ECE3050 Assignment 18

1. Sketch and label the Bode magnitude and phase plots for the transfer functions given. Use log-log scales for the magnitude plots and linear-log scales for the phase plots. On the magnitude plots, label the slopes of all asymptotes in dec/dec, label the break frequencies in rad/s, label the gain magnitude on all zero-slope asymptotes, and label the approximate gain magnitude for the actual plot at the break frequencies. You can check your plots with a convenient computer program, e.g. *PSpice*, Mathcad, MatLab, etc., to obtain computer generated plots. An example *PSpice* deck is given for one of the transfer functions. The output voltage is `V(2)`, i.e. the voltage at node 2. It is obtained by the `LAPLACE` statement, which multiplies the voltage `V(1)` by the transfer function after the word `LAPLACE`. The y-axis in the *PSpice* Probe graphics routine must be changed to a log scale to see the correct slopes on the Bode magnitude plot. The number of decades displayed on the y-axis should be changed to no more than 3 or 4 to get the best looking plot.

$$T(s) = 10\frac{1}{1+s/100} \qquad T(s) = 10\frac{s/100}{1+s/100}$$

$$T(s) = \frac{10}{(1+s/100)(1+s/1000)} \qquad T(s) = 10\frac{(s/1000)^2}{(1+s/100)(1+s/10000)}$$

$$T(s) = \frac{10}{(s/100)^2 + 2(s/100) + 1} \qquad T(s) = 10\frac{(s/100)^2}{(s/100)^2 + 2(s/100) + 1}$$

$$T(s) = 100\frac{2(s/100)}{(s/100)^2 + 2(s/100) + 1} \qquad T(s) = 10\frac{(s/100)^2 + 1}{(s/100)^2 + 2(s/100) + 1}$$

$$T(s) = 10\frac{1}{(s/100)^2 + \sqrt{2}(s/100) + 1} \qquad T(s) = 10\frac{(s/100)^2}{(s/100)^2 + \sqrt{2}(s/100) + 1}$$

$$T(s) = 100\frac{\sqrt{2}(s/100)}{(s/100)^2 + \sqrt{2}(s/100) + 1} \qquad T(s) = 10\frac{(s/100)^2 + 1}{(s/100)^2 + \sqrt{2}(s/100) + 1}$$

$$T(s) = 10\frac{1}{(s/100)^2 + 0.5(s/100) + 1} \qquad T(s) = 10\frac{(s/100)^2}{(s/100)^2 + 0.5(s/100) + 1}$$

$$T(s) = 100\frac{0.5(s/100)}{(s/100)^2 + 0.5(s/100) + 1} \qquad T(s) = 10\frac{(s/100)^2 + 1}{(s/100)^2 + 0.5(s/100) + 1}$$

$$T(s) = 10\frac{s/10 - 1}{s/10 + 1} \qquad T(s) = 10\frac{(s/100)^2 - \sqrt{2}(s/100) + 1}{(s/100)^2 + \sqrt{2}(s/100) + 1}$$

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EXAMPLE TRANSFER FUNCTION BODE PLOT
VS 1 0 AC 1
E1 2 0 LAPLACE {V(1)}={10*PWR(S/100,2)/(PWR(S/100,2)+SQRT(2)*S/100+1)}
.AC DEC 50 1 100K
.PROBE
.END
```

2. The figure shows an *RLC* circuit. Show that the voltage gain transfer function is of the form

$$T(s) = \frac{(s/\omega_0)^2}{(s/\omega_0)^2 + s/(Q\omega_0) + 1}$$

where you must give the equations for ω_0 and Q. For $Q = 0.5$, show that the transfer function becomes

$$T(s) = \frac{(s/\omega_0)^2}{(s/\omega_0 + 1)^2}$$

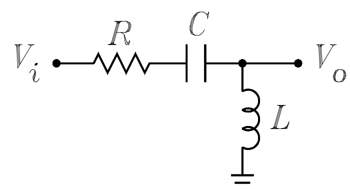
For $Q < 0.5$, show that the transfer function becomes

$$T(s) = \frac{s/\omega_1}{s/\omega_1 + 1} \frac{s/\omega_2}{s/\omega_2 + 1}$$

where

$$\omega_{1,2} = \omega_0 \left[\frac{1}{2Q} \pm \sqrt{\left(\frac{1}{2Q}\right)^2 - 1} \right]$$

Sketch the Bode magnitude and phase plots as a function of ω for the cases $Q < 0.5$, $Q = 0.5$, and $Q > 0.5$. Label the slopes in dec/dec on the magnitude plot and label all break frequencies in the asymptotes.



