Effects of Op-Amp Finite Gain and Bandwidth

Open-Loop Transfer Function

In our analysis of op-amp circuits this far, we have considered the op-amps to have an infinite gain and an infinite bandwidth. This is not true for physical op-amps. In this section, we examine the effects of a non-infinite gain and non-infinite bandwidth on the inverting and the non-inverting amplifier circuits. Fig. 1 shows the circuit symbol of an op-amp having an open-loop voltage-gain transfer function $A(s)$. The output voltage is given by

$$V_o = A(s) (V_+ - V_-)$$

where complex variable notation is used. We assume here that $A(s)$ can be modeled by a single-pole low-pass transfer function of the form

$$A(s) = \frac{A_0}{1 + s/\omega_0}$$

where $A_0$ is the dc gain constant and $\omega_0$ is the pole frequency. Most general purpose op-amps have a voltage-gain transfer function of this form for frequencies such that $|A(j\omega)| \geq 1$.

Gain-Bandwidth Product

Figure 2 shows the Bode magnitude plot for $A(j\omega)$. The radian gain-bandwidth product is defined as the frequency $\omega_x$ for which $|A(j\omega)| = 1$. It is given by

$$\omega_x = \omega_0 \sqrt{A_0^2 - 1} \simeq A_0 \omega_0$$

where we assume that $A_0 >> 1$. This equation illustrates why $\omega_x$ is called a gain-bandwidth product. It is given by the product of the dc gain constant $A_0$ and the radian bandwidth $\omega_0$. It is commonly specified in Hz with the symbol $f_x$, where $f_x = \omega_x/2\pi$. Many general purpose op-amps have a gain-bandwidth product $f_x \simeq 1\text{MHz}$ and a dc gain constant $A_0 \simeq 2 \times 10^5$. It follows from Eq. (3) that the corresponding pole frequency in the voltage-gain transfer function for the general purpose op-amp is $f_0 \simeq 1 \times 10^8/(2 \times 10^5) = 5\text{Hz}$.

Non-Inverting Amplifier

Figure 3 shows the circuit diagram of a non-inverting amplifier. For this circuit, we can write by inspection

$$V_o = A(s) (V_i - V_-)$$

$$V_- = V_o \frac{R_1}{R_1 + R_F}$$

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Simultaneous solution for the voltage-gain transfer function yields

\[
\frac{V_o}{V_i} = \frac{A(s)}{1 + A(s) R_1/(R_1 + R_F)} = \frac{1 + R_F/R_1}{1 + (1 + R_F/R_1)/A(s)} \tag{6}
\]

For \( s = j\omega \) and \(|(1 + R_F/R_1)/A(j\omega)| << 1\), this reduces to \( V_o/V_i \simeq (1 + R_F/R_1) \). This is the gain which would be predicted if the op-amp is assumed to be ideal.

When Eq. (2) is used for \( A(s) \), it is straightforward to show that Eq. (6) can be written

\[
\frac{V_o}{V_i} = \frac{A_{0\text{f}}}{1 + s/\omega_{0\text{f}}} \tag{7}
\]

where \( A_{0\text{f}} \) is the gain constant with feedback and \( \omega_{0\text{f}} \) is the radian pole frequency with feedback. These are given by

\[
A_{0\text{f}} = \frac{A_0}{1 + A_0 R_1/(R_1 + R_F)} = \frac{1 + R_F/R_1}{1 + (1 + R_F/R_1)/A_0} \tag{8}
\]

\[
\omega_{0\text{f}} = \omega_0 \left(1 + \frac{A_0 R_1}{R_1 + R_F}\right) \tag{9}
\]

It follows from these two equations that the radian gain-bandwidth product of the non-inverting amplifier with feedback is given by \( A_{0\text{f}}\omega_{0\text{f}} = A_0\omega_0 = \omega_x \). This is the same as for the op-amp without feedback. Fig. 4 shows the Bode magnitude plots for both \( V_o/V_i \) and \( A(j\omega) \). The figure shows that the break frequency on the plot for \( V_o/V_i \) lies on the negative-slope asymptote of the plot for \( A(j\omega) \).
Example 1 At very low frequencies, an op-amp has the frequency independent open-loop gain 

\[ A(s) = A_0 = 2 \times 10^5 \]. The op-amp is to be used in a non-inverting amplifier. The theoretical gain is calculated assuming that the op-amp is ideal. What is the highest theoretical gain that gives an error between the theoretical gain and the actual gain that is less than 1%?

