The BJT Differential Amplifier

Basic Circuit

Figure 1 shows the circuit diagram of a differential amplifier. The tail supply is modeled as a current source $I_Q$. The object is to solve for the small-signal output voltages and output resistances. It will be assumed that the transistors are identical.

![Circuit Diagram](image)

Figure 1: Circuit diagram of the differential amplifier.

DC Solution

Zero both base inputs. For identical transistors, the current $I_Q$ divides equally between the two emitters.

(a) The dc currents are given by

\[ I_{E1} = I_{E2} = \frac{I_Q}{2} \quad I_{B1} = I_{B2} = \frac{I_Q}{2(1 + \beta)} \]

\[ I_{C1} = I_{C2} = \frac{\alpha I_Q}{2} \]

(b) Verify that $V_{CB} > 0$ for the active mode.

\[ V_{CB} = V_C - V_B = (V^+ - \alpha I_E R_C) - \left( -\frac{I_E}{1+\beta} R_B \right) = V^+ - \alpha I_E R_C + \frac{I_E}{1+\beta} R_B \]

(e) Calculate the collector-emitter voltage.

\[ V_{CE} = V_C - V_E = V_C - (V_B - V_{BE}) = V_{CB} + V_{BE} \]
Small-Signal AC Solution using the Emitter Equivalent Circuit

This solution uses the \( r_0 \) approximations and assumes that the base spreading resistance \( r_x \) is not zero.

(a) Calculate \( g_m, r_\pi, r_e, \) and \( r_{ie} \).

\[
\begin{align*}
g_m &= \frac{I_C}{V_T} \\
r_\pi &= \frac{V_T}{I_B} \\
r_e &= \frac{V_T}{I_E} \\
r_{ie} &= \frac{R_B + r_x + r_\pi}{1 + \beta} \\
r_0 &= \frac{V_A + V_{CE}}{I_E}
\end{align*}
\]

(b) Redraw the circuit with \( V^+ = V^- = 0 \). Replace the two BJTs with the emitter equivalent circuit. The emitter part of the circuit obtained is shown in Fig. 2(a).

![Emitter equivalent circuit for \( i_{e1} \) and \( i_{e2} \).](image)

(c) Using Ohm’s Law, solve for \( i_{e1} \) and \( i_{e2} \).

\[
i_{e1} = \frac{v_{i1} - v_{i2}}{2(r_{ie} + R_E)} \\
i_{e2} = -i_{e1}
\]

(d) The circuit for \( v_{o1}, v_{o2}, \) and \( r_{out} \) is shown in Fig. 2(b).

\[
\begin{align*}
v_{o1} &= -i_{e1(sc)} \times r_{ic} || R_C = -\alpha \times i_{e1} \times r_{ic} || R_C = \frac{-\alpha \times r_{ic} || R_C}{2(r_{ie} + R_E)} (v_{i1} - v_{i2}) \\
v_{o2} &= -i_{e2(sc)} \times r_{ic} || R_C = -\alpha \times i_{e2} \times r_{ic} || R_C = \frac{-\alpha \times r_{ic} || R_C}{2(r_{ie} + R_E)} (v_{i2} - v_{i1}) \\
r_{out1} &= r_{out2} = r_{ic} || R_C \\
r_{ic} &= r_0 \left[ 1 + \frac{\beta (2R_E + r_{ie})}{R_B + r_\pi + r_x + 2R_E + r_{ie}} \right] + (R_B + r_x + r_\pi) || (2R_E + r_{ie})
\end{align*}
\]

(e) The resistance seen looking into either input with the other input zeroed is

\[
r_{in} = R_B + r_x + r_\pi + (1 + \beta) (2R_E + r_{ie})
\]

The differential input resistance \( r_{ind} \) is the resistance between the two inputs for differential input signals. For an ideal current source tail supply, this is the same as the input resistance \( r_{in} \) above.