3. Repeat problem 2 for the circuit given. Show that the transfer function is given by

$$T(s) = \frac{s/(Q\omega_0)}{(s/\omega_0)^2 + s/(Q\omega_0) + 1}$$

For $Q = 0.5$, show that the transfer function becomes

$$T(s) = \frac{s/\omega_0}{(s/\omega_0 + 1)^2}$$

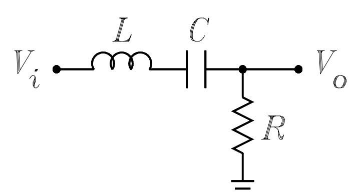
For $Q < 0.5$, show that the transfer function becomes

$$T(s) = \sqrt{\frac{\omega_1}{\omega_2}} \frac{s/\omega_1}{s/\omega_1 + 1} \frac{1}{s/\omega_2 + 1}$$

where

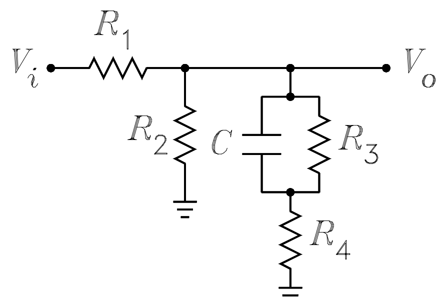
$$\omega_{1,2} = \omega_0 \left[\frac{1}{2Q} \pm \sqrt{\left(\frac{1}{2Q}\right)^2 - 1} \right]$$

Sketch the Bode magnitude and phase plots as a function of ω for the cases $Q < 0.5$, $Q = 0.5$, and $Q > 0.5$. Label the slopes in dec/dec on the magnitude plot and label all break frequencies in the asymptotes.



4. Show that the voltage gain transfer function for the circuit is given by

$$T(s) = \frac{R_2 \| (R_3 + R_4)}{R_1 + R_2 \| (R_3 + R_4)} \frac{1 + R_3 \| R_4 C s}{1 + (R_1 \| R_2 + R_4) \| R_3 C s}$$

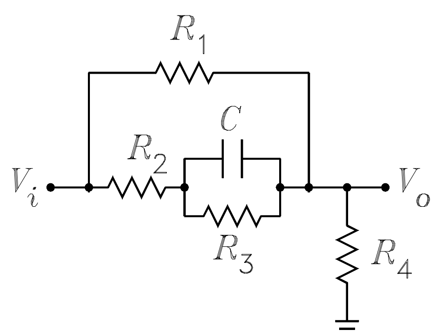


Show that the input impedance is given by

$$Z_{in} = [R_1 + R_2 \| (R_3 + R_4)] \frac{1 + [R_3 \| (R_1 \| R_2 + R_4)] C s}{1 + [R_3 \| (R_2 + R_4)] C s}$$

5. Show that the voltage gain transfer function for the circuit is given by

$$T(s) = \frac{R_4}{R_1 \| (R_2 + R_3) + R_4} \frac{1 + (R_1 + R_2) \| R_3 C s}{1 + (R_1 \| R_4 + R_2) \| R_3 C s}$$



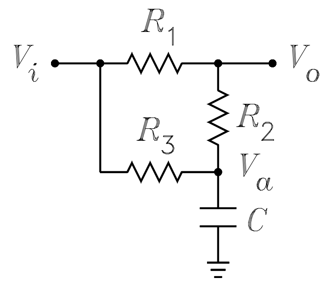
Show that the input impedance is given by

$$Z_{in} = [R_1 \| (R_2 + R_3) + R_4] \frac{1 + [R_3 \| (R_1 \| R_4 + R_2)] C s}{1 + [R_3 \| (R_1 + R_2)] C s}$$