Solution. The theoretical gain is given by \( 1 + R_F/R_1 \). The actual gain is always less than the theoretical gain. For an error less than 1%, we can use Eq. (6) to write

\[
1 - 0.01 < \frac{1}{1 + (1 + R_F/R_1)/(2 \times 10^5)}
\]

This can be solved for the upper bound on the theoretical gain to obtain

\[
1 + \frac{R_F}{R_1} < 2 \times 10^5 \left( \frac{1}{0.99} - 1 \right) = 2020
\]

Example 2 An op-amp has a gain-bandwidth product of 1MHz. The op-amp is to be used in a non-inverting amplifier circuit. Calculate the highest gain that the amplifier can have if the half-power or \(-3\)dB bandwidth is to be 20kHz or more.

Solution. The minimum bandwidth occurs at the highest gain. For a bandwidth of 20kHz, we can write \( A_{0f} \times 20 \times 10^3 = 10^6 \). Solution for \( A_{0f} \) yields \( A_{0f} = 50 \).

Example 3 Two non-inverting op-amp amplifiers are operated in cascade. Each amplifier has a gain of 10. If each op-amp has a gain-bandwidth product of 1MHz, calculate the half-power or \(-3\)dB bandwidth of the cascade amplifier.

Solution. Each amplifier by itself has a pole frequency of \( 10^6/10 = 100 \)kHz, corresponding to a radian frequency \( \omega_{0f} = 2\pi \times 100,000 \). The cascade combination has the voltage-gain transfer function given by

\[
\frac{V_o}{V_i} = 100 \left( \frac{1}{1 + s/\omega_{0f}} \right)^2
\]

The half-power frequency is obtained by setting \( s = j\omega \) and solving for the frequency for which \( |V_o/V_i|^2 = 100^2/2 \). If we let \( x = \omega/\omega_{0f} \), the resulting equation is

\[
100^2 \left( \frac{1}{1 + x^2} \right)^2 = \frac{100^2}{2}
\]
This equation reduces to $1 + x^2 = \sqrt{2}$. Solution for $x$ yields $x = 0.644$. It follows that the half-power frequency is $0.644 \times 100\text{kHz} = 64.4\text{kHz}$.

**Inverting Amplifier**

Figure 5(a) shows the circuit diagram of an inverting amplifier. Fig. 5(b) shows an equivalent circuit which can be used to solve for $V_-$. By inspection, we can write

\[ V_o = -A(s) V_- \quad (10) \]

\[ V_- = \left( \frac{V_i}{R_1} + \frac{V_o}{R_F} \right) (R_1||R_F) \quad (11) \]

These equations can be solved for the voltage-gain transfer function to obtain

\[ \frac{V_o}{V_i} = -\frac{A_0 f}{1 + s/\omega_{0f}} \quad (12) \]

where $A_0 f$ is the gain constant with feedback and $\omega_{0f}$ is the radian pole frequency with feedback. These are given by

\[ A_0 f = \frac{(1/R_1) A_0 (R_1||R_F)}{1 + (1/R_F) A_0 (R_1||R_F)} = \frac{R_F/R_1}{1 + (1 + R_F/R_1)/A_0} \quad (13) \]

\[ \omega_{0f} = \omega_0 \left( 1 + A_0 \frac{R_1||R_F}{R_F} \right) = \omega_0 \left( 1 + A_0 \frac{R_1}{R_1 + R_F} \right) \quad (14) \]

Note that $A_0 f$ is defined here as a positive quantity so that the negative sign for the inverting gain is retained in the transfer function for $V_o/V_i$.

