The Common-Emitter Amplifier

Basic Circuit

Fig. 1 shows the circuit diagram of a single stage common-emitter amplifier. The object is to solve for the small-signal voltage gain, input resistance, and output resistance.

DC Solution

(a) Replace the capacitors with open circuits. Look out of the 3 BJT terminals and make Thévenin equivalent circuits as shown in Fig. 2.

\[ V_{BB} = \frac{V^+ R_2 + V^- R_1}{R_1 + R_2} \quad R_{BB} = R_1 \parallel R_2 \quad V_{EE} = V^- \quad R_{EE} = R_E \]

(b) Make an “educated guess” for \( V_{BE} \). Write the loop equation between the \( V_{BB} \) and the \( V_{EE} \) nodes.

\[ V_{BB} - V_{EE} = I_B R_{BB} + V_{BE} + I_E R_{EE} = \frac{I_C}{\beta} R_{BB} + V_{BE} + \frac{I_C}{\alpha} R_{EE} \]

(c) Solve the loop equation for the currents.

\[ I_C = \alpha I_E = \beta I_B = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/\beta + R_{EE}/\alpha} \]

(d) Verify that \( V_{CB} > 0 \) for the active mode.

\[ V_{CB} = V_C - V_B = (V_{CC} - I_C R_{CC}) - (V_{BB} - I_B R_{BB}) = V_{CC} - V_{BB} - I_C (R_{CC} - R_{BB}/\beta) \]
Figure 2: Bias circuit.

Figure 3: Signal circuit.
Small-Signal or AC Solutions

(a) Redraw the circuit with $V^+ = V^- = 0$ and all capacitors replaced with short circuits as shown in Fig. 3.

(b) Calculate $g_m$, $r_\pi$, $r_e$, and $r_0$ from the DC solution.

$$g_m = \frac{I_C}{V_T}, \quad r_\pi = \frac{V_T}{I_B}, \quad r_e = \frac{V_T}{I_E}, \quad r_0 = \frac{V_A + V_{CE}}{I_C}$$

(c) Replace the circuits looking out of the base and emitter with Thévenin equivalent circuits as shown in Fig. 4.

$$v_{tb} = v_s \frac{R_1 || R_2}{R_s + R_1 || R_2}, \quad R_{tb} = R_1 || R_2, \quad v_{te} = 0, \quad R_{te} = R_E || R_3$$

![Figure 4: Signal circuit with Thévenin base circuit.](image)

Exact Solution

This solution is based on the exact equivalent circuits developed in the more advanced notes on the BJT. It treats $r_0$ as a resistor from collector to emitter without the $r_0$ approximations.

(a) Replace the BJT in Fig. 4 with the Thévenin base circuit and the Norton collector circuit as shown in Fig. 5.

![Figure 5: Base and collector equivalent circuits.](image)
(b) Solve for $i_{c(sc)}$.

\[ i_{c(sc)} = G_{mb}v_{ib} = G_{mb}v_s \frac{R_1 \parallel R_2}{R_s + R_1 \parallel R_2} \]

\[ G_{mb} = \frac{\alpha}{r'_e + R_{te}} \frac{r_0 - R_{te}/\beta}{r_0 + R_{te}} \]

\[ r'_e = \frac{R_{ib} + r_x}{1 + \beta} + r_e \]

(c) Solve for $v_o$.

\[ v_o = -i_{c(sc)}r_{ic} \parallel R_C \| R_L = -G_{mb}v_s \frac{R_1 \parallel R_2}{R_s + R_1 \parallel R_2} r_{ic} \parallel R_C \| R_L \]

\[ r_{ic} = \frac{r_0 + r'_e R_{te}}{1 - \alpha R_{te} / (r'_e + R_{te})} \]

(d) Solve for the voltage gain.

\[ A_v = \frac{v_o}{v_s} = -G_{mb} \frac{R_1 \parallel R_2}{R_s + R_1 \parallel R_2} r_{ic} \parallel R_C \| R_L \]

(e) Solve for $r_{in}$.

\[ r_{in} = R_1 \parallel R_2 \parallel r_{ib} \]

\[ r_{ib} = r_x + r_\pi + R_{te} \frac{(1 + \beta) r_0 + R_{te}}{r_0 + R_{te} + R_{te}} \]

(f) Solve for $r_{out}$.

\[ r_{out} = r_{ic} \parallel R_C \]

(g) Special Case for $R_{te} = 0$.

\[ G_{mb} = \frac{\alpha}{r'_e} \]

\[ r_{ic} = r_0 \]

\[ r_{ib} = r_x + r_\pi \]