6. Solve for the transfer function for V_o/V_i for the circuit below. The short cut method we covered in class does not work with this circuit. However, the short cut method can be used to solve for V_a/V_i. Once this is obtained, superposition of V_i and V_a can be used to solve for V_o. This eliminates writing node equations, but there is some algebra involved in combining terms to put the transfer function into the ratio of two polynomials. Sketch the Bode magnitude plot, label the break frequencies, and label the gain on the zero-slope asymptotes. Answer:

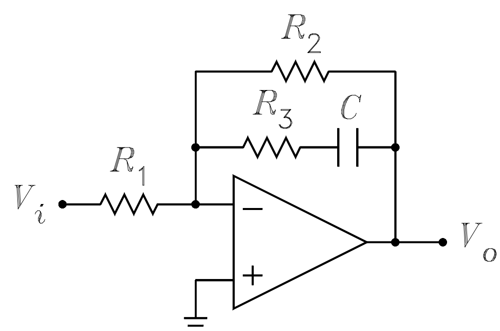
$$\frac{V_o}{V_i} = \frac{1 + [R_2 R_3 / (R_1 + R_2 + R_3)] C s}{1 + [(R_1 + R_2) \| R_3] C s}$$

The element values are to be chosen so that the high-frequency asymptotic gain is 0.05 and the high-frequency asymptotic output resistance (with $V_i = 0$) is $100\,\Omega$. The frequency of the zero in the transfer function is to be 100 Hz. If $C = 220\,\mu$F, specify the element values in the circuit and calculate the pole frequency. Answers: $R_1 = 2\,\text{k}\Omega$, $R_2 = 105.26\,\Omega$, $R_3 = 155.36\,\Omega$, $f_p = 5\,\text{Hz}$.



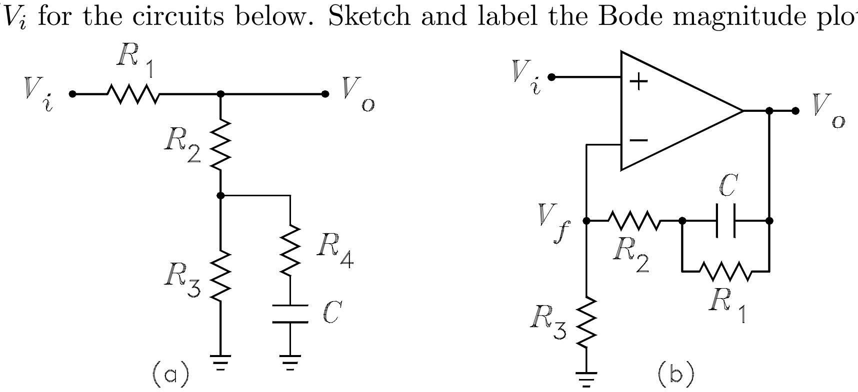
7. Solve for the transfer function for V_o/V_i for the circuit below. Sketch the Bode plot, label the break frequencies, and label the gain on the zero-slope asymptotes. Answer:

$$\frac{V_o}{V_i} = -\frac{Z_F}{R_1} = -\frac{R_2}{R_1}\frac{1 + R_3Cs}{1 + (R_2 + R_3)\,Cs}$$



The circuit is to be designed as a lag-lead compensator for a motor control system. The specifications are low-frequency asymptotic gain: -2, input resistance: $10\,\text{k}\Omega$, pole frequency: $1\,\text{Hz}$, zero frequency: $10\,\text{Hz}$. Specify the element values. Answers: $R_1 = 10\,\text{k}\Omega$, $R_2 = 20\,\text{k}\Omega$, $R_3 = 2222.2\,\Omega$, and $C = 7.1620\,\mu$F.