Let the radian gain-bandwidth product of the inverting amplifier with feedback be denoted by $\omega_x'$. It follows from Eqs. (14) and (15) that this is given by $\omega_x' = A_0 f \omega_{0f} = \omega_0 R_F/(R_F + R_1)$. This is less than the gain-bandwidth product of the op-amp without feedback by the factor $R_F/(R_F + R_1)$. Fig. 6 shows the Bode magnitude plots for $V_o/V_i$ and for $A(j\omega)$. The frequency labeled $\omega_{0f}$ is the break frequency for the non-inverting amplifier with the same gain magnitude as the inverting amplifier. The non-inverting amplifier with the same gain has a bandwidth that is greater by the factor $(1 + R_1/R_F)$. The bandwidth of the inverting and the non-inverting amplifiers is approximately the same if $R_1/R_F << 1$. This is equivalent to the condition that $A_0 f >> 1$. 
Example 4 An op-amp has a gain-bandwidth product of 1 MHz. Compare the bandwidths of an inverting and a non-inverting amplifier which use the op-amp for \( A_{of} = 1, 2, 5, \) and 10.

Solution. The non-inverting amplifier has a bandwidth of \( f_x/A_{of} \). The inverting amplifier has a bandwidth of \( (f_x/A_{of}) \times R_F/(R_1 + R_F) \). If we approximate \( A_{of} \) of the inverting amplifier by \( A_{of} \simeq R_F/R_1 \), its bandwidth reduces to \( f_x/(1 + A_{of}) \). The calculated bandwidths of the two amplifiers are summarized in the following table.

<table>
<thead>
<tr>
<th>( A_{of} )</th>
<th>Non-Inverting</th>
<th>Inverting</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 MHz</td>
<td>500 kHz</td>
</tr>
<tr>
<td>2</td>
<td>500 kHz</td>
<td>333 kHz</td>
</tr>
<tr>
<td>5</td>
<td>200 kHz</td>
<td>167 kHz</td>
</tr>
<tr>
<td>10</td>
<td>100 kHz</td>
<td>91 kHz</td>
</tr>
</tbody>
</table>

For the case of the ideal op-amp, the \( V_- \) input to the inverting amplifier is a virtual ground so that the input impedance \( Z_{in} \) is resistive and equal to \( R_1 \). For the op-amp with finite gain and bandwidth, the \( V_- \) terminal is not a virtual ground so that the input impedance differs from \( R_1 \). We use the circuit in Fig. 5(a) to solve for the input impedance as follows:

\[
Z_{in} = \frac{V_i}{I_1} = \frac{V_i}{(V_i - V_-)/R_1} = \frac{R_1}{1 - (V_-/V_o)/(V_o/V_i)}
\]

(16)

To put this into the desired form, we let \( V_-/V_o = -1/A(s) \) and use Eq. (12) for \( V_o/V_i \). The equation for \( Z_{in} \) reduces to

\[
Z_{in} = R_1 + \frac{R_F}{1 + A(s)} = R_1 + \left[ \frac{1}{R_F} + \left( \frac{R_F}{A_0} + \frac{R_F}{A_0 \omega_0 s} \right)^{-1} \right]^{-1}
\]

(17)

where Eq. (2) has been used. It follows from this equation that \( Z_{in} \) consists of the resistor \( R_1 \) in series with an impedance that consists of the resistor \( R_F \) in parallel with the series combination of a resistor \( R_2 \) and an inductor \( L \) given by

\[
R_2 = \frac{R_F}{A_0}
\]

(18)

\[
L = \frac{R_F}{A_0 \omega_0}
\]

