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The BJT

Notation

The notations used here for voltages and currents correspond to the following conventions: Dc bias values are indicated by an upper case letter with upper case subscripts, e.g. V_{DS}, I_C. Instantaneous values of small-signal variables are indicated by a lower-case letter with lower-case subscripts, e.g. v_s, i_c. Total values are indicated by a lower-case letter with upper-case subscripts, e.g. v_{BE}, i_D. Circuit symbols for independent sources are circular and symbols for controlled sources have a diamond shape. Voltage sources have a $\pm$ sign within the symbol and current sources have an arrow.

Device Equations

Figure 1 shows the circuit symbols for the npn and pnp BJTs. In the active mode, the collector-base junction is reverse biased and the base-emitter junction is forward biased. For the npn device, the active-mode collector and base currents are given by

$$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right) \qquad i_B = \frac{i_C}{\beta} \tag{1}$$

where V_T is the thermal voltage, I_S is the saturation current, and β is the base-to-collector current gain. These are given by

$$V_T = \frac{kT}{q} = 0.025\,\text{V for } T = 290\,\text{K} = 25.86\,\text{mV for } T = 300\,\text{K} \tag{2}$$

$$I_S = I_{S0}\left(1 + \frac{v_{CE}}{V_A}\right) \tag{3}$$

$$\beta = \beta_0\left(1 + \frac{v_{CE}}{V_A}\right) \tag{4}$$

where V_A is the Early voltage and I_{S0} and β_0, respectively, are the zero bias values of I_S and β. Because $I_S/\beta = I_{S0}/\beta_0$, it follows that i_B is not a function of v_{CE}. The equations apply to the pnp device if the subscripts BE and CE are reversed.

The emitter-to-collector current gain α is defined as the ratio i_C/i_E. To solve for this, we can write

$$i_E = i_B + i_C = \left(\frac{1}{\beta} + 1\right) i_C = \frac{1+\beta}{\beta} i_C \tag{5}$$

It follows that

$$\alpha = \frac{i_C}{i_E} = \frac{\beta}{1+\beta} \qquad \beta = \frac{i_C}{i_B} = \frac{\alpha}{1-\alpha} \tag{6}$$

Thus the currents are related by the equations

$$i_C = \beta i_B = \alpha i_E \tag{7}$$

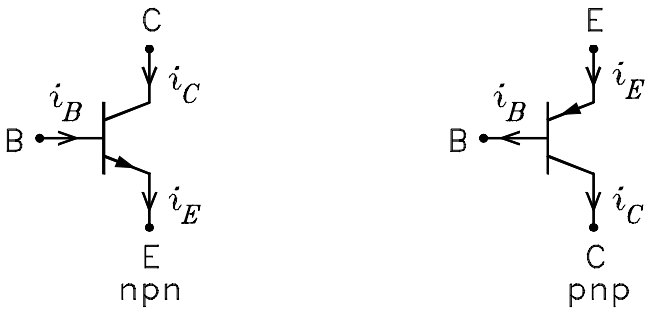

Figure 1: BJT circuit symbols.

Transfer and Output Characteristics

The transfer characteristics are a plot of the collector current i_C as a function of the base-to-emitter voltage v_{BE} with the collector-to-emitter voltage v_{CE} held constant. From Eqs. 1 and 3, we can write

$$i_C = I_{S0}\left(1 + \frac{v_{CE}}{V_A}\right)\exp\left(\frac{v_{BE}}{V_T}\right) \tag{8}$$

It follows that i_C varies exponentially with v_{BE}. A plot of this variation is given in Fig. 2. It can be seen from the plot that the collector current is essentially zero until the base-to-emitter voltage reaches a threshold value. Above this value, the collector current increases rapidly. The threshold value is typically in the range of 0.5 to 0.6 V. For high current transistors, it is usually smaller. The plot shows a single curve. If v_{CE} is increased, the current for a given v_{BE} is larger. However, the displacement between the curves is so small that it can be difficult to distinguish between them. The small-signal transconductance g_m defined below is the slope of the transfer characteristics curve evaluated at the quiescent or dc operating point.

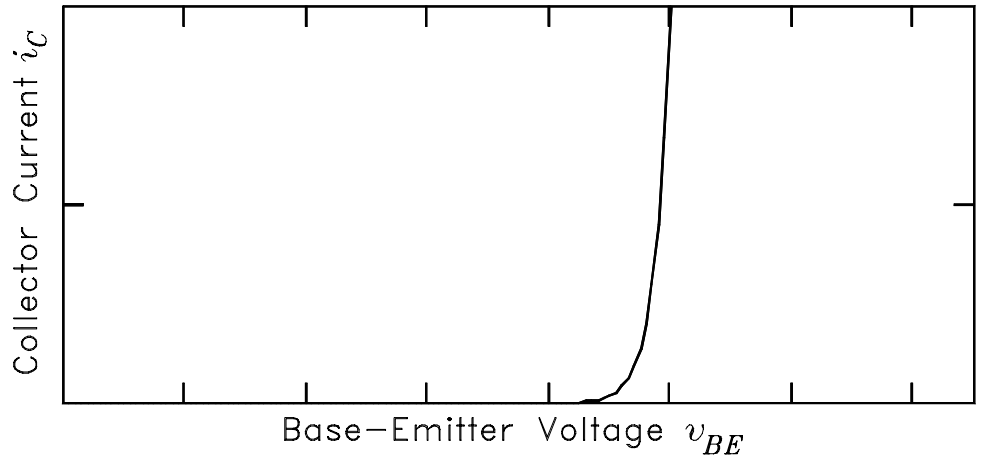

Figure 2: BJT transfer characteristics.

The output characteristics are a plot of the collector current i_C as a function of the collector-

to-emitter voltage v_{CE} with the base current i_B held constant. From Eqs. 1 and 4, we can write

$$i_C = \beta_0 \left(1 + \frac{v_{CE}}{V_A}\right) i_B \tag{9}$$

It follows that i_C varies linearly with v_{CE}. A plot of this variation is given in Fig. 3. For small v_{CE} such that $0 \leq v_{CE} < v_{BE}$, Eq. (9) does not hold. This is the region on the left in Fig. 3. In this region, the BJT is saturated. The small-signal collector-to-emitter resistance r_0 defined below is the reciprocal of the slope of the transfer characteristics curve evaluated at the quiescent or dc operating point to the right of the saturation region in Fig. 3.

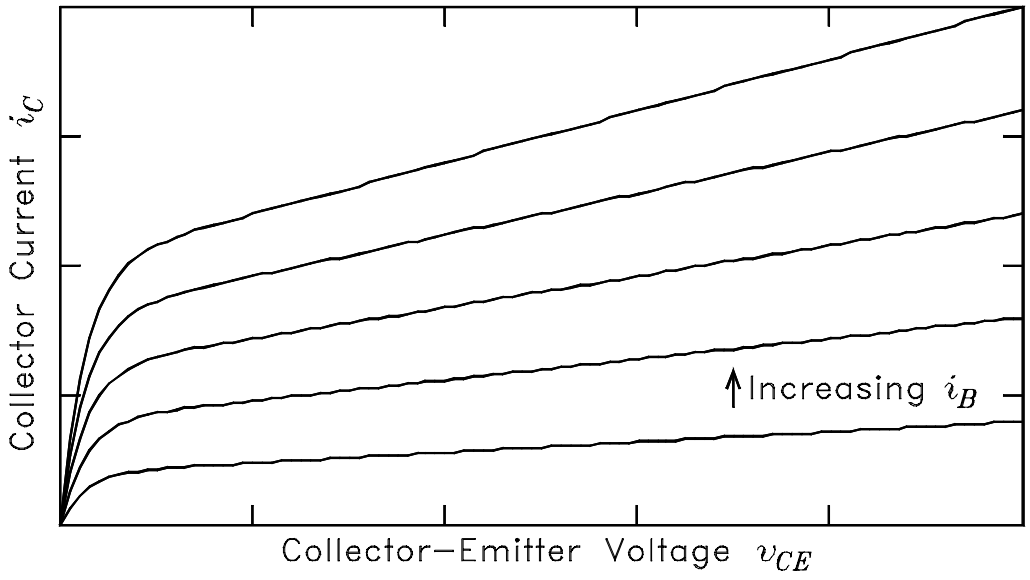

Figure 3: BJT output characteristics.