**Diff Amp with Non-Perfect Tail Supply**

Fig. 3 shows the circuit diagram of a differential amplifier. The tail supply is modeled as a current source \( I_Q' \) having a parallel resistance \( R_Q \). In the case of an ideal current source, \( R_Q \) is an open circuit. Often a diff amp is designed with a resistive tail supply. In this case, \( I_Q' = 0 \). The solutions below are valid for each of these connections. The object is to solve for the small-signal output voltages and output resistances.
DC Solutions

This solution assumes that $I'_Q$ is known. If $I_Q$ is known, the solutions are the same as above.

(a) Zero both inputs. Divide the tail supply into two equal parallel current sources having a current $I'_Q/2$ in parallel with a resistor $2R_Q$. The circuit obtained for $Q_1$ is shown on the left in Fig. 4. The circuit for $Q_2$ is identical. Now make a Thévenin equivalent as shown in on the right in Fig. 4. This is the basic bias circuit.

(b) Make an “educated guess” for $V_{BE}$. Write the loop equation between the ground node to the left of $R_B$ and $V^-$. To solve for $I_E$, this equation is

$$0 - (V^- - I'_QR_Q) = \frac{I_E}{1+\beta}R_B + V_{BE} + I_E(R_E + 2R_Q)$$

(c) Solve the loop equation for the currents.

$$I_E = \frac{I_C}{\alpha} = (1 + \beta)I_B = \frac{-V^- + I'_QR_Q - V_{BE}}{R_B/(1 + \beta) + R_E + 2R_Q}$$

(d) Verify that $V_{CB} > 0$ for the active mode.

$$V_{CB} = V_C - V_B = (V^+ - \alpha I_E R_C) - \left(-\frac{I_E}{1+\beta}R_B\right) = V^+ - \alpha I_E R_C + \frac{I_E}{1+\beta}R_B$$

(e) Calculate the collector-emitter voltage.

$$V_{CE} = V_C - V_E = V_C - (V_B - V_{BE}) = V_{CB} + V_{BE}$$

(f) If $R_Q = \infty$, it follows that $I_{E1} = I_{E2} = I'_Q/2$. If the current source is replaced with a resistor $R_Q$ only, the currents are given by

$$I_E = \frac{I_C}{\alpha} = (1 + \beta)I_B = \frac{-V^- - V_{BE}}{R_B/(1 + \beta) + R_E + 2R_Q}$$
Small-Signal or AC Solutions

This solutions use the $r_0$ approximations.

(a) Calculate $g_m$, $r_\pi$, and $r_{ie}$.

$$g_m = \frac{\alpha I_E}{V_T}, \quad r_\pi = \frac{(1 + \beta) V_T}{I_E}, \quad r_{ie} = \frac{R_B + r_x + r_\pi}{1 + \beta}, \quad r_0 = \frac{V_A + V_{CE}}{\alpha I_E}$$

(b) Redraw the circuit with $V^+ = V^- = 0$ and $I'_Q = 0$. Replace the two BJTs with the emitter equivalent circuit. The emitter part of the circuit obtained is shown in 5(a).

(c) Using superposition, Ohm's Law, and current division, solve for $i_{e1}$ and $i_{e2}$.

$$i_{e1} = \frac{v_{i1}}{r_{ie} + R_E + R_Q \| (r_{ie} + R_E)} - \frac{v_{i2}}{r_{ie} + R_E + R_Q \| (r_{ie} + R_E)} \times \frac{R_Q}{R_Q + r_{ie} + R_E}$$
\[ i_{c2} = \frac{v_{i2}}{r_{ie} + R_E + R_Q \| (r_{ie} + R_E)} - \frac{v_{i1}}{r_{ie} + R_E + R_Q \| (r_{ie} + R_E)} \times \frac{R_Q}{R_Q + r_{ie} + R_E} \]

For \( R_Q = \infty \), these become

\[ i_{e1} = \frac{v_{i1} - v_{i2}}{2(r_{ie} + R_E)} \quad i_{e2} = \frac{v_{i2} - v_{i1}}{2(r_{ie} + R_E)} \]

(d) The circuit for \( v_{o1}, v_{o2}, r_{out1}, \) and \( r_{out2} \) is shown in Fig. 6.

