Effects of Op-Amp Finite Gain and Bandwidth

Open-Loop Transfer Function

In our analysis of op-amp circuits this far, we have considered the op-amps to have an infinite gain and an infinite bandwidth. This is not true for physical op-amps. In this section, we examine the effects of a non-infinite gain and non-infinite bandwidth on the inverting and the non-inverting amplifier circuits. Fig. 1 shows the circuit symbol of an op-amp having an open-loop voltage-gain transfer function $A(s)$. The output voltage is given by

$$V_o = A(s)(V_+ - V_-) \tag{1}$$

where complex variable notation is used. We assume here that $A(s)$ can be modeled by a single-pole low-pass transfer function of the form

$$A(s) = \frac{A_0}{1 + s/\omega_0} \tag{2}$$

where A_0 is the dc gain constant and ω_0 is the pole frequency. Most general purpose op-amps have a voltage-gain transfer function of this form for frequencies such that $|A(j\omega)| \geq 1$.

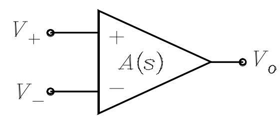

Figure 1: Op-amp symbol.

Gain-Bandwidth Product

Figure 2 shows the Bode magnitude plot for $A(j\omega)$. The radian gain-bandwidth product is defined as the frequency ω_x for which $|A(j\omega)| = 1$. It is given by

$$\omega_x = \omega_0\sqrt{A_0^2 - 1} \simeq A_0\omega_0 \tag{3}$$

where we assume that $A_0 >> 1$. This equation illustrates why ω_x is called a gain-bandwidth product. It is given by the product of the dc gain constant A_0 and the radian bandwidth ω_0. It is commonly specified in Hz with the symbol f_x, where $f_x = \omega_x/2\pi$. Many general purpose op-amps have a gain-bandwidth product $f_x \simeq 1\,\text{MHz}$ and a dc gain constant $A_0 \simeq 2 \times 10^5$. It follows from Eq. (3) that the corresponding pole frequency in the voltage-gain transfer function for the general purpose op-amp is $f_0 \simeq 1 \times 10^6/\left(2 \times 10^5\right) = 5\,\text{Hz}$.

Non-Inverting Amplifier

Figure 3 shows the circuit diagram of a non-inverting amplifier. For this circuit, we can write by inspection

$$V_o = A(s)(V_i - V_-) \tag{4}$$

$$V_- = V_o\frac{R_1}{R_1 + R_F} \tag{5}$$

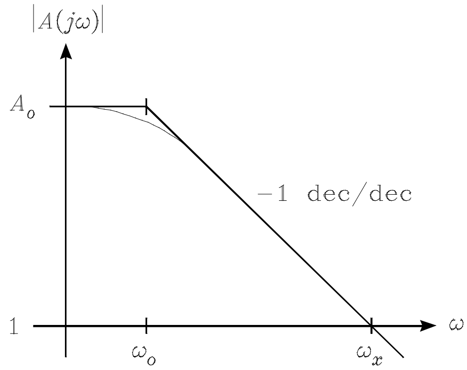

Figure 2: Bode plot of $|A(j\omega)|$.

Simultaneous solution for the voltage-gain transfer function yields

$$\frac{V_o}{V_i} = \frac{A(s)}{1 + A(s) R_1 / (R_1 + R_F)} = \frac{1 + R_F/R_1}{1 + (1 + R_F/R_1)/A(s)} \tag{6}$$

For $s = j\omega$ and $|(1 + R_F/R_1)/A(j\omega)| << 1$, this reduces to $V_o/V_i \simeq (1 + R_F/R_1)$. This is the gain which would be predicted if the op-amp is assumed to be ideal.

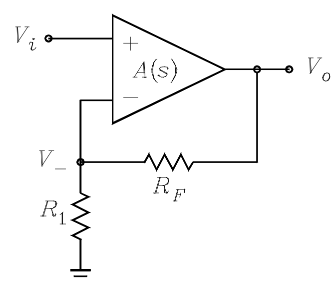

Figure 3: Non-inverting amplifier.