Bias Equation

Figure 4(a) shows the BJT with the external circuits represented by Thévenin dc circuits. If the BJT is biased in the active region, we can write

$$\begin{aligned} V_{BB} - V_{EE} &= I_B R_{BB} + V_{BE} + I_E R_{EE} \\ &= \frac{I_C}{\beta} R_{BB} + V_{BE} + \frac{I_C}{\alpha} R_{EE} \end{aligned} \tag{10}$$

This equation can be solved for I_C to obtain

$$I_C = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/\beta + R_{EE}/\alpha} \tag{11}$$

It can be seen from Fig. 2 that large changes in I_C are associated with small changes in V_{BE}. This makes it possible to calculate I_C by assuming typical values of V_{BE}. Values in the range from 0.6 V to 0.7 V are commonly used. Two sets of values for α and β are convenient for hand calculations. One is $\alpha = 0.99$ and $\beta = 99$. The other is $\alpha = 0.995$ and $\beta = 199$.

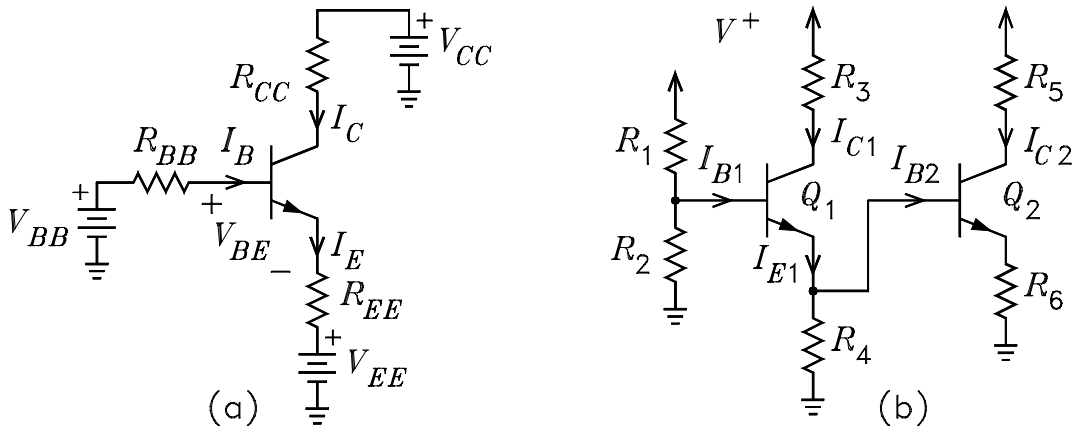

Figure 4: (a) BJT dc bias circuit. (b) Circuit for Example 1.

Example 1 *Figure 4(b) shows a BJT dc bias circuit. It is given that* $V^+ = 15\,\text{V}$, $R_1 = 20\,\text{k}\Omega$, $R_2 = 10\,\text{k}\Omega$, $R_3 = R_4 = 3\,\text{k}\Omega$, $R_5 = R_6 = 2\,\text{k}\Omega$. *Solve for* I_{C1} *and* I_{C2}. *Assume* $V_{BE} = 0.7\,\text{V}$ *and* $\beta = 100$ *for each transistor.*

Solution. For Q_1, we have $V_{BB1} = V^+R_2/(R_1 + R_2)$, $R_{BB1} = R_1 \| R_2$, $V_{EE1} = -I_{B2}R_4 = -I_{C2}R_4/\beta$, $V_{EE1} = 0$, and $R_{EE1} = R_4$. For Q_2, we have $V_{BB2} = I_{E1}R_4 = I_{C1}R_4/\alpha$, $R_{BB2} = R_4$, $V_{EE2} = 0$, $R_{EE2} = R_6$. Thus the bias equations are

$$V^+\frac{R_2}{R_1 + R_2} + \frac{I_{C2}}{\beta}R_4 = V_{BE} + \frac{I_{C1}}{\beta}R_1 \| R_2 + \frac{I_{C1}}{\alpha}R_4$$

$$\frac{I_{C1}}{\alpha}R_4 = V_{BE} + \frac{I_{C2}}{\beta}R_4 + \frac{I_{C2}}{\alpha}R_6$$

These equations can be solved simultaneously to obtain $I_{C1} = 1.41\,\text{mA}$ and $I_{C2} = 1.74\,\text{mA}$.

Hybrid-π Model

Let each current and voltage be written as the sum of a dc component and a small-signal ac component as follows:

$$i_C = I_C + i_c \qquad i_B = I_B + i_b \tag{12}$$

$$v_{BE} = V_{BE} + v_{be} \qquad v_{CE} = V_{CE} + v_{ce} \tag{13}$$

If the ac components are sufficiently small, we can write

$$i_c = \frac{\partial I_C}{\partial V_{BE}}v_{be} + \frac{\partial I_C}{\partial V_{CE}}v_{ce} \qquad i_b = \frac{\partial I_B}{\partial V_{BE}}v_{be} \tag{14}$$

where the derivatives are evaluated at the dc bias values. Let us define the transconductance g_m, the collector-to-emitter resistance r_0, and the base-to-emitter resistance r_π as follows:

$$g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{I_S}{V_T}\exp\left(\frac{V_{BE}}{V_T}\right) = \frac{I_C}{V_T} \tag{15}$$

$$r_0 = \left(\frac{\partial I_C}{\partial V_{CE}}\right)^{-1} = \left[\frac{I_{S0}}{V_A}\exp\left(\frac{V_{BE}}{V_T}\right)\right]^{-1} = \frac{V_A + V_{CE}}{I_C} \tag{16}$$

$$r_\pi = \left(\frac{\partial I_B}{\partial V_{BE}}\right)^{-1} = \left[\frac{I_{S0}}{\beta_0 V_T}\exp\left(\frac{V_{BE}}{V_T}\right)\right]^{-1} = \frac{V_T}{I_B} \tag{17}$$

The collector and base currents can thus be written

$$i_c = i_c' + \frac{v_{ce}}{r_0} \qquad i_b = \frac{v_\pi}{r_\pi} \tag{18}$$

where

$$i_c' = g_m v_\pi \qquad v_\pi = v_{be} \tag{19}$$

The small-signal circuit which models these equations is given in Fig. 5(a). This is called the hybrid-π model. The resistor r_x, which does not appear in the above equations, is called the base spreading resistance. It represents the resistance of the connection to the base region inside the device. Because the base region is very narrow, the connection exhibits a resistance which often cannot be neglected.

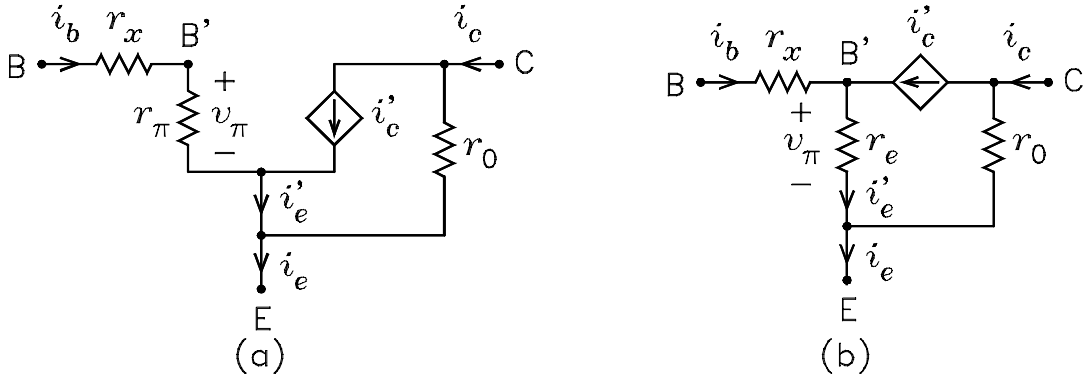

Figure 5: (a) Hybrid-π model. (b) T model.