\[ r_{out1} = r_{out2} = r_{ic} R_C \]

\[ r_{ic} = r_0 \left[ 1 + \frac{\beta R_{te}}{R_B + r_x + R_{te}} \right] + \left( R_B + r_x + r_\pi \right) R_{te} \quad R_{te} = R_E + R_Q \| (r_{ie} + R_E) \]

(e) The resistance seen looking into the \( v_{i1} (v_{i2}) \) input with \( v_{i2} = 0 \) (\( v_{i1} = 0 \)) is

\[ r_{ib} = R_B + r_x + r_\pi + (1 + \beta) R_{te} \]

(f) Special case for \( R_Q = \infty \).

\[ v_{o1} = -\alpha \times r_{ic} R_C \left( v_{i1} - v_{i2} \right) \quad v_{o2} = -\alpha \times r_{ic} R_C \left( v_{i2} - v_{i1} \right) \]

(g) The equivalent circuit seen looking into the two inputs is shown in Fig. 7. The resistors labeled \( r_\pi' \) are given by

\[ r_\pi' = r_x + r_\pi + (1 + \beta) R_E \]

The differential input resistance \( r_{id} \) is defined the same way that it is defined for Fig. 6. That is, it is the resistance seen between the two inputs when \( v_{i1} = v_{id}/2 \) and \( v_{i2} = -v_{id}/2 \), where \( v_{id} \) is the differential input voltage. In this case, the small-signal voltage at the upper node of the resistor \( (1 + \beta) R_Q \) is zero so that no current flows it. It follows that \( r_{id} \) is given by

\[ r_{id} = 2 \left( R_B + r_\pi' \right) \]
Figure 7: Equivalent circuits for calculating \(i_{b1}\) and \(i_{b2}\).

**Differential and Common-Mode Gains**

(a) Define the common-mode and differential input voltages as follows:

\[
v_{id} = v_{i1} - v_{i2} \quad v_{icm} = \frac{v_{i1} + v_{i2}}{2}
\]

With these definitions, \(v_{i1}\) and \(v_{i2}\) can be written

\[
v_{i1} = v_{icm} + \frac{v_{id}}{2} \quad v_{i2} = v_{icm} - \frac{v_{id}}{2}
\]

By linearity, it follows that superposition of \(v_{icm}\) and \(v_{id}\) can be used to solve for the currents and voltages.

(b) Redraw the emitter equivalent circuit as shown in Fig. 8.

(c) For \(v_{i1} = v_{id}/2\) and \(v_{i2} = -v_{id}/2\), it follows by superposition that \(v_a = 0\) and

\[
i_{e1} = \frac{v_{id}/2}{r_{ie} + R_E} \quad i_{e2} = \frac{-v_{id}/2}{r_{ie} + R_E}
\]

\[
v_{o1} = -\alpha \times i_{e1} \times r_{ic} || R_C = \frac{-\alpha \times r_{ic} || R_C}{r_{ie} + R_E} \left( v_{i1} - v_{i2} \right) = \frac{-\alpha \times r_{ic} || R_C}{r_{ie} + R_E} \left( \frac{v_{i1} - v_{i2}}{2} \right)
\]

\[
v_{o2} = -\alpha \times i_{e2} \times r_{ic} || R_C = \frac{+\alpha \times r_{ic} || R_C}{r_{ie} + R_E} \left( v_{i1} - v_{i2} \right) = \frac{+\alpha \times r_{ic} || R_C}{r_{ie} + R_E} \left( \frac{v_{i1} - v_{i2}}{2} \right)
\]
The common-mode voltage gain is given by

$$A_{cm} = \frac{v_{o1}}{v_{icm}} = \frac{v_{o2}}{v_{icm}} = -\frac{\alpha \times r_{ie}||R_C}{r_{ie} + R_E + 2R_Q}$$

(e) If the output is taken from the collector of $Q_1$ or $Q_2$, the common-mode rejection ratio is given by

$$CMRR = \left| \frac{v_{o1}/v_{id}}{v_{o2}/v_{icm}} \right| = \left| \frac{v_{o2}/v_{id}}{v_{o2}/v_{icm}} \right| = \frac{1}{2} + \frac{R_Q}{r_{ie} + R_E}$$