When Eq. (2) is used for $A(s)$, it is straightforward to show that Eq. (6) can be written

$$\frac{V_o}{V_i} = \frac{A_{0f}}{1 + s/\omega_{0f}} \tag{7}$$

where A_{0f} is the gain constant with feedback and ω_{0f} is the radian pole frequency with feedback. These are given by

$$A_{0f} = \frac{A_0}{1 + A_0 R_1 / (R_1 + R_F)} = \frac{1 + R_F/R_1}{1 + (1 + R_F/R_1)/A_0} \tag{8}$$

$$\omega_{0f} = \omega_0 \left(1 + \frac{A_0 R_1}{R_1 + R_F}\right) \tag{9}$$

It follows from these two equations that the radian gain-bandwidth product of the non-inverting amplifier with feedback is given by $A_{0f}\omega_{0f} = A_0\omega_0 = \omega_x$. This is the same as for the op-amp without feedback. Fig. 4 shows the Bode magnitude plots for both V_o/V_i and $A(j\omega)$. The figure shows that the break frequency on the plot for V_o/V_i lies on the negative-slope asymptote of the plot for $A(j\omega)$.

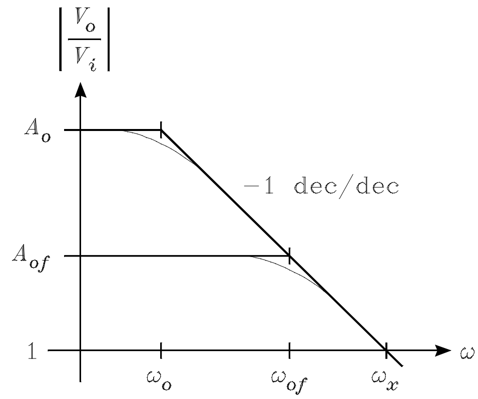

Figure 4: Bode plot for $|V_o/V_i|$.

Example 1 *At very low frequencies, an op-amp has the frequency independent open-loop gain* $A(s) = A_0 = 2 \times 10^5$. *The op-amp is to be used in a non-inverting amplifier. The theoretical gain is calculated assuming that the op-amp is ideal. What is the highest theoretical gain that gives an error between the theoretical gain and the actual gain that is less than 1%?*

Solution. The theoretical gain is given by $(1 + R_F/R_1)$. The actual gain is always less than the theoretical gain. For an error less than 1%, we can use Eq. (6) to write

$$1 - 0.01 < \frac{1}{1 + (1 + R_F/R_1) / (2 \times 10^5)}$$

This can be solved for the upper bound on the theoretical gain to obtain

$$1 + \frac{R_F}{R_1} < 2 \times 10^5 \left(\frac{1}{0.99} - 1 \right) = 2020$$

Example 2 *An op-amp has a gain-bandwidth product of* 1 MHz. *The op-amp is to be used in a non-inverting amplifier circuit. Calculate the highest gain that the amplifier can have if the half-power or* $-3\,\mathrm{dB}$ *bandwidth is to be* 20 kHz *or more.*

Solution. The minimum bandwidth occurs at the highest gain. For a bandwidth of 20 kHz, we can write $A_{0f} \times 20 \times 10^3 = 10^6$. Solution for A_{0f} yields $A_{0f} = 50$.

Example 3 *Two non-inverting op-amp amplifiers are operated in cascade. Each amplifier has a gain of* 10. *If each op-amp has a gain-bandwidth product of* 1 MHz, *calculate the half-power or* $-3\,\mathrm{dB}$ *bandwidth of the cascade amplifier.*

Solution. Each amplifier by itself has a pole frequency of $10^6/10 = 100\,\mathrm{kHz}$, corresponding to a radian frequency $\omega_{0f} = 2\pi \times 100{,}000$. The cascade combination has the voltage-gain transfer function given by

$$\frac{V_o}{V_i} = 100 \left(\frac{1}{1 + s/\omega_{0f}} \right)^2$$

The half-power frequency is obtained by setting $s = j\omega$ and solving for the frequency for which $|V_o/V_i|^2 = 100^2/2$. If we let $x = \omega/\omega_{0f}$, the resulting equation is

$$100^2 \left(\frac{1}{1 + x^2} \right)^2 = \frac{100^2}{2}$$

This equation reduces to $1+x^2 = \sqrt{2}$. Solution for x yields $x = 0.644$. It follows that the half-power frequency is $0.644 \times 100\,\text{kHz} = 64.4\,\text{kHz}$.