The small-signal base-to-collector ac current gain β is defined as the ratio i_c'/i_b. It is given by

$$\beta = \frac{i_c'}{i_b} = \frac{g_m v_\pi}{i_b} = g_m r_\pi = \frac{I_C}{V_T}\frac{V_T}{I_B} = \frac{I_C}{I_B} \tag{20}$$

Note that i_c differs from i_c' by the current through r_0. Therefore, $i_c/i_b \neq \beta$ unless $r_0 = \infty$.

T Model

The T model replaces the resistor r_π in series with the base with a resistor r_e in series with the emitter. This resistor is called the emitter intrinsic resistance. The current i_e' can be written

$$i_e' = i_b + i_c' = \left(\frac{1}{\beta} + 1\right) i_c' = \frac{1+\beta}{\beta} i_c' = \frac{i_c'}{\alpha} \tag{21}$$

where α is the small-signal emitter-to-collector ac current gain given by

$$\alpha = \frac{\beta}{1+\beta} \tag{22}$$

Thus the current i_c' can be written

$$i_c' = \alpha i_e' \tag{23}$$

The voltage v_π can be related to i'_e as follows:

$$v_\pi = i_b r_\pi = \frac{i'_c}{\beta} r_\pi = \frac{\alpha i'_e}{\beta} r_\pi = i'_e \frac{\alpha r_\pi}{\beta} = i'_e \frac{r_\pi}{1+\beta} = i'_e r_e \tag{24}$$

It follows that the intrinsic emitter resistance r_e is given by

$$r_e = \frac{v_\pi}{i'_e} = \frac{r_\pi}{1+\beta} = \frac{V_T}{(1+\beta)\, I_B} = \frac{V_T}{I_E} \tag{25}$$

The T model of the BJT is shown in Fig. 5(b). The currents in both models are related by the equations

$$i'_c = g_m v_\pi = \beta i_b = \alpha i'_e \tag{26}$$

An Emitter Equivalent Circuit

Figure 6 shows the T model with a Thévenin source in series with the base. We wish to solve for an equivalent circuit in which the source i'_c connects from the collector node to ground rather than from the collector node to the B' node.

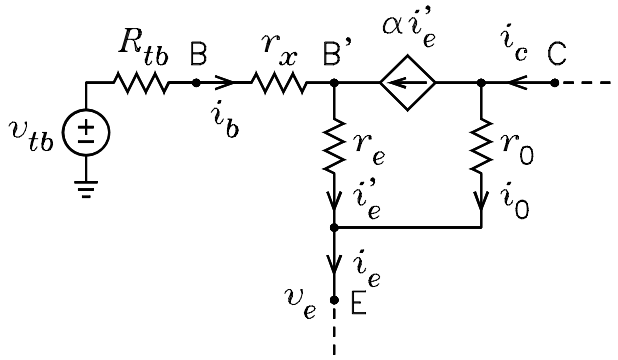

Figure 6: T model with Thévenin source connected to the base.

The first step is to replace the source $\alpha i'_e$ with two identical series sources with the common node grounded. The circuit is shown in Fig. 7(a). The object is to absorb the left $\alpha i'_e$ source into the base-emitter circuit. For the circuit, we can write

$$v_e = v_{tb} - \frac{i'_e}{1+\beta}\,(R_{tb} + r_x) - i'_e r_e = v_{tb} - i'_e \left(\frac{R_{tb} + r_x}{1+\beta} + r_e \right) \tag{27}$$

Let us define the resistance r'_e by

$$r'_e = \frac{R_{tb} + r_x}{1+\beta} + r_e \overset{\text{or}}{=} \frac{R_{tb} + r_x + r_\pi}{1+\beta} \tag{28}$$

With this definition, v_e is given by

$$v_e = v_{tb} - i'_e r'_e \tag{29}$$

The circuit which models Eq. (29) is shown in Fig. 7(b). We will call this the emitter equivalent circuit. It predicts the same emitter and collector currents as the circuit in Fig. 6. Note that the resistors R_{tb} and r_x do not appear in this circuit because they are contained in r'_e.

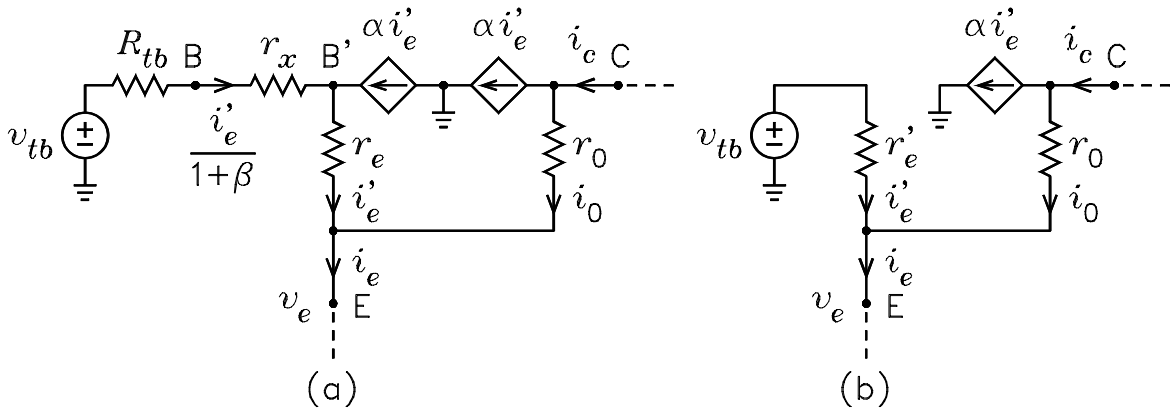

Figure 7: (a) Circuit with the i_c' source replaced by identical series sources. (b) Simplified T model.

Norton Collector Circuit

The Norton equivalent circuit seen looking into the collector can be used to solve for the response of the common-emitter and common-base stages. It consists of a parallel current source $i_{c(sc)}$ and a resistor r_{ic} from the collector to signal ground. Fig. 8(a) shows the BJT with Thévenin sources connected to its base and emitter. To solve for the Norton equivalent circuit seen looking into the collector, we use the emitter equivalent circuit in Fig. 8(b).

By superposition of v_c, $\alpha i_e'$, v_{tb}, and v_{te}, the following equations for i_c and i_e' can be written

$$\begin{aligned} i_c &= \frac{v_c}{r_0 + r_e' \| R_{te}} + \alpha i_e' - \frac{v_{tb}}{r_e' + R_{te} \| r_0} \frac{R_{te}}{R_{te} + r_0} \\ &\quad - \frac{v_{te}}{R_{te} + r_e' \| r_0} \frac{r_e'}{r_e' + r_0} \end{aligned} \tag{30}$$

$$\begin{aligned} i_e' &= \frac{v_{tb}}{r_e' + R_{te} \| r_0} - \frac{v_{te}}{R_{te} + r_e' \| r_0} \\ &\quad - \frac{v_c}{r_0 + r_e' \| R_{te}} \frac{R_{te}}{r_e' + R_{te}} \end{aligned} \tag{31}$$