(h) Special Case for $r_0 = \infty$.

\[ G_{mb} = \frac{\alpha}{r'_e + R_{te}} \]

\[ r_{ic} = \infty \]

\[ r_{ib} = r_x + r_\pi + (1 + \beta) R_{te} \]

**Example 1** For the CE amplifier of Fig. 1, it is given that $R_s = 5 \, \text{k}\Omega$, $R_1 = 120 \, \text{k}\Omega$, $R_2 = 100 \, \text{k}\Omega$, $R_C = 4.3 \, \text{k}\Omega$, $R_E = 5.6 \, \text{k}\Omega$, $R_3 = 100 \, \Omega$, $R_L = 20 \, \text{k}\Omega$, $V^+ = 15 \, \text{V}$, $V^- = -15 \, \text{V}$, $V_{BE} = 0.65 \, \text{V}$, $\beta = 99$, $\alpha = 0.99$, $r_x = 20 \, \Omega$, $V_A = 100 \, \text{V}$ and $V_T = 0.025 \, \text{V}$. Solve for the gain $A_v = v_o/v_s$, the input resistance $r_{in}$, and the output resistance $r_{out}$. The capacitors can be assumed to be ac short circuits at the operating frequency.

**Solution.** For the dc bias solution, replace all capacitors with open circuits. The Thévenin voltage and resistance seen looking out of the base are

\[ V_{BB} = \frac{V^+ R_2 + V^- R_1}{R_1 + R_2} = -1.364 \, \text{V} \]

\[ R_{BB} = R_1 \parallel R_2 = 54.55 \, \text{k}\Omega \]

The Thévenin voltage and resistance seen looking out of the emitter are $V_{EE} = V^-$ and $R_{EE} = R_E$. The bias equation for $I_E$ is

\[ I_E = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/(1 + \beta) + R_{EE}} = 2.113 \, \text{mA} \]
To test for the active mode, we calculate the collector-base voltage

\[ V_{CB} = V_C - V_B = (V^+ - \alpha I_E R_C) - \left( V_{BB} - \frac{I_E}{1 + \beta} R_{BB} \right) = 8.521 \text{ V} \]

Because this is positive, the BJT is biased in its active mode.

For the small-signal ac analysis, we need \( r_0 \) and \( r_e \). To calculate \( r_0 \), we first calculate the collector-emitter voltage

\[ V_{CE} = V_{CB} + V_{BE} = 9.171 \text{ V} \]

It follows that \( r_0 \) and \( r_e \) have the values

\[ r_0 = \frac{V_A + V_{CE}}{\alpha I_E} = 52.18 \text{ k} \Omega \quad r_e = \frac{V_T}{I_E} = 11.83 \text{ k} \Omega \]

For the small-signal analysis, \( V^+ \) and \( V^- \) are zeroed and the three capacitors are replaced with ac short circuits. The Thévenin voltage and resistance seen looking out of the base are given by

\[ v_{tb} = v_s \frac{R_1 || R_2}{R_s + R_1 || R_2} = 0.916 v_s \quad R_{tb} = R_s || R_1 || R_2 = 4.58 \text{ k} \Omega \]

The Thévenin resistances seen looking out of the emitter and the collector are

\[ R_{te} = R_E || R_3 = 98.25 \text{ k} \Omega \quad R_{tc} = R_C || R_L = 3.539 \text{ k} \Omega \]

Next, we calculate \( r'_{e} \), \( G_{mb} \), \( r_{ic} \), and \( r_{ib} \).

\[ r'_{e} = \frac{R_{tb} + r_x}{1 + \beta} + r_e = 57.83 \text{ k} \Omega \]

\[ G_{mb} = \frac{\alpha}{r'_{e} + R_{te} || r_0} \frac{r_0 - R_{te} / \beta}{r_0 + R_{te}} = \frac{1}{157.8} \text{ S} \]

\[ r_{ic} = \frac{r_0 + r'_{e} || R_{te}}{1 - \alpha R_{te} / (r'_{e} + R_{te})} = 138.6 \text{ k} \Omega \]

\[ r_{ib} = r_x + (1 + \beta) r_e + R_{te} \frac{(1 + \beta) r_0 + R_{te}}{r_0 + R_{te} + R_{tc}} = 10.39 \text{ k} \Omega \]

The output voltage is given by

\[ v_o = -G_{mb} \times (r_{ic} || R_{tc}) v_{tb} = -G_{mb} \times (r_{ic} || R_{tc}) \times 0.916 v_s = -20.04 v_s \]

Thus the voltage gain is

\[ A_v = -20.04 \]

The input and output resistances are given by

\[ r_{in} = R_1 || R_2 || r_{ib} = 8.73 \text{ k} \Omega \quad r_{out} = r_{ic} || R_C = 3.539 \text{ k} \Omega \]

**Approximate Solutions**

These solutions use the \( r_0 \) approximations. That is, it is assumed that \( r_0 = \infty \) except in calculating \( r_{ic} \). In this case, \( i_{c(sc)} = i'_{c} = \alpha i'_e = \beta i_b \).
Figure 6: Simplified T model circuit.