This can be expressed in dB.

$$CMRR_{dB} = 20 \log \left( \frac{1}{2} + \frac{R_Q}{r_{ie} + R_E} \right)$$

Example 1 For $I_Q = 2$ mA, $R_Q = 50$ k$\Omega$, $R_B = 1$ k$\Omega$, $R_E = 100$ $\Omega$, $R_C = 10$ k$\Omega$, $V^+ = 20$ V, $V^- = -20$ V, $V_T = 0.025$ V, $r_x = 20$ $\Omega$, $\beta = 99$, $V_{BE} = 0.65$ V, and $V_A = 50$ V, calculate $v_{o1}$, $v_{o2}$, $v_{od}$, $r_{out}$, and $CMRR$.

Solution.

$$I_E = \frac{0 - (V^- - I_Q R_Q)}{R_B/(1 + \beta) + R_E + 2R_Q} = 1.192 \text{ mA}$$

$$V_{CB} = V_C - V_B = (V^+ - \alpha I_E R_C) - \left( -\frac{I_E}{1 + \beta} R_B \right) = 8.209 \text{ V}$$

$$g_m = \frac{\alpha I_E}{V_T} = 0.0472 \text{ S} \quad r_\pi = \frac{(1 + \beta) V_T}{I_E} = 2.097 \text{ k$\Omega$}$$

$$r_e = \frac{V_T}{I_E} = 20.97 \text{ $\Omega$} \quad r_{ie} = \frac{R_B + r_x}{1 + \beta} + r_e = 31.17 \text{ $\Omega$}$$

$$r_0 = \frac{V_A + V_{CE}}{I_C} = 49.869 \text{ k$\Omega$} \quad R_{te} = R_E + R_Q \parallel (r_{ie} + R_E) = 230.83 \text{ $\Omega$}$$

$$r_{ie} = r_0 \left[ 1 + \frac{\beta (2R_E + r_{ie})}{R_B + r_\pi + 2R_E + r_{ie}} \right] + (R_B + r_\pi) \parallel (2R_E + r_{ie}) = 390.5 \text{ k$\Omega$}$$

$$v_{o1} = \frac{-\alpha \times r_{ie}||R_C}{r_{ie} + R_E + R_Q \parallel (r_{ie} + R_E)} \left( v_{i1} - v_{i2} \frac{R_Q}{R_Q + r_{ie} + R_E} \right) = -36.84 v_{i1} + 36.75 v_{i2}$$

$$v_{o2} = -36.84 v_{i2} + 36.75 v_{i1}$$
\[ r_{\text{out}} = r_{\text{iC}}R_C = 9.75 \text{k}\Omega \]
\[ A_{\text{vd}} = -\frac{1}{2} \frac{\alpha \times r_{\text{iC}}R_C}{r_{\text{ie}} + R_E} = -36.80 \]
\[ A_{\text{vcm}} = -\frac{\alpha \times r_{\text{iC}}R_C}{r_{\text{ie}} + R_E + 2R_Q} = -0.0964 \]
\[ CMRR_{\text{dB}} = 20 \log \left| \frac{A_{\text{vd}}}{A_{\text{vcm}}} \right| = 51.63 \text{ dB} \]

The Diff Amp with an Active Load

Figure 9 shows a BJT diff amp with an active load formed by a current mirror with base current compensation. Similar circuits are commonly seen as the input stages of operational amplifiers and audio amplifiers. The object is to solve for the open-circuit output voltage \( v_{\text{oc}} \), the short-circuit output current \( i_{\text{sc}} \), and the output resistance \( r_{\text{out}} \). By Thévenin’s theorem, these are related by the equation \( v_{\text{oc}} = i_{\text{sc}}r_{\text{out}} \). It will be assumed that the current mirror consisting of transistors \( Q_3 - Q_5 \) is perfect so that its output current is equal to its input current, i.e. \( i_{\text{c4}} = i_{\text{c1}} \). In addition, the \( r_0 \) approximations will be used in solving for the currents. That is, the Early effect will be neglected except in solving for \( r_{\text{out}} \). For the bias solution, it will be assumed that the tail bias current \( I_Q \) splits equally between \( Q_1 \) and \( Q_2 \) so that \( I_{E1} = I_{E2} = I_Q/2 \).