These can be solved to obtain

$$\begin{aligned} i_c &= \frac{v_{tb}}{r_e' + R_{te} \| r_0} \left(\alpha - \frac{R_{te}}{R_{te} + r_0} \right) - \frac{v_{te}}{R_{te} + r_e' \| r_0} \left(\alpha + \frac{r_e'}{r_e' + r_0} \right) \\ &\quad + \frac{v_c}{r_0 + r_e' \| R_{te}} \left(1 - \frac{\alpha R_{te}}{r_e' + R_{te}} \right) \end{aligned} \tag{32}$$

This equation is of the form

$$i_c = i_{c(sc)} + \frac{v_c}{r_{ic}} \tag{33}$$

where $i_{c(sc)}$ and r_{ic} are given by

$$i_{c(sc)} = G_{mb} v_{tb} - G_{me} v_{te} \tag{34}$$

$$r_{ic} = \frac{r_0 + r_e' \| R_{te}}{1 - \alpha R_{te} / (r_e' + R_{te})} \tag{35}$$

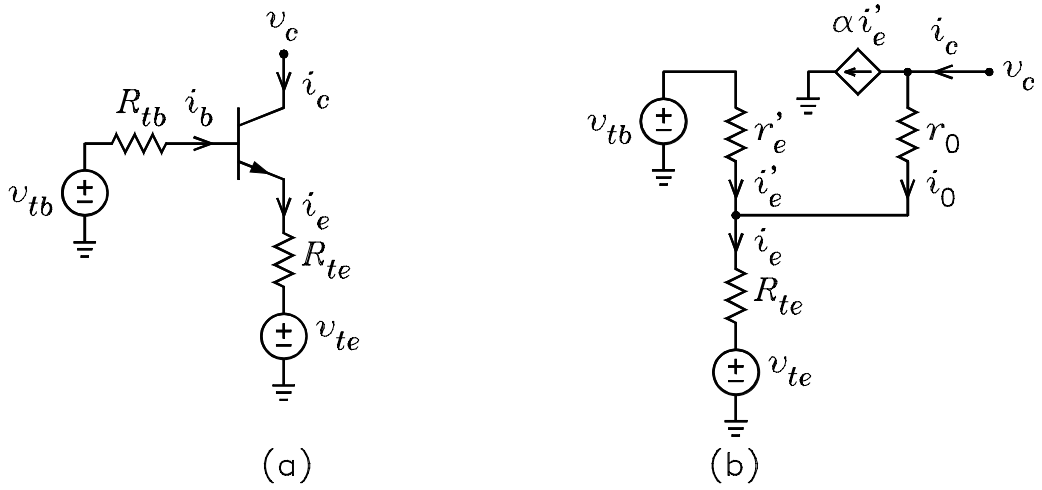

Figure 8: (a) BJT with Thevenin sources connected to the base and the emitter. (b) Simplified T model.

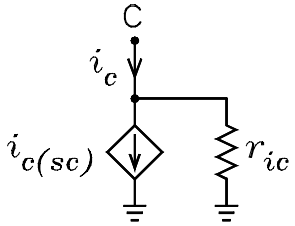

Figure 9: (a) Circuit for calculating r_{ic}. (b) Norton collector circuit.

and G_{mb} and G_{me} are given by

$$\begin{aligned} G_{mb} &= \frac{1}{r'_e + R_{te} \| r_0} \left(\alpha - \frac{R_{te}}{R_{te} + r_0} \right) \\ &\stackrel{\text{or}}{=} \frac{\alpha}{r'_e + R_{te} \| r_0} \, \frac{r_0 - R_{te}/\beta}{r_0 + R_{te}} \end{aligned} \tag{36}$$

$$\begin{aligned} G_{me} &= \frac{1}{R_{te} + r'_e \| r_0} \left(\alpha + \frac{r'_e}{r'_e + r_0} \right) \\ &\stackrel{\text{or}}{=} \frac{\alpha}{r'_e + R_{te} \| r_0} \, \frac{r_0 + r'_e/\alpha}{r_0 + R_{te}} \end{aligned} \tag{37}$$

The Norton equivalent circuit seen looking into the collector is shown in Fig. 9.

For the case $r_0 \gg R_{te}$ and $r_0 \gg r'_e$, we can write

$$i_{c(sc)} = G_m \left(v_{tb} - v_{te} \right) \tag{38}$$

where

$$G_m = \frac{\alpha}{r'_e + R_{te}} \tag{39}$$

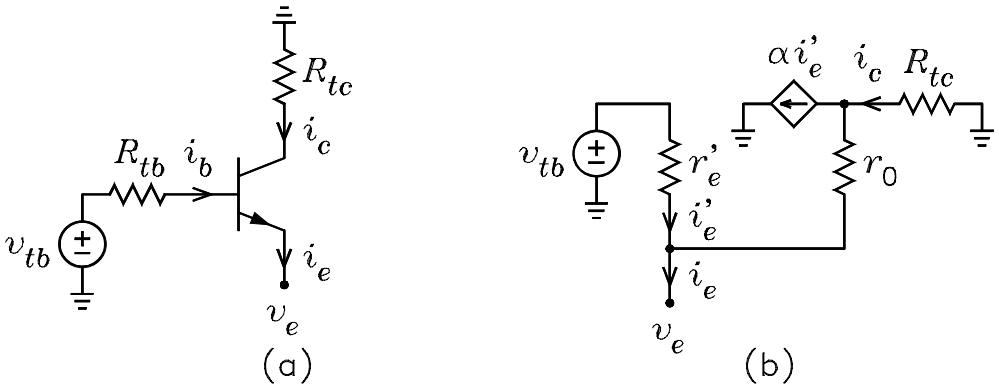

Figure 10: (a) BJT with Thévenin source connected to the base. (b) T model circuit for calculating $v_{e(oc)}$.

The value of $i_{c(sc)}$ calculated with this approximation is simply the value of $\alpha i'_e$, where i'_e is calculated with r_0 considered to be an open circuit. The term "r_0 approximations" is used in the following when r_0 is neglected in calculating $i_{c(sc)}$ but not neglected in calculating r_{ic}.

Thévenin Emitter Circuit

The Thévenin equivalent circuit seen looking into the emitter is useful in calculating the response of common-collector stages. It consists of a voltage source $v_{e(oc)}$ in series with a resistor r_{ie} from the emitter node to signal ground. Fig. 10(a) shows the BJT symbol with a Thévenin source connected to the base. The resistor R_{tc} represents the external load resistance in series with the collector. To solve for the Thévenin equivalent circuit seen looking into the emitter, we use the emitter equivalent circuit in Fig. 10(b).

By superposition of v_{tb}, i_e, and $\alpha i'_e$, the following equations for v_e and i'_e can be written

$$\begin{aligned} v_e &= v_{tb}\frac{r_0 + R_{tc}}{r'_e + r_0 + R_{tc}} - i_e\left[r'_e \| (r_0 + R_{tc})\right] \\ &\quad -\alpha i'_e \frac{R_{tc} r'_e}{r'_e + r_0 + R_{tc}} \end{aligned} \tag{40}$$

$$i'_e = \frac{v_{tb} - v_e}{r'_e} \tag{41}$$

These can be solved to obtain

$$\begin{aligned} v_e\left(1 - \frac{\alpha R_{tc}}{r'_e + r_0 + R_{tc}}\right) &= \frac{v_{tb}}{r'_e + r_0 + R_{tc}}\left[r_0 + (1-\alpha) R_{tc}\right] \\ &\quad -i_e r'_e \| (r_0 + R_{tc}) \end{aligned} \tag{42}$$

which simplifies to

$$v_e = v_{tb}\frac{r_0 + (1-\alpha) R_{tc}}{r'_e + r_0 + (1-\alpha) R_{tc}} - i_e \frac{r'_e (r_0 + R_{tc})}{r'_e + r_0 + (1-\alpha) R_{tc}} \tag{43}$$

This equation is of the form

$$v_e = v_{e(oc)} - i_e r_{ie} \tag{44}$$

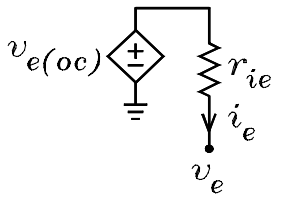

Figure 11: (a) Circuit for calculating $i_{e(sc)}$. (b) Thévenin emitter circuit.

where $v_{e(oc)}$ and r_{ie} are given by

$$\begin{aligned} v_{e(oc)} &= v_{tb}\frac{r_0+(1-\alpha)\,R_{tc}}{r'_e+r_0+(1-\alpha)\,R_{tc}} \\ &\overset{\text{or}}{=} v_{tb}\frac{r_0+R_{tc}/\,(1+\beta)}{r'_e+r_0+R_{tc}/\,(1+\beta)} \end{aligned} \tag{45}$$

$$\begin{aligned} r_{ie} &= r'_e\frac{r_0+R_{tc}}{r'_e+r_0+(1-\alpha)\,R_{tc}} \\ &\overset{\text{or}}{=} r'_e\frac{r_0+R_{tc}}{r'_e+r_0+R_{tc}/\,(1+\beta)} \end{aligned} \tag{46}$$

The Thévenin equivalent circuit seen looking into the emitter is shown in Fig. 10.