Inverting Amplifier

Figure 5(a) shows the circuit diagram of an inverting amplifier. Fig. 5(b) shows an equivalent circuit which can be used to solve for V_-. By inspection, we can write

$$V_o = -A(s)\,V_- \tag{10}$$

$$V_- = \left(\frac{V_i}{R_1} + \frac{V_o}{R_F}\right)(R_1 \| R_F) \tag{11}$$

These equations can be solved for the voltage-gain transfer function to obtain

$$\frac{V_o}{V_i} = \frac{-(1/R_1)\,A(s)\,(R_1\|R_F)}{1+(1/R_F)\,A(s)\,(R_1\|R_F)} = \frac{-R_F/R_1}{1+(1+R_F/R_1)/A(s)} \tag{12}$$

For $s = j\omega$ and $|(1+R_F/R_1)/A(j\omega)| << 1$, the voltage-gain transfer function reduces to $V_o/V_i \simeq -R_F/R_1$. This is the gain which would be predicted if the op-amp is assumed to be ideal.

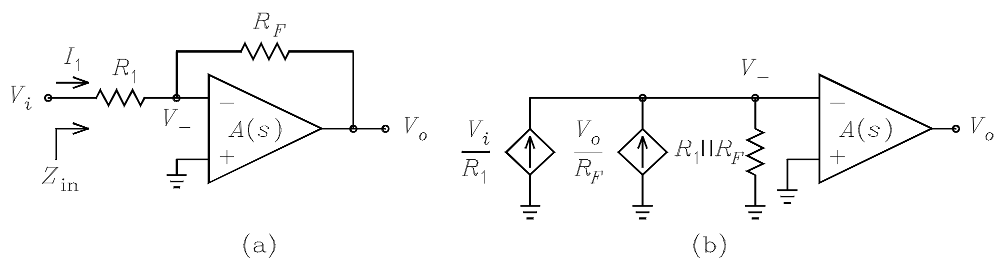

Figure 5: (a) Inverting amplifier. (b) Equivalent circuit for calculating V_o.

When Eq. (2) is used for $A(s)$, it is straightforward to show that the voltage-gain transfer function reduces to

$$\frac{V_o}{V_i} = \frac{-A_{0f}}{1+s/\omega_{0f}} \tag{13}$$

where A_{0f} is the gain constant with feedback and ω_{0f} is the radian pole frequency with feedback. These are given by

$$A_{0f} = \frac{(1/R_1)\,A_0\,(R_1\|R_F)}{1+(1/R_F)\,A_0\,(R_1\|R_F)} = \frac{R_F/R_1}{1+(1+R_F/R_1)/A_0} \tag{14}$$

$$\omega_{0f} = \omega_0\left(1 + A_0\frac{R_1\|R_F}{R_F}\right) = \omega_0\left(1 + A_0\frac{R_1}{R_1+R_F}\right) \tag{15}$$

Note that A_{0f} is defined here as a positive quantity so that the negative sign for the inverting gain is retained in the transfer function for V_o/V_i.

Let the radian gain-bandwidth product of the inverting amplifier with feedback be denoted by ω_x'. It follows from Eqs. (14) and (15) that this is given by $\omega_x' = A_{0f}\omega_{0f} = \omega_x R_F/(R_F+R_1)$. This is less than the gain-bandwidth product of the op-amp without feedback by the factor $R_F/(R_F+R_1)$. Fig. 6 shows the Bode magnitude plots for V_o/V_i and for $A(j\omega)$. The frequency labeled ω_{0f}' is the break frequency for the non-inverting amplifier with the same gain magnitude as the inverting amplifier. The non-inverting amplifier with the same gain has a bandwidth that is greater by the factor $(1+R_1/R_F)$. The bandwidth of the inverting and the non-inverting amplifiers is approximately the same if $R_1/R_F << 1$. This is equivalent to the condition that $A_{0f} >> 1$.