8. Solve for V_o/V_i for the circuits below. Sketch and label the Bode magnitude plots.



Answers: (a) The transfer function is a low-pass shelving function with a dc gain of

$$K_{dc} = \frac{R_2 + R_3}{R_1 + R_2 + R_3}$$

and a high-frequency gain of

$$K_\infty = \frac{R_2 + R_3 \| R_4}{R_1 + R_2 + R_3 \| R_4}$$

The transfer function is

$$\frac{V_o}{V_i} = \frac{R_2 + R_3}{R_1 + R_2 + R_3} \frac{1 + (R_2 \| R_3 + R_4)\, Cs}{1 + [(R_1 + R_2) \| R_3 + R_4]\, Cs}$$

(b) The transfer function is a high-pass shelving function. The zero-frequency gain is

$$K_{dc} = 1 + \frac{R_1 + R_2}{R_3}$$

The high-frequency gain is

$$K_\infty = 1 + \frac{R_2}{R_3}$$

The transfer function is given by

$$\frac{V_o}{V_i} = \left(\frac{V_f}{V_o}\right)^{-1} = \left(1 + \frac{R_1 + R_2}{R_3}\right) \frac{1 + [(R_2 + R_3) \| R_1]\, Cs}{1 + R_1 Cs}$$

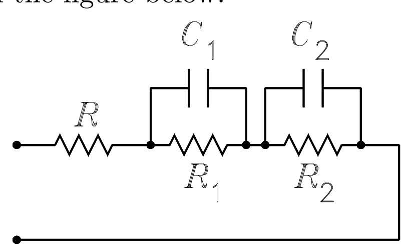
9. It is desired to design a circuit that realizes the following impedance transfer function:

$$Z = 1000 \frac{(1 + s/2\pi f_2)(1 + s/2\pi f_4)}{(1 + s/2\pi f_1)(1 + s/2\pi f_3)}$$

where $f_1 = 10\,\text{Hz}$, $f_2 = 100\,\text{Hz}$, $f_3 = 1\,\text{kHz}$, and $f_4 = 10\,\text{kHz}$.

(a) Sketch the Bode magnitude plot for Z. Note that the impedance starts at $1000\,\Omega$, shelves at $100\,\Omega$, then shelves again at $10\,\Omega$. Label the break frequencies and label the magnitude of the impedance on each zero-slope asymptote.

(b) A possible circuit realization is in the figure below.



At low frequencies, the impedance starts at the value $R + R_1 + R_2$. As frequency is increased, suppose that C_1 becomes a short circuit well before C_2 becomes a short circuit. When C_1 becomes a short, the impedance shelves at the value $R + R_2$. Therefore, C_1 causes both a pole and a zero. As frequency is increased further, C_2 becomes a short and the impedance shelves at the value R. Thus C_2 also causes a pole and a zero. With this information show that the input impedance is approximately given by

$$Z_{in}(s) \simeq (R + R_1 + R_2) \frac{1 + R_1 \| (R + R_2)\, C_1 s}{1 + R_1 C_1 s} \frac{1 + (R_2 \| R)\, C_2 s}{1 + R_2 C_2 s}$$

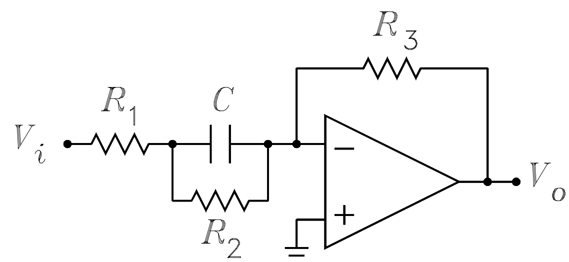
where you consider C_2 to be an open in calculating the effect of C_1 and you consider C_1 to be a short in calculating the effect of C_2. 0Write the three equations for the resistors and solve for their values. Answers: $R = 10\,\Omega$, $R_1 = 900\,\Omega$, and $R_2 = 90\,\Omega$.