Thévenin Base Circuit

Although the base is not an output terminal, the Thévenin equivalent circuit seen looking into the base is useful in calculating the base current. It consists of a voltage source $v_{b(oc)}$ in series with a resistor r_{ib} from the base node to signal ground. Fig. 12(a) shows the BJT symbol with a Thévenin source connected to its emitter. Fig. 12(b) shows the Pi model for calculating the base voltage.

By superposition of v_{te}, i_b, and βi_b, treating each branch of βi_b separately in the superposition, we can write the following equation for v_b

$$\begin{aligned} v_b &= v_{te}\frac{r_0+R_{tc}}{R_{te}+r_0+R_{tc}} + i_b\left[r_x+r_\pi+R_{te}\|\,(r_0+R_{tc})\right] \\ &\quad +\beta i_b\frac{r_0R_{te}}{R_{te}+r_0+R_{tc}} \\ &= v_{te}\frac{r_0+R_{tc}}{R_{te}+r_0+R_{tc}} + i_b\left\{\, r_x+r_\pi+R_{te}\|\,(r_0+R_{tc})\right. \\ &\quad \left.+\beta\frac{r_0R_{te}}{R_{te}+r_0+R_{tc}}\,\right\} \end{aligned} \tag{47}$$

This equation is of the form

$$v_b = v_{b(oc)} + i_b r_{ib} \tag{48}$$

where $v_{b(oc)}$ and r_{ib} are given by

$$v_{b(oc)} = v_{te}\frac{r_0+R_{tc}}{R_{te}+r_0+R_{tc}} \tag{49}$$

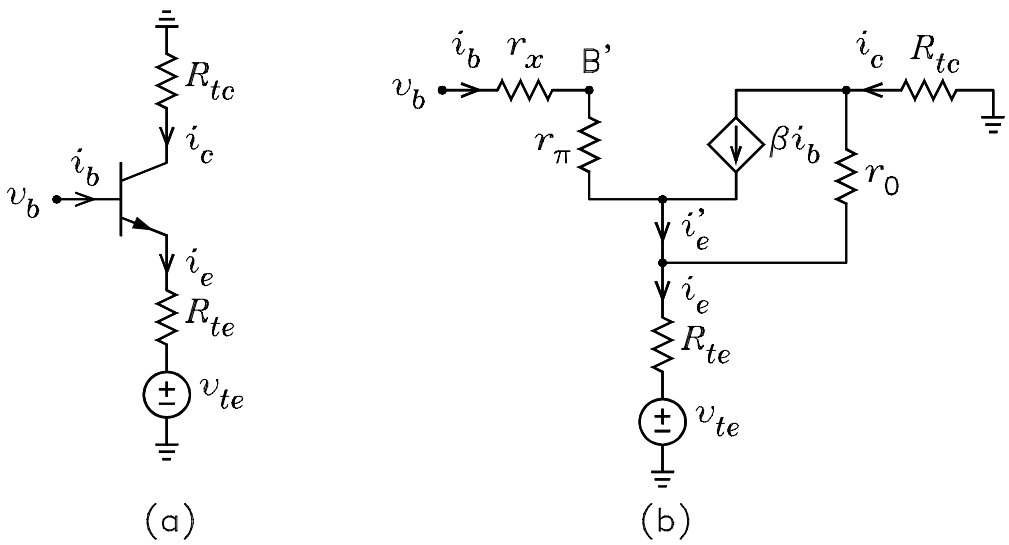

Figure 12: (a) BJT with Thevenin source connected to the emitter. (b) T model for calculating $v_{b(oc)}$.

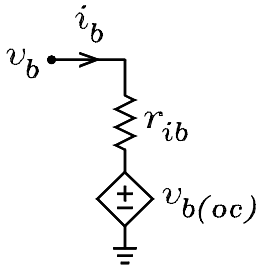

Figure 13: (a) Circuit for calculating v_b. (b) Thévenin base circuit.

$$\begin{aligned} r_{ib} &= r_x + r_\pi + R_{te} \| (r_0 + R_{tc}) \\ &\quad + \beta \frac{r_0 R_{te}}{R_{te} + r_0 + R_{tc}} \\ &\stackrel{\text{or}}{=} r_x + (1+\beta) r_e + R_{te} \| (r_0 + R_{tc}) \\ &\quad + \beta \frac{r_0 R_{te}}{R_{te} + r_0 + R_{tc}} \end{aligned} \tag{50}$$

The equivalent circuit which models these equations is shown in Fig. 13.

The r_0 Approximations

The r_0 approximations approximate r_0 as an open circuit in all equations except the one for r_{ic}. In this case, the equations for $i_{c(sc)}$, G_m, r_{ic}, $v_{e(oc)}$, r_{ie}, $v_{b(oc)}$, and r_{ib} are

$$i_{c(sc)} = i_c' = G_m (v_{tb} - v_{te}) \qquad G_m = \frac{\alpha}{r_e' + R_{te}} \tag{51}$$

$$r_{ic} = \frac{r_0 + r'_e \| R_{te}}{1 - \alpha R_{te} / (r'_e + R_{te})} \tag{52}$$

$$v_{e(oc)} = v_{tb} \qquad r_{ie} = r'_e \tag{53}$$

$$v_{b(oc)} = v_{te} \qquad r_{ib} = r_x + r_\pi + (1+\beta) R_{te} \overset{\text{or}}{=} r_x + (1+\beta)(r_e + R_{te}) \tag{54}$$

The emitter equivalent circuit, the hybrid π model, and the T model, respectively, are given Figs. 14 and 15. If $r_0 = \infty$, then r_{ic} is an open circuit in each. Because r_0 no longer connects to the emitter, there is only one emitter current and $i_e = i'_e$.

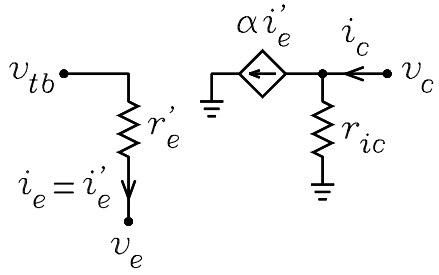

Figure 14: T model with the r_0 approximations.

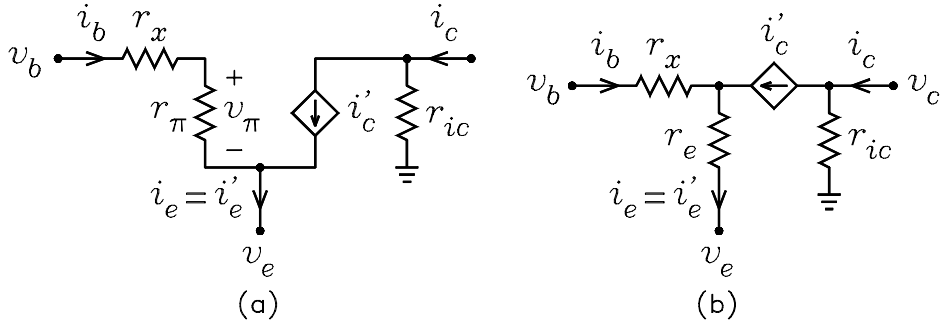

Figure 15: (a) Hybrid π and (b) T models with the r_0 approximations.