(19)
The equivalent circuit for $Z_{in}$ is shown in Fig. 7(a). If $A_0 \to \infty$, it follows that $R_2 \to 0$ and $L_2 \to 0$ so that $Z_{in} \to R_1$. The impedance transfer function for $Z_{in}$ is of the form of a high-pass shelving transfer function given by

$$Z_{in}(s) = R_{DC} \frac{1 + s/\omega_z}{1 + s/\omega_p}$$  \hspace{1cm} (20)

where $R_{DC}$ is the dc resistance, $\omega_p$ is the pole frequency, and $\omega_z$ is the zero frequency. These are given by

$$R_{DC} = R_1 + \frac{R_F}{1 + A_0}$$ \hspace{1cm} (21)

$$\omega_p = \frac{R_2 + R_F}{L} = \omega_0 (1 + A_0)$$ \hspace{1cm} (22)

$$\omega_z = \frac{R_2 + R_F||R_1}{L} = \omega_0 \left( 1 + \frac{A_0 R_1}{R_1 + R_F} \right)$$  \hspace{1cm} (23)

The Bode magnitude plot for $Z_{in}$ is shown in Fig. 7(b).

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**Example 5** At very low frequencies, an op-amp has the frequency independent open-loop gain $A(s) = A_0 = 2 \times 10^5$. The op-amp is to be used in an inverting amplifier with a gain of $-1000$. What is the required ratio $R_F/R_1$? For the value of $R_F/R_1$, how much larger is the input resistance than $R_1$?

**Solution.** By Eq. (12), we have

$$-1000 = \frac{R_F/R_1}{1 + (1 + R_F/R_1)/(2 \times 10^5)}$$

This can be solved for $R_F/R_1$ to obtain

$$\frac{R_F}{R_1} = \frac{2 \times 10^5 + 1}{(2 \times 10^5/1000) - 1} = 1005$$

By Eq. (17), the input resistance can be written

$$R_{in} = R_1 \left( 1 + \frac{R_F/R_1}{1 + A_0}\right) = R_1 \left( 1 + \frac{1005}{1 + 2 \times 10^5}\right) = 1.005R_1$$
Example 6 An op-amp has a dc gain $A_0 = 2 \times 10^5$ and a gain bandwidth product $f_x = 1$ MHz. The op-amp is used in the inverting amplifier of Fig. 5(a). The circuit element values are $R_1 = 1 \text{k} \Omega$ and $R_F = 100 \text{k} \Omega$. Calculate the dc gain of the amplifier, the upper cutoff frequency, and the value of the elements in the equivalent circuit for the input impedance. In addition, calculate the zero and the pole frequencies in Hz for the impedance Bode plot of Fig. 7(b).

Solution. The dc voltage gain is $-A_{0f}$. Eq. (14) can be used to calculate $A_{0f}$ to obtain

$$A_{0f} = \frac{2 \times 10^5 \times (1k||100k) / 1k}{1 + 2 \times 10^5 \times (1k||100k) / 100k} = 99.95$$

By Eqs. (3) and (15), the upper cutoff frequency $f_{0f}$ is given by

$$f_{0f} = \frac{\omega_{0f}}{2\pi} = \frac{10^6}{2 \times 10^5} \left( 1 + 2 \times 10^5 \frac{1k||100k}{100k} \right) = 9.91 \text{kHz}$$

The element values in the equivalent circuit of Fig. 7(a) for the input impedance are as follows:

$$R_1 = 1 \text{k} \Omega, \quad R_F = 100 \text{k} \Omega, \quad R_2 = 0.5 \Omega, \quad \text{and} \quad L = 15.9 \text{mH}$$

where Eqs. (18) and (19) have been used for $R_2$ and $L$. The pole and zero frequencies in the Bode impedance plot are

$$f_p = f_0 \left( 1 + A_0 \right) = \frac{f_x}{A_0} \left( 1 + A_0 \right) = 1.00 \text{MHz}$$

$$f_z = f_0 \left( 1 + A_0 \frac{R_1}{R_1 + R_F} \right) = 9.91 \text{kHz}$$