(c) Solve for the two time constants for C_1 assuming C_2 is an open circuit. What must be the value of C_1? Answer: The pole time constant is calculated with the inputs open circuited. The zero time constant is calculated with the inputs short circuited. The value of C_1 is $C_1 = 17.68\,\mu\text{F}$.

(d) Solve for the two time constants for C_2 assuming C_1 is a short circuit. What must be the value of C_2? Answer: The pole time constant is calculated with the inputs open circuited. The zero time constant is calculated with the inputs short circuited. The value of C_2 is $C_2 = 1.77\,\mu\text{F}$.

(e) Use SPICE to plot the magnitude of the impedance as a function of frequency. To do this, drive the circuit with an ac current source of 1 A. The voltage across the terminals is the impedance. Use a log scale for the vertical axis to display the correct Bode plot. (`.AC DEC 50 1 100K`)

10. Use the inverting gain formula to solve for the voltage-gain transfer function for the circuit below Sketch and label the Bode magnitude plot.



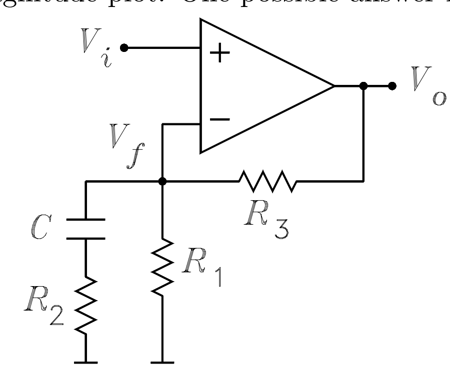
Answer:

$$\frac{V_o}{V_i} = -\frac{R_3}{Z_1} = -\frac{R_3}{R_1 + R_2}\,\frac{1 + R_2 C s}{1 + (R_1 \| R_2)\, C s}$$

11. Using a single $100\,\mu\text{F}$ capacitor, design a single op amp circuit which has the voltage-gain transfer function

$$\frac{V_o}{V_i} = 10\frac{1 + s/10}{1 + s/100}$$

Sketch and label the Bode magnitude plot. One possible answer is the circuit below.



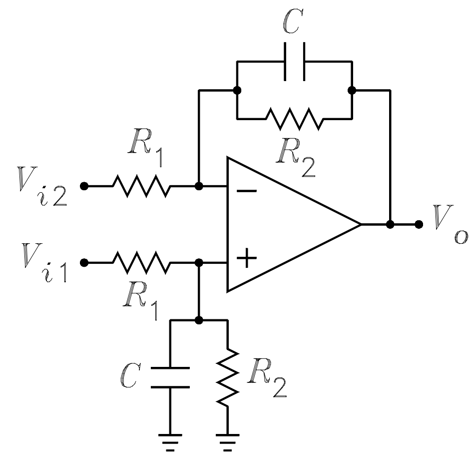
where

$$\frac{V_o}{V_i} = \left(\frac{V_f}{V_o}\right)^{-1} = \left(1 + \frac{R_3}{R_1}\right)\frac{1 + (R_1 \| R_3 + R_2)\, C s}{1 + R_2 C s}$$

The element values are $R_1 = 1\,\text{k}\Omega$, $R_2 = 100\,\Omega$, $R_3 = 9\,\text{k}\Omega$, and $C = 15.92\,\mu\text{F}$.