Summary of Models

Examples

This section describes several examples which illustrate the use of the small-signal equivalent circuits derived above to write by inspection the voltage gain, the input resistance, and the output resistance of both single-stage and two-stage amplifiers.

The Common-Emitter Amplifier

Figure 17(a) shows the ac signal circuit of a common-emitter amplifier. We assume that the bias solution and the small-signal resistances r'_e and r_0 are known. The output voltage and output

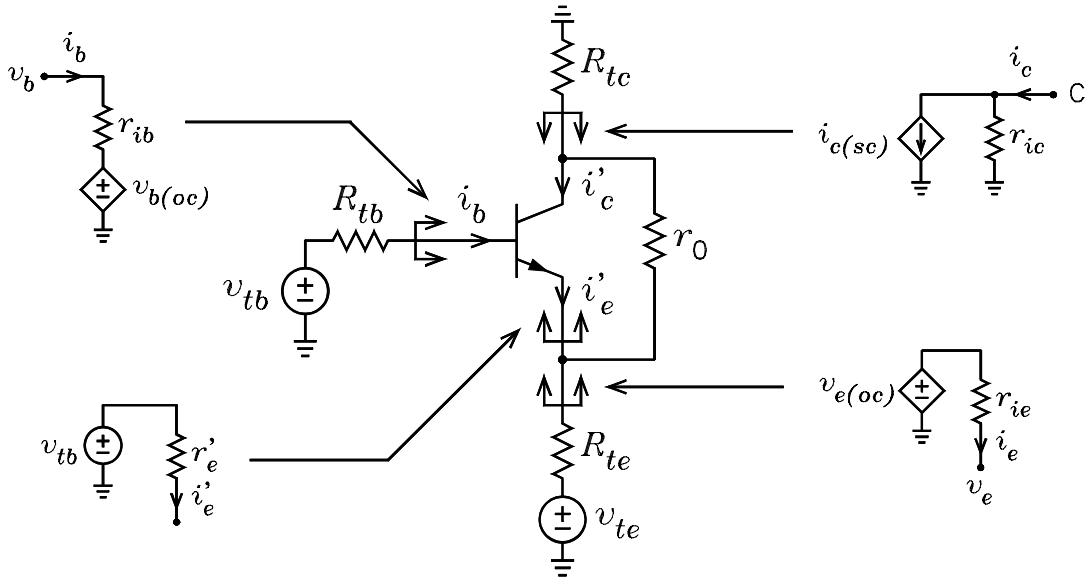

Figure 16: Summary of the small-signal equivalent circuits.

resistance can be calculated by replacing the circuit seen looking into the collector by the Norton equivalent circuit of Fig. 9(b). With the aid of this circuit, we can write

$$v_o = -i_{c(sc)} \left(r_{ic} \| R_{tc} \right) = -G_{mb} \left(r_{ic} \| R_{tc} \right) v_{tb} \tag{55}$$

$$r_{out} = r_{ic} \| R_{tc} \tag{56}$$

where G_{mb} and r_{ic}, respectively, are given by Eqs. (36) and (35). The input resistance is given by

$$r_{in} = R_{tb} + r_{ib} \tag{57}$$

where r_{ib} is given by Eq. (50).

The Common-Collector Amplifier

Figure 17(b) shows the ac signal circuit of a common-collector amplifier. We assume that the bias solution and the small-signal resistances r'_e and r_0 are known. The output voltage and output resistance can be calculated by replacing the circuit seen looking into the emitter by the Thévenin equivalent circuit of Fig. 10(b). With the aid of this circuit, we can write

$$v_o = v_{e(oc)} \frac{R_{te}}{r_{ie} + R_{te}} = \frac{r_0 + R_{tc}/\left(1+\beta\right)}{r'_e + r_0 + R_{tc}/\left(1+\beta\right)} \frac{R_{te}}{r_{ie} + R_{te}} v_{tb} \tag{58}$$

$$r_{out} = r_{ie} \| R_{te} \tag{59}$$

where r_{ie} is given by Eq. (46). The input resistance is given by

$$r_{in} = R_{tb} + r_{ib} \tag{60}$$

where r_{ib} is given by Eq. (50).

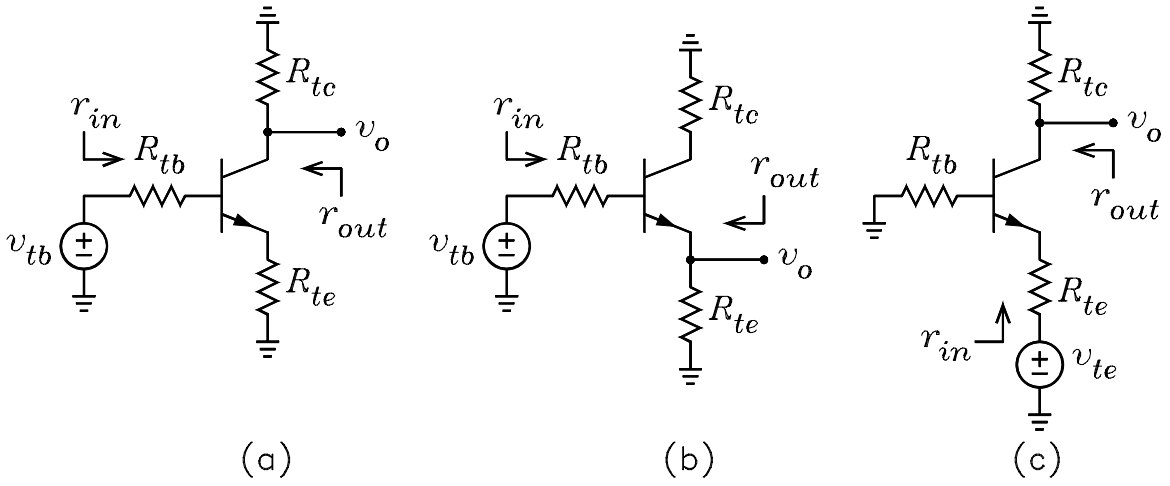

Figure 17: (a) Common-emitter amplifier. (b) Common-collector amplifier. (c) Common-base amplifier.

The Common-Base Amplifier

Figure 17(c) shows the ac signal circuit of a common-base amplifier. We assume that the bias solution and the small-signal parameters r'_e and r_0 are known. The output voltage and output resistance can be calculated by replacing the circuit seen looking into the collector by the Norton equivalent circuit of Fig. 9(b). The input resistance can be calculated by replacing the circuit seen looking into the emitter by the Thévenin equivalent circuit of Fig. 10(b) with $v_{e(oc)} = 0$. With the aid of these circuits, we can write

$$v_o = -i_{c(sc)} \left(r_{ic} \| R_{tc}\right) = G_{me} \left(r_{ic} \| R_{tc}\right) v_{te} \tag{61}$$

$$r_{out} = r_{ic} \| R_{tc} \tag{62}$$

$$r_{in} = R_{te} + r_{ie} \tag{63}$$

where G_{me}, r_{ic}, and r_{ie}, respectively, are given by Eqs. (37), (35), and (46).

