $R_2 = 1.36\,\text{k}\Omega$.

   (f) Repeat part (d) for $I_1 = 0.9I$. Answers: $R_1 = 395\,\Omega$, $R_2 = 137\,\Omega$.

   (g) Calculate the new values of R_1 and R_2 if the base current in the BJT is neglected. Answers: $R_1 = 395\,\Omega$, $R_2 = 138\,\Omega$.

2. The figure shows a circuit for calculating the small-signal resistance of the V_{BE} multiplier. It is given by $r = v_t/i_t$.



(a) If r_0 is neglected in the small-signal model, use superposition of v_t, i_c', and i_b to show that

$$\begin{aligned} i_t &= \frac{v_t}{R_1+R_2} + i_c' + i_b \frac{R_2}{R_1+R_2} \\ &= \frac{v_t}{R_1+R_2} + \alpha i_e' + \frac{i_e'}{1+\beta}\frac{R_2}{R_1+R_2} \end{aligned}$$

(b) With r_0 neglected, replace the BJT with its simplified T model and show that i_e' is given by

$$i_e' = \frac{v_{tb}}{r_e'} = \frac{v_t R_2}{R_1+R_2}\frac{1}{r_e'}$$

where r_e' is calculated with $R_{tb} = R_1 \| R_2$.

(c) Combine the above results to show that

$$\begin{aligned} i_t &= \frac{v_t}{R_1+R_2}\left[1 + \frac{R_2}{r_e'}\left(\alpha + \frac{1}{1+\beta}\frac{R_2}{R_1+R_2}\right)\right] \\ &= \frac{v_t}{R_1+R_2}\left[1 + \frac{\alpha R_2}{r_e'}\left(1 + \frac{R_2}{\beta\left(R_1+R_2\right)}\right)\right] \end{aligned}$$

(d) Add r_0 to the circuit and show that the small-signal resistance reduces to

$$r = \frac{v_t}{i_t} = r_0 \| r_c \qquad \text{where} \qquad r_c = \frac{R_1+R_2}{1 + \dfrac{\alpha R_2}{r_e'}\left(1 + \dfrac{R_2}{\beta\left(R_1+R_2\right)}\right)}$$

(e) If $\beta \to \infty$, show that the expression for r_c reduces to

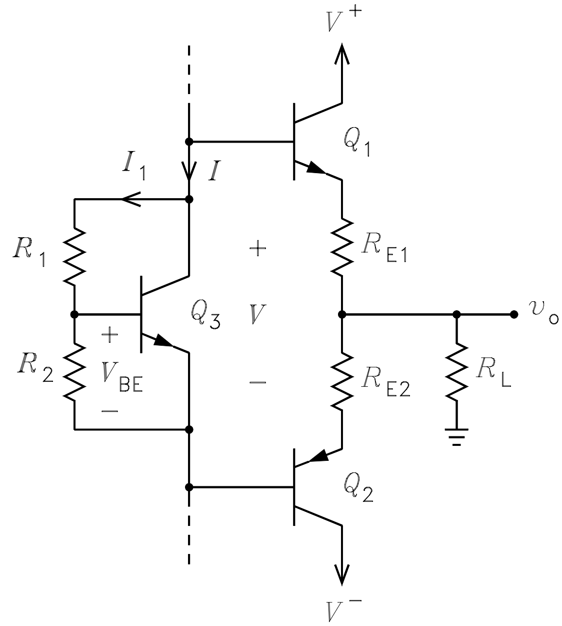
$$r_c = \frac{R_1+R_2}{1+R_2/r_e} = \frac{R_1+R_2}{1+g_m R_2}$$

(f) For the values in Problem 1 part (e) and $r_x = 40\,\Omega$, calculate r_c. Answer: $r_c = 55.1\,\Omega$.

(g) For the values in Problem 1 part (f) and $r_x = 40\,\Omega$, calculate r_c. Answer: $r_c = 144\,\Omega$.

(h) For the lowest small-signal resistance of the V_{BE} multiplier, would it be better to bias the BJT at a small fraction of the current I or at a large fraction? Answer: A large fraction. However, the larger the BJT current, the more sensitive the value of R_2 becomes to the β of the transistor.

3. The figure shows a V_{BE} multiplier used as a bias source for a complementary CC amplifier. The transistor parameters are the same as those specified in Problem **??**.



(a) For $v_O = 0$, solve for V_{BE1} and V_{EB2} for $I_{E1} = I_{E2} = 5\,\text{mA}$. Answer: $V_{BE1} = V_{EB2} = 0.68\,\text{V}$.

(b) For $v_O = 0$, solve for R_{E1} and R_{E2} so the voltage across each is equal to 0.1 of the voltage calculated in part (a). Answer: $R_{E1} = R_{E2} = 13.6\,\Omega$.

(c) Calculate the required voltage V across the V_{BE} multiplier. Answer: $V = 1.50\,\text{V}$.

(d) If $I = 3\,\text{mA}$ and $I_1 = 0.1I$, calculate V_{BE3}, R_1, and R_2 in the V_{BE} multiplier. Answers: $V_{BE3} = 0.665\,\text{V}$, $R_1 = 2.77\,\text{k}\Omega$, $R_2 = 2.72\,\text{k}\Omega$.

(e) If $V_A = 70\,\text{V}$ for Q_3, calculate r_0, r_c, and the small-signal resistance r of the V_{BE} multiplier. Answer: $r_0 = 26.5\,\text{k}\Omega$, $r'_e = 37.3\,\Omega$, $r_c = 75.1\,\Omega$, $r = 74.9\,\Omega$.

(f) If $R_L = 100\,\Omega$, the V_{BE} multiplier can be approximated by an ac short circuit, and Q_1 and Q_2 have the parameters $r_0 = 40\,\text{k}\Omega$ and $r_x = 10\,\Omega$, solve for the voltage gain of the CC stage. Answer: $A_v = 0.914$.