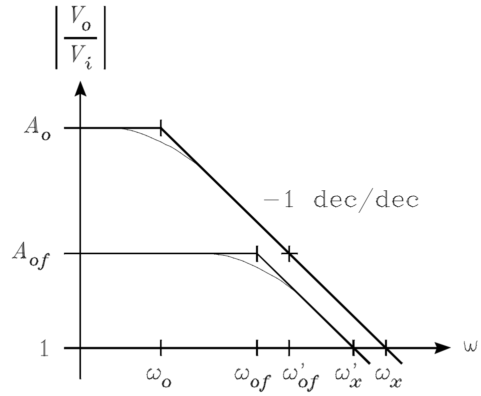

Figure 6: Bode plot for $|V_o/V_i|$.

Example 4 *An op-amp has a gain-bandwidth product of* 1 MHz*. Compare the bandwidths of an inverting and a non-inverting amplifier which use the op-amp for* $A_{0f} = 1$*,* 2*,* 5*, and* 10*.*

Solution. The non-inverting amplifier has a bandwidth of f_x/A_{0f}. The inverting amplifier has a bandwidth of $(f_x/A_{0f}) \times R_F/(R_1 + R_F)$. If we approximate A_{0f} of the inverting amplifier by $A_{0f} \simeq R_F/R_1$, its bandwidth reduces to $f_x/(1 + A_{0f})$. The calculated bandwidths of the two amplifiers are summarized in the following table.

A_{0f}	Non-Inverting	Inverting
1	1 MHz	500 kHz
2	500 kHz	333 kHz
5	200 kHz	167 kHz
10	100 kHz	91 kHz

For the case of the ideal op-amp, the V_- input to the inverting amplifier is a virtual ground so that the input impedance Z_{in} is resistive and equal to R_1. For the op-amp with finite gain and bandwidth, the V_- terminal is not a virtual ground so that the input impedance differs from R_1. We use the circuit in Fig. 5(a) to solve for the input impedance as follows:

$$Z_{in} = \frac{V_i}{I_1} = \frac{V_i}{(V_i - V_-)/R_1} = \frac{R_1}{1 - (V_-/V_o)(V_o/V_i)} \tag{16}$$

To put this into the desired form, we let $V_-/V_o = -1/A(s)$ and use Eq. (12) for V_o/V_i. The equation for Z_{in} reduces to

$$Z_{in} = R_1 + \frac{R_F}{1 + A(s)} = R_1 + \left[\frac{1}{R_F} + \left(\frac{R_F}{A_0} + \frac{R_F}{A_0\omega_0}s\right)^{-1}\right]^{-1} \tag{17}$$

where Eq. (2) has been used. It follows from this equation that Z_{in} consists of the resistor R_1 in series with an impedance that consists of the resistor R_F in parallel with the series combination of a resistor R_2 and an inductor L given by

$$R_2 = \frac{R_F}{A_0} \tag{18}$$

$$L = \frac{R_F}{A_0\omega_0} \tag{19}$$

The equivalent circuit for Z_{in} is shown in Fig. 7(a). If $A_0 \to \infty$, it follows that $R_2 \to 0$ and $L_2 \to 0$ so that $Z_{in} \to R_1$. The impedance transfer function for Z_{in} is of the form of a high-pass shelving transfer function given by

$$Z_{in}(s) = R_{DC} \frac{1 + s/\omega_z}{1 + s/\omega_p} \tag{20}$$

where R_{DC} is the dc resistance, ω_p is the pole frequency, and ω_z is the zero frequency. These are given by

$$R_{DC} = R_1 + \frac{R_F}{1 + A_0} \tag{21}$$

$$\omega_p = \frac{R_2 + R_F}{L} = \omega_0 (1 + A_0) \tag{22}$$

$$\omega_z = \frac{R_2 + R_F \| R_1}{L} = \omega_0 \left(1 + \frac{A_0 R_1}{R_1 + R_F}\right) \tag{23}$$

The Bode magnitude plot for Z_{in} is shown in Fig. 7(b).