The CE/CC Amplifier

Figure 18(a) shows the ac signal circuit of a two-stage amplifier consisting of a CE stage followed by a CC stage. Such a circuit is used to obtain a high voltage gain and a low output resistance. The voltage gain can be written

$$\begin{aligned} \frac{v_o}{v_{tb1}} &= \frac{i_{c1(sc)}}{v_{tb1}} \times \frac{v_{tb2}}{i_{c1(sc)}} \times \frac{v_{e2(oc)}}{v_{tb2}} \times \frac{v_o}{v_{e2(oc)}} \\ &= G_{mb1} \times \left[-\left(r_{ic1} \| R_{C1}\right)\right] \times \frac{r_0}{r'_{e2} + r_0} \times \frac{R_{te2}}{r_{ie2} + R_{te2}} \end{aligned} \tag{64}$$

where r'_{e2} is calculated with $R_{tb2} = r_{ic1} \| R_{C1}$. The input and output resistances are given by

$$r_{in} = R_{tb1} + r_{ib1} \tag{65}$$

$$r_{out} = r_{ie2} \| R_{te2} \tag{66}$$

Although not a part of the solution, the resistance seen looking out of the collector of Q_1 is $R_{tc1} = R_{C1} \| r_{ib2}$.

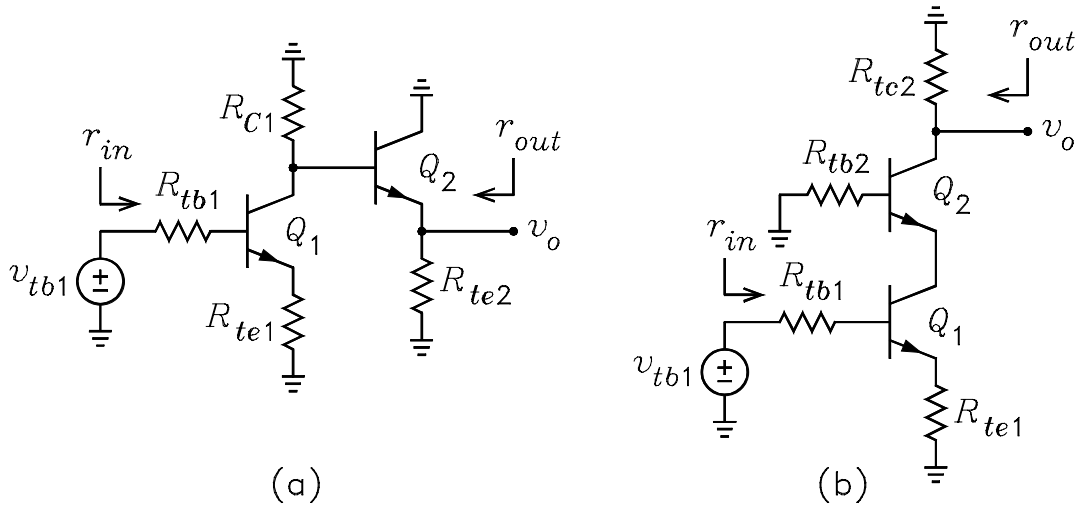

Figure 18: (a) CE-CC amplifier. (b) Cascode amplifier.

The Cascode Amplifier

Figure 18(b) shows the ac signal circuit of a cascode amplifier. The voltage gain can be written

$$\begin{aligned}\frac{v_o}{v_{tb1}} &= \frac{i_{c1(sc)}}{v_{tb1}} \times \frac{v_{te2}}{i_{c1(sc)}} \times \frac{i_{c2(sc)}}{v_{te2}} \times \frac{v_o}{i_{c2(sc)}} \\ &= G_{m1} \times (-r_{ic1}) \times (-G_{me2}) \times (-r_{ic2} \| R_{tc2})\end{aligned}$$

where G_{me2} and r_{ic2} are calculated with $R_{te2} = r_{ic1}$. The input and output resistances are given by

$$r_{in} = R_{tb1} + r_{ib1}$$

$$r_{out} = R_{tc2} \| r_{ic2}$$

The resistance seen looking out of the collector of Q_1 is $R_{tc1} = r_{ie2}$.

A second cascode amplifier is shown in Fig. 19(a) where a pnp transistor is used for the second stage. The voltage gain is given by

$$\begin{aligned}\frac{v_o}{v_{tb1}} &= \frac{i_{c1(sc)}}{v_{tb1}} \times \frac{v_{te2}}{i_{c1(sc)}} \times \frac{i_{c2(sc)}}{v_{te2}} \times \frac{v_o}{i_{c2(sc)}} \\ &= G_{m1} \times (-r_{ic1} \| R_{C1}) \times (-G_{me2}) \times (-r_{ic2} \| R_{tc2})\end{aligned}$$

The expressions for r_{in} and r_{out} are the same as for the cascode amplifier in Fig. 18(b). The resistance seen looking out of the collector of Q_1 is $R_{tc1} = R_{C1} \| r_{ie2}$.

The Differential Amplifier

Figure 19(b) shows the ac signal circuit of a differential amplifier. For the case of an active tail bias supply, the resistor R_Q represents its small-signal ac resistance. We assume that the transistors are identical, biased at the same currents and voltages, and have identical small-signal parameters.

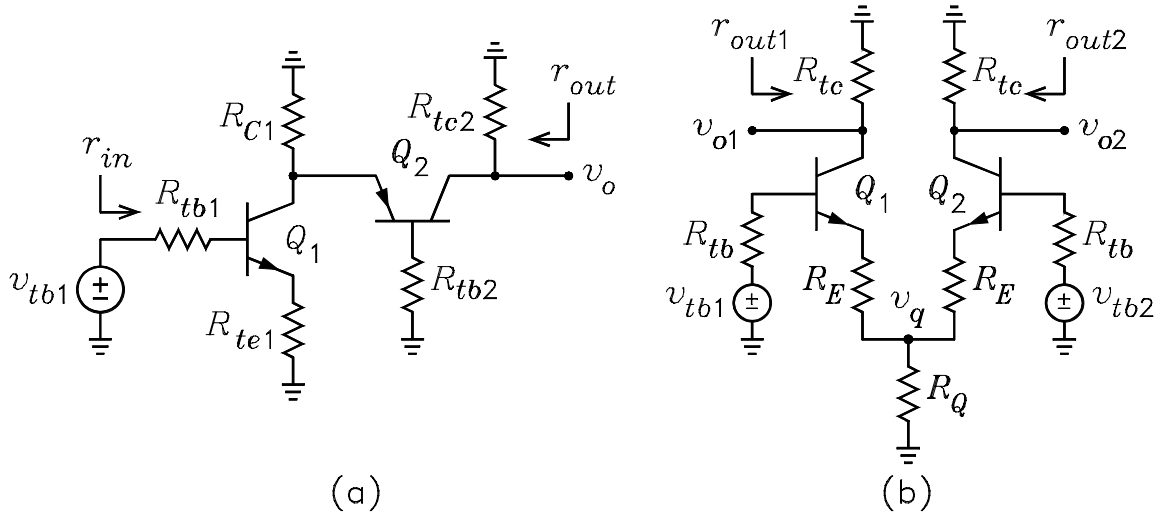

Figure 19: (a) Second cascode amplifier. (b) Differential amplifier.