**Simplified T Model Solution**

(a) After making the Thévenin equivalent circuits looking out of the base and emitter, replace the BJT with the simplified T model as shown in Fig. 6.

(b) Solve for \( i'_e \).

\[
i'_e = \frac{v_{tb}}{r'_e + R_{te}} = v_s \frac{R_1||R_2}{R_s + R_1||R_2 r'_e + R_{te}} \frac{1}{\alpha}
\]

(b) Solve for \( i'_c \) and \( r_{ic} \).

\[
i'_c = \alpha i'_e = v_s \frac{R_1||R_2}{R_s + R_1||R_2 r'_e + R_{te}} \frac{\alpha}{\alpha}
\]

\[
r_{ic} = \frac{r_0 + r'_e R_{te}}{1 - \alpha R_{te}/(r'_e + R_{te})}
\]

(c) Solve for \( v_o \) and \( A_v = v_o/v_s \).

\[
v_o = -i_{c(sc)} r_{ic} R_C||R_L = v_s \frac{R_1||R_2}{R_s + R_1||R_2 r'_e + R_{te}} \frac{\alpha}{\alpha} \times -r_{ic} R_C||R_L
\]

\[
A_v = \frac{v_o}{v_s} = \frac{v_{ib}}{v_s} \times \frac{i'_e}{i'_e} \times \frac{v_o}{i'_c}
\]

(d) Solve for \( r_{out} \).

\[
r_{out} = r_{ic} R_C
\]

(d) Solve for \( r_{ib} \) and \( r_{in} \). Because the base node is absorbed, use the formula for \( r_{ib} \).

\[
r_{ib} = r_x (1 + \beta) (r_e + R_{te}) \quad r_{in} = R_1||R_2||r_{ib}
\]

**Example 2** Use the simplified T-model solutions to calculate the values of \( A_v \), \( r_{in} \), and \( r_{out} \) for Example 1.

\[
A_v = 0.916 \times (6.343 \times 10^{-3}) \times (-3.451 \times 10^3) = -20.05
\]

\[
r_{ib} = 1.103 \, \text{kΩ} \quad r_{in} = 9.173 \, \text{kΩ}
\]

\[
r_{ic} = 138.6 \, \text{kΩ} \quad r_{out} = 4.171 \, \text{kΩ}
\]
(a) After making the Thévenin equivalent circuits looking out of the base and emitter, replace the BJT with the π model as shown in Fig. 7.

\[ v_{tb} = i_b (R_{tb} + r_x) + v_{\pi} + i_c R_{te} = \frac{v_c'}{\beta} (R_{tb} + r_x) + \frac{v_c'}{g_m} + \frac{v_c'}{\alpha} R_{te} \implies i_c' = \frac{v_{tb}}{\beta} + \frac{1}{g_m} + \frac{R_{te}}{\alpha} \]

\[ r_{ic} = \frac{r_0 + v_{e'} R_{te}}{1 - \alpha R_{te}/(r_{e'} + R_{te})} \]

(b) Solve for \( i_c' \) and \( r_{ic} \).

(c) Solve for \( v_o \).

\[ v_o = i_c' R_C || R_L = \frac{v_{tb}}{\beta} + \frac{1}{g_m} + \frac{R_{te}}{\alpha} \times -r_{ic} || R_C || R_L \]

\[ = v_o' \frac{R_1 || R_2}{R_s + R_1 || R_2} \frac{1}{\beta} \frac{1}{R_{tb} + r_x} + \frac{1}{g_m} + \frac{R_{te}}{\alpha} \times -r_{ic} || R_C || R_L \]

(d) Solve for the voltage gain.

\[ A_v = \frac{v_o}{v_s} = \frac{R_1 || R_2}{R_s + R_1 || R_2} \frac{1}{\beta} \frac{1}{R_{tb} + r_x} + \frac{1}{g_m} + \frac{R_{te}}{\alpha} \times -r_{ic} || R_C || R_L \]