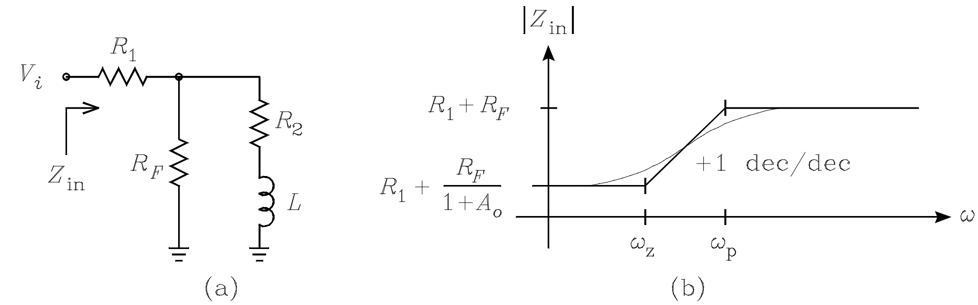

Figure 7: (a) Equivalent input impedance. (b) Bode plot for $|Z_{in}|$.

Example 5 *At very low frequencies, an op-amp has the frequency independent open-loop gain* $A(s) = A_0 = 2 \times 10^5$. *The op-amp is to be used in an inverting amplifier with a gain of* -1000. *What is the required ratio* R_F/R_1*? For the value of* R_F/R_1*, how much larger is the input resistance than* R_1*?*

Solution. By Eq. (12), we have

$$-1000 = -\frac{R_F/R_1}{1 + (1 + R_F/R_1)/(2 \times 10^5)}$$

This can be solved for R_F/R_1 to obtain

$$\frac{R_F}{R_1} = \frac{2 \times 10^5 + 1}{(2 \times 10^5/1000) - 1} = 1005$$

By Eq. (17), the input resistance can be written

$$R_{in} = R_1 \left(1 + \frac{R_F/R_1}{1 + A_0}\right) = R_1 \left(1 + \frac{1005}{1 + 2 \times 10^5}\right) = 1.005 R_1$$

Example 6 *An op-amp has a dc gain* $A_0 = 2\times10^5$ *and a gain bandwidth product* $f_x = 1\,\mathrm{MHz}$*. The op-amp is used in the inverting amplifier of Fig. 5(a). The circuit element values are* $R_1 = 1\,\mathrm{k}\Omega$ *and* $R_F = 100\,\mathrm{k}\Omega$*. Calculate the dc gain of the amplifier, the upper cutoff frequency, and the value of the elements in the equivalent circuit for the input impedance. In addition, calculate the zero and the pole frequencies in Hz for the impedance Bode plot of Fig. 7(b).*

Solution. The dc voltage gain is $-A_{0f}$. Eq. (14) can be used to calculate A_{0f} to obtain

$$A_{0f} = \frac{2\times10^5 \times (1k\|100k)/1k}{1 + 2\times10^5 \times (1k\|100k)/100k} = 99.95$$

By Eqs. (3) and (15), the upper cutoff frequency f_{0f} is given by

$$f_{of} = \frac{\omega_{0f}}{2\pi} = \frac{10^6}{2\times10^5}\left(1 + 2\times10^5 \frac{1k\|100k}{100k}\right) = 9.91\,\mathrm{kHz}$$

The element values in the equivalent circuit of Fig. 7(a) for the input impedance are as follows:

$$R_1 = 1\,\mathrm{k}\Omega,\ R_F = 100\,\mathrm{k}\Omega,\ R_2 = 0.5\,\Omega,\ \text{and}\ \ L = 15.9\,\mathrm{mH}$$

where Eqs. (18) and (19) have been used for R_2 and L. The pole and zero frequencies in the Bode impedance plot are

$$f_p = f_0\left(1 + A_0\right) = \frac{f_x}{A_0}\left(1 + A_0\right) = 1.00\,\mathrm{MHz}$$

$$f_z = f_0\left(1 + A_0\frac{R_1}{R_1 + R_F}\right) = 9.91\,\mathrm{kHz}$$