Looking out of the emitter of Q_1, the Thévenin voltage and resistance are given by

$$\begin{aligned} v_{te1} &= v_{e2(oc)} \frac{R_Q}{R_Q + R_E + r_{ie}} \\ &= v_{tb2} \frac{r_0 + R_{tc}/(1+\beta)}{r'_e + r_0 + R_{tc}/(1+\beta)} \frac{R_Q}{R_Q + R_E + r_{ie}} \end{aligned} \tag{67}$$

$$R_{te1} = R_E + R_Q \| (R_E + r_{ie}) \tag{68}$$

The small-signal collector voltage of Q_1 is given by

$$\begin{aligned} v_{o1} &= -i_{c1(sc)} (r_{ic} \| R_{tc}) = -(G_{mb} v_{tb1} - G_{me} v_{te1}) (r_{ic} \| R_{tc}) \\ &= -G_{mb} (r_{ic} \| R_{tc}) v_{tb1} \\ &\quad + G_{me} \frac{r_0 + R_{tc}/(1+\beta)}{r'_e + r_0 + R_{tc}/(1+\beta)} \frac{R_Q}{r_{ie} + R_E + R_Q} v_{tb2} \end{aligned} \tag{69}$$

By symmetry, v_{o2} is obtained by interchanging the subscripts 1 and 2 in this equation. The small-signal resistance seen looking into either output is

$$r_{out} = R_{tc} \| r_{ic} \tag{70}$$

where r_{ic} calculated from Eq. (35) with $R_{te} = R_E + R_Q \| (R_E + r_{ie})$. Although not labeled on the circuit, the input resistance seen by both v_{tb1} and v_{tb2} is $r_{in} = r_{ib}$.

A second solution of the diff amp can be obtained by replacing v_{tb1} and v_{tb2} with differential and common-mode components as follows:

$$v_{tb1} = v_{i(cm)} + \frac{v_{i(d)}}{2} \tag{71}$$

$$v_{tb2} = v_{i(cm)} - \frac{v_{i(d)}}{2} \tag{72}$$

where $v_{i(d)} = v_{tb1} - v_{tb2}$ and $v_{i(cm)} = (v_{tb1} + v_{tb2})/2$. Superposition of $v_{i(d)}$ and $v_{i(cm)}$ can be used to solve for v_{o1} and v_{o2}. With $v_{i(cm)} = 0$, the effects of $v_{tb1} = v_{i(d)}/2$ and $v_{tb2} = -v_{i(d)}/2$ are to cause

$v_q = 0$. Thus the v_q node can be grounded and the circuit can be divided into two common-emitter stages in which $R_{te(d)} = R_E$ for each transistor. In this case, $v_{o1(d)}$ can be written

$$\begin{aligned} v_{o1(d)} &= \frac{i_{c1(sc)}}{v_{tb1(d)}} \times \frac{v_{o1(d)}}{i_{c1(sc)}} v_{tb1(d)} = G_{m(d)} \times (-r_{ic} \| R_{tc}) \frac{v_{i(d)}}{2} \\ &= G_{m(d)} \times (-r_{ic} \| R_{tc}) \frac{v_{tb1} - v_{tb2}}{2} \end{aligned} \tag{73}$$

By symmetry $v_{o2(d)} = -v_{o1(d)}$.

With $v_{i(d)} = 0$, the effects of $v_{tb1} = v_{tb2} = v_{i(cm)}$ are to cause the emitter currents in Q_1 and Q_2 to change by the same amounts. If R_Q is replaced by two parallel resistors of value $2R_Q$, it follows by symmetry that the circuit can be separated into two common-emitter stages each with $R_{te(cm)} = R_E + 2R_Q$. In this case, $v_{o1(cm)}$ can be written

$$\begin{aligned} v_{o1(cm)} &= \frac{i_{c1(sc)}}{v_{tb1(cm)}} \times \frac{v_{o1(cm)}}{i_{c1(sc)}} v_{tb1(cm)} = G_{m(cm)} \left(-r_{ic} \| R_{tc}\right) v_{i(cm)} \\ &= G_{m(cm)} \times (-r_{ic} \| R_{tc}) \frac{v_{tb1} + v_{tb2}}{2} \end{aligned} \tag{74}$$

By symmetry $v_{o2(cm)} = v_{o1(cm)}$.

Because R_{te} is different for the differential and common-mode circuits, G_m and r_{ib} are different. However, the total solution $v_{o1} = v_{o1(d)} + v_{o1(cm)}$ is the same as that given by Eq. (69), and similarly for v_{o2}. Note that r_{ic} is the same for both solutions and is calculated with $R_{te} = R_E + R_Q \| (R_E + r_{ie})$. The small-signal base currents can be written $i_{b1} = v_{i(cm)}/r_{ib(cm)} + v_{i(d)}/r_{ib(d)}$ and $i_{b2} = v_{i(cm)}/r_{ib(cm)} - v_{i(d)}/r_{ib(d)}$. If $R_Q \to \infty$, the common-mode gain is very small, approaching 0 as $r_0 \to \infty$. In this case, the differential solutions can be used for the total solutions. If $R_Q \gg R_E + r_{ie}$, the common-mode solutions are often approximated by zero.

Small-Signal High-Frequency Models

Figure 20 shows the hybrid-π and T models for the BJT with the base-emitter capacitance c_π and the base-collector capacitance c_μ added. The capacitor c_{cs} is the collector-substrate capacitance which in present in monolithic integrated-circuit devices but is omitted in discrete devices. These capacitors model charge storage in the device which affects its high-frequency performance. The capacitors are given by

$$c_\pi = c_{je} + \frac{\tau_F I_C}{V_T} \tag{75}$$

$$c_\mu = \frac{c_{jc}}{[1 + V_{CB}/\phi_C]^{m_c}} \tag{76}$$

$$c_{cs} = \frac{c_{jcs}}{[1 + V_{CS}/\phi_C]^{m_c}} \tag{77}$$

where I_C is the dc collector current, V_{CB} is the dc collector-base voltage, V_{CS} is the dc collector-substrate voltage, c_{je} is the zero-bias junction capacitance of the base-emitter junction, τ_F is the forward transit time of the base-emitter junction, c_{jc} is the zero-bias junction capacitance of the base-collector junction, c_{jcs} is the zero-bias collector-substrate capacitance, ϕ_C is the built-in potential, and m_c is the junction exponential factor. For integrated circuit lateral pnp transistors, c_{cs} is replaced with a capacitor c_{bs} from base to substrate, i.e. from the B node to ground.

In these models, the currents are related by

$$i_c' = g_m v_\pi = \beta i_b' = \alpha i_e' \tag{78}$$

These relations are the same as those in Eq. (26) with i_b replaced with i_b'.

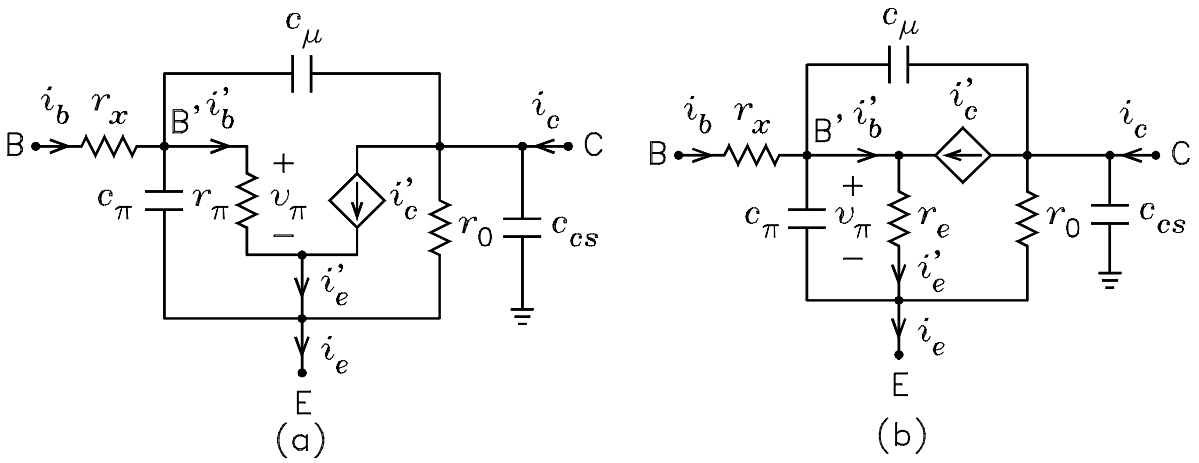

Figure 20: High-frequency small-signal models of the BJT. (a) Hybrid-π model. (b) T model.