This is of the form

\[ A_v = \frac{v_o}{v_s} = \frac{v_{tb}}{v_s} \times \frac{i_c'}{v_{tb}} \times \frac{v_o}{i_c'} \]

(e) Solve for \( r_{ib} \) and \( r_{in} \).

\[ v_b = i_b (r_x + r_{\pi}) + i_c' R_{te} = i_b (r_x + r_{\pi}) + (1 + \beta) i_b R_{te} = i_b [r_x + r_{\pi} + (1 + \beta) R_{te}] \]

\[ r_{ib} = \frac{v_b}{i_b} = r_x + r_{\pi} + (1 + \beta) R_{te} \]
\[ r_{in} = R_1 \| R_2 \| r_{ib} \]

(f) Solve for \( r_{out} \).

\[ r_{out} = r_{ic} \| R_C \]

**Example 3** Use the \( \pi \)-model solutions to calculate the values of \( A_v, r_{in}, \) and \( r_{out} \) for Example 1.

\[ g_m = 0.0837 \quad r_\pi = 1.183 \, \text{k}\Omega \]

\[ A_v = 0.916 \times (6.343 \times 10^{-3}) \times (-3.451 \times 10^3) = -20.05 = -20.05 \]

\[ r_{ib} = 11.03 \, \text{k}\Omega \quad r_{in} = 9.173 \, \text{k}\Omega \]

\[ r_{ic} = 138.6 \, \text{k}\Omega \quad r_{out} = 4.171 \, \text{k}\Omega \]

**T Model Solution**

(a) After making the Thévenin equivalent circuits looking out of the base and emitter, replace the BJT with the T model as shown in Fig. 8.

\[ \text{Figure 8: T model circuit.} \]

(b) Solve for \( \dot{i}_c' \) and \( r_{ic} \).

\[ v_{tb} = i_b (R_{tb} + r_x) + i_e' (r_e + R_{te}) = \frac{\dot{i}_c'}{\beta} (R_{tb} + r_x) + \frac{\dot{i}_e'}{\alpha} (r_e + R_{te}) \implies \dot{i}_c' = \frac{v_{tb}}{R_{tb} + r_x} + \frac{r_e + R_{te}}{\alpha} \]

\[ r_{ic} = \frac{r_0 + r_e' \| R_{te}}{1 - \alpha R_{te} / (r_e + R_{te})} \]

(c) Solve for \( v_o \).

\[ v_o = -\dot{i}_c' R_C \| R_L = \frac{v_{tb}}{R_{tb} + r_x} + \frac{r_e + R_{te}}{\alpha} \times -r_{ic} \| R_C \| R_L \]

\[ = v_s \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \frac{1}{\frac{R_{tb} + r_x}{\beta} + \frac{r_e + R_{te}}{\alpha}} \times -r_{ic} \| R_C \| R_L \]
(d) Solve for the voltage gain.

\[ A_v = \frac{v_o}{v_s} = \frac{R_1 R_2}{R_s + R_1 R_2} \frac{1}{\beta} + \frac{r_x + R_{te}}{\alpha} \times -r_{ic} R_C R_L \]

Note that this is of the form

\[ A_v = \frac{v_o}{v_s} = \frac{v_{tb}}{v_s} \times \frac{i_c'}{i_c} \times \frac{v_o}{v_t} \]

(e) Solve for \( r_{ib} \) and \( r_{in} \).

\[ v_b = i_b r_x + i_c' (r_e + R_{te}) = i_b r_x + (1 + \beta) i_b (r_e + R_{te}) = i_b [r_x + (1 + \beta) (r_e + R_{te})] \]

\[ r_{ib} = \frac{v_b}{i_b} = r_x + (1 + \beta) (r_e + R_{te}) \]

\[ r_{in} = R_1 R_2 r_{ib} \]

(f) Solve for \( r_{out} \).

\[ r_{out} = r_{ic} R_C \]

Example 4 Use the T-model solutions to calculate the values of \( A_v \), \( r_{in} \), and \( r_{out} \) for Example 1.

\[ A_v = 0.916 \times (6.343 \times 10^{-3}) \times (-3.451 \times 10^3) = -20.05 = -20.05 \]

\[ r_{ib} = 11.03 \text{ k\Omega} \quad r_{in} = 9.173 \text{ k\Omega} \]

\[ r_{ic} = 138.6 \text{ k\Omega} \quad r_{out} = 4.171 \text{ k\Omega} \]