Figure 9: Diff amp with active current-mirror load.

Because the tail supply is assumed to be a current source, the common-mode gain of the circuit is zero when the \( r_0 \) approximations are used. In this case, it can be assumed that the two input signals are pure differential signals that can be written \( v_{i1} = v_{id}/2 \) and \( v_{i2} = -v_{id}/2 \). For differential input signals, it follows by symmetry that the signal voltage is zero at the node above the tail current.
supply $I_Q$. Following the analysis above, the small-signal collector currents in $Q_1$ and $Q_2$ are given by

$$i_{c1(sc)} = \frac{\alpha}{r_{ie} + R_E} v_{id} \quad i_{c2(sc)} = -\frac{\alpha}{r_{ie} + R_E} v_{id}$$

where

$$r_{ie} = \frac{R_B + r_x}{1 + \beta}$$

The short-circuit output current is given by

$$i_{sc} = i_{c4} - i_{c2}$$

With $i_{c4} = i_{c3} = i_{c1}$ and $i_{c2} = -i_{c1}$, this becomes

$$i_{sc} = 2i_{c1(sc)} = \frac{\alpha}{r_{ie} + R_E} v_{id} = \frac{\alpha}{r_{ie} + R_E} (v_{i1} - v_{i2})$$

The output resistance is given by

$$r_{out} = r_{04} || r_{ie2}$$

where $r_{ie2}$ is given by

$$r_{ie2} = r_0 \left[ 1 + \frac{\beta (2R_E + r_{ie})}{R_B + r_x + 2R_E + r_{ie}} \right] + (R_B + r_x) \| (2R_E + r_{ie})$$

By Thévenin’s theorem, the small-signal open-circuit output voltage is given by

$$v_{oc} = i_{sc} r_{out} = \frac{\alpha (r_{04} || r_{ie2})}{r_{ie} + R_E} (v_{i1} - v_{i2})$$

**Example 2** For $I_Q = 2\, mA$, $R_B = 100\, \Omega$, $R_E = 51\, \Omega$, $V^+ = 15\, V$, $V^- = -15\, V$, $V_T = 0.025\, V$, $r_x = 50\, \Omega$, $\beta = 99$, $\alpha = 0.99$ $V_{BE1} = V_{BE2} = 0.65\, V$, $V_{EB3} = V_{EB4} = V_{EB5} = 0.65\, V$, $V_C2 = V_C4 = 13.7\, V$, and $V_A = 50\, V$, calculate $i_{sc}$, $r_{out}$, and $v_{oc}$. Because $r_x > 0$, we add it to $R_B$ in the equations above.

**Solution.**

$$r_{e1} = r_{e2} = \frac{2V_T}{I_Q} = 25\, \Omega \quad r_{ie1} = r_{ie2} = \frac{R_B + r_x}{1 + \beta} + r_e = 26.5\, \Omega$$

$$R_{te2} = 2R_E + r_{ie1} = 128.5\, \Omega \quad i_{sc} = \frac{\alpha}{r_{ie} + R_E} (v_{i1} - v_{i2}) = 0.0128 (v_{i1} - v_{i2})$$

$$r_{02} = \frac{V_A + (V_C2 + V_{BE})}{\alpha I_Q/2} = 65\, k\Omega \quad r_{ie2} = r_0 \left[ 1 + \frac{\beta R_{te2}}{R_B + r_x + R_{te2}} \right] + (R_B + r_x) || R_{te2} = 362.7\, k\Omega$$

$$r_{04} = \frac{V_A + (V^+ - V_C4)}{I_Q} = 51.82\, k\Omega$$

$$r_{out} = r_{04} || r_{ie2} = 45.34\, k\Omega$$

$$v_{oc} = i_{sc} r_{out} = 579.2 (v_{i1} - v_{i2})$$

This is a dB gain of 55.3 dB.