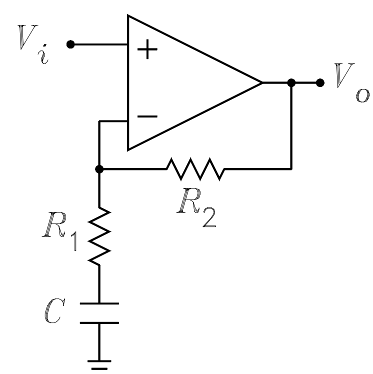
12. For the circuit shown, show that

$$V_o = \frac{R_2}{R_1}\,\frac{V_{i1} - V_{i2}}{1 + R_2 C s}$$



13. For the circuit shown, show that

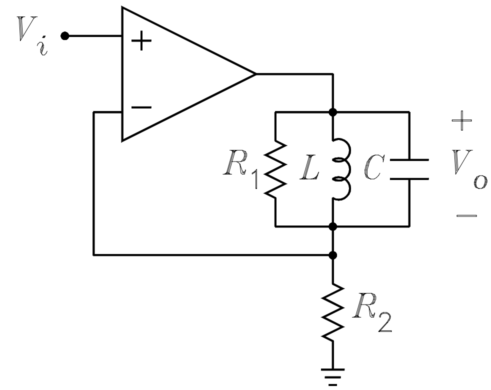
$$\frac{V_o}{V_i} = \frac{1 + (R_1 + R_2)\, C s}{1 + R_1 C s}$$



14. For the circuit shown, show that

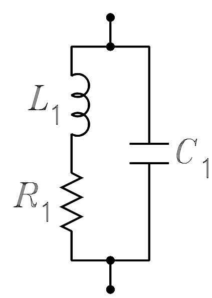
$$\frac{V_o}{V_i} = \frac{R_1}{R_2}\,\frac{(s/\omega_0)}{(s/\omega_0)^2 + (1/Q)\,(s/\omega_0) + 1}$$

where $\omega_0 = 1/\sqrt{LC}$ and $Q = \omega_0 R_1 C$. Sketch and label the Bode magnitude and phase plots for $Q = 0.5$, $Q = 1$, and $Q = 2$.



15. For the circuit shown, show that the impedance is real at the frequency

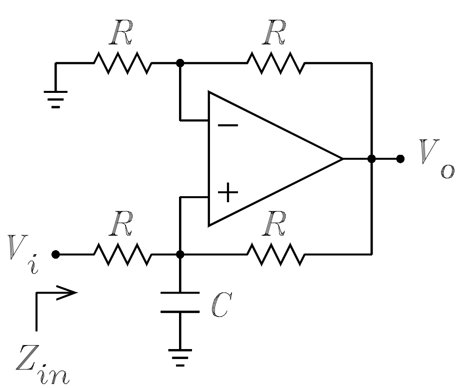
$$\omega = \sqrt{\frac{R_1^2}{L_1^2} + \frac{1}{L_1 C_1}}$$



16. For the circuit shown, show that

$$\frac{V_o}{V_i} = \frac{2}{RCs}$$

Show that the equivalent circuit for Z_{in} is the resistor $R_1 = R$ in parallel with the negative inductor $L_1 = -R^2 C$.

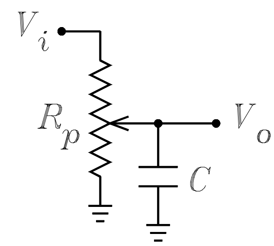


17. For the potentiometer circuit shown, let the resistance below the wiper be xR_p and the resistance above the wiper be $(1-x)\,R_p$.Show that

$$\frac{V_o}{V_i} = \frac{x}{1 + x\,(1-x)\,R_p C s}$$

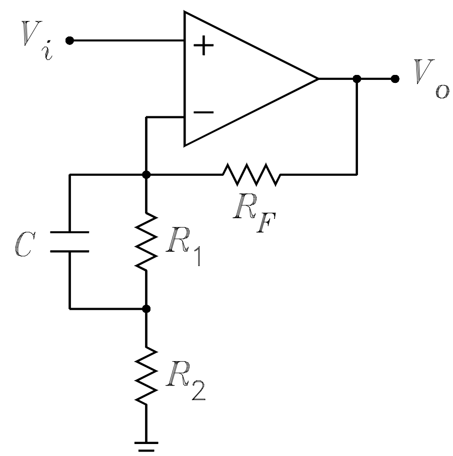
Show that the circuit has a worst-case minimum bandwidth when $x = 0.5$ and that the corresponding pole frequency is given by

$$f_{pole} = \frac{1}{\pi R_p C}$$

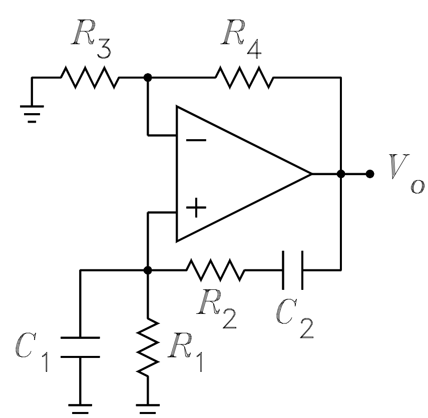


18. For the circuit shown, show that

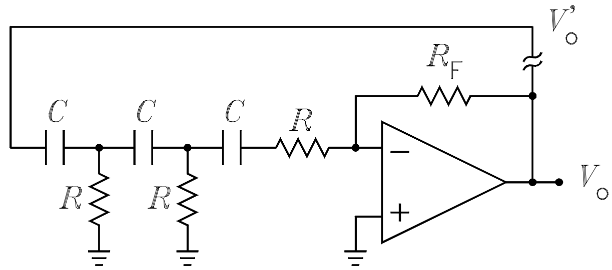
$$\frac{V_o}{V_i} = \left(1 + \frac{R_F}{R_1 + R_2}\right) \frac{1 + [R_1 \| (R_2 + R_F)] Cs}{1 + (R_1 \| R_2) Cs}$$



19. The circuit shows a Wein bridge oscillator. If $R_1 = R_2$, $C_1 = 0.1\,\mu$F, $C_2 = 0.22\,\mu$F, and $R_4 = 10\,\text{k}\Omega$, specify R_1, R_2, and R_3 for the circuit to have stable oscillations at $f = 1000\,\text{Hz}$. Answers: $R_1 = R_2 = 1073\,\Omega$ and $R_3 = 6875\,\Omega$.



20. The figure shows a phase-shift oscillator with the feedback loop broken.

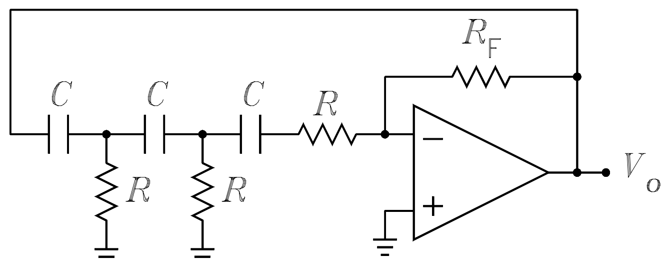


By writing node equations, it can be shown that the loop-gain transfer function is given by

$$\frac{V_o'}{V_o} = -\frac{R_F}{R} \frac{(RCs)^3}{(RCs)^3 + 6(RCs)^2 + 5(RCs) + 1}$$

(a) To solve for the frequency of oscillation, what do you set V_o'/V_o equal to? Answer: $1\angle 0\,°$. (b) Use the transfer function to solve for the frequency of oscillation. Answer: $f_0 = 1/\left(2\pi\sqrt{6}RC\right)$. (c) Use the transfer function to solve for value of R_F/R in order for $|V_o'/V_o| = 1\angle 0\,°$ at $f = f_0$. Answer: $R_F/R_1 = 29$.

21. The figure shows a phase shift oscillator. If $C = 0.1\,\mu$F, specify R and R_F for the circuit to have stable oscillations at $f = 250\,$Hz. Answers: $R = 3249\,$kΩ and $R_F = 94.21\,$kΩ.



22. The loop-gain transfer function of a particular oscillator circuit is given by

$$\frac{V_o}{V_o'} = K \frac{s}{(s/100)^2 + 0.5\,(s/100) + 1}$$

At what frequency does the circuit oscillate and what must be the value of K for stable oscillations? Answers: $f = 15.9\,$Hz and $K = 0.005$.