The Common-Emitter Amplifier

Basic Circuit

Fig. 1 shows the circuit diagram of a single stage common-emitter amplifier. The object is to solve for the small-signal voltage gain, input resistance, and output resistance.

DC Solution

(a) Replace the capacitors with open circuits. Look out of the 3 BJT terminals and make Thévenin equivalent circuits as shown in Fig. 2.

\[ V_{BB} = \frac{V^+R_2 + V^-R_1}{R_1 + R_2} \quad R_{BB} = R_1 || R_2 \quad V_{EE} = V^- \quad R_{EE} = R_E \]

(b) Make an “educated guess” for \( V_{BE} \). Write the loop equation between the \( V_{BB} \) and the \( V_{EE} \) nodes.

\[ V_{BB} - V_{EE} = I_B R_{BB} + V_{BE} + I_E R_{EE} = \frac{I_C}{\beta} R_{BB} + V_{BE} + \frac{I_C}{\alpha} R_{EE} \]

(c) Solve the loop equation for the currents.

\[ I_C = \alpha I_E = \beta I_B = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/\beta + R_{EE}/\alpha} \]

(d) Verify that \( V_{CB} > 0 \) for the active mode.

\[ V_{CB} = V_C - V_B = (V_{CC} - I_C R_{CC}) - (V_{BB} - I_B R_{BB}) = V_{CC} - V_{BB} - I_C (R_{CC} - R_{BB}/\beta) \]
Small-Signal Solution

It will be assumed that the base spreading resistance \( r_x \) is non zero. This is a resistance in series with the base lead in the small signal models.

(a) Redraw the circuit with \( V^+ = V^- = 0 \) and all capacitors replaced with short circuits as shown in Fig. 3.

(b) Calculate \( g_m, r_\pi, r_e, \) and \( r_0 \) from the DC solution.

\[
\begin{align*}
g_m &= \frac{I_C}{V_T} \\
r_\pi &= \frac{V_T}{I_B} \\
r_e &= \frac{V_T}{I_E} \\
r_0 &= \frac{V_A + V_{CE}}{I_C}
\end{align*}
\]

(c) Replace the circuits looking out of the base and emitter with Thévenin equivalent circuits as shown in Fig. 4.

\[
v_{tb} = v_s \frac{R_1 R_2}{R_s + R_1 R_2} \\
R_{tb} = R_s R_1 R_2 \\
v_{te} = 0 \\
R_{te} = R_E R_3
\]

(a) Replace the BJT in Fig. 4 with the Thévenin base circuit and the Norton collector circuit as shown in Fig. 5.
Figure 4: Signal circuit with Thévenin base circuit.

Figure 5: Base and collector equivalent circuits.
(b) Solve for $G_m$.

$$G_m = \frac{i_{c(sc)}}{v_{tb}} = \frac{i'_c}{v_{tb}} = \frac{1}{\frac{R_{tb} + r_x}{\beta} + \frac{1 + R_{tc}}{\alpha}}$$

(c) Solve for the voltage gain $A_v = v_o/v_s$. A flow graph is shown in Figure 6.

![Flow graph](image)

Figure 6: Flow graph for the voltage gain.

$$A_v = \frac{v_o}{v_s} = \frac{v_{tb}}{v_s} \times \frac{i'_e}{v_{tb}} \times \frac{v_o}{i'_c} = \frac{R_1||R_2}{R_s + R_1||R_2} \times G_m \times -\left(r_{ic}||R_{tc}\right)$$

(d) Solve for $r_{out}$.

$$r_{out} = r_{ic}||R_C \quad r_{ic} = r_0 \left(1 + \frac{\beta R_{te}}{R_{tb} + r_x + r_\pi + R_{te}}\right) + (R_{tb} + r_x + r_\pi) || R_{te}$$

(e) Solve for $r_{in}$.

$$r_{in} = R_1||R_2||r_{ib} \quad r_{ib} = r_x + r_\pi + (1 + \beta) \left(R_E||R_3\right)$$

Second Small-Signal Solution for $A_v$

The voltage gain can be written

$$A_v = \frac{v_o}{v_s} = \frac{v_{tb}}{v_s} \times \frac{i'_e}{v_{tb}} \times \frac{v_o}{i'_c} = \frac{R_1||R_2}{R_s + R_1||R_2} \times \frac{1}{R_{tb} + r_{ib}} \times \beta \times -\left(r_{ic}||R_{tc}\right)$$

Third Small-Signal Solution for $A_v$

The voltage gain can be written

$$A_v = \frac{v_o}{v_s} = \frac{v_{tb}}{v_s} \times \frac{i'_e}{v_{tb}} \times \frac{v_o}{i'_c} = \frac{R_1||R_2}{R_s + R_1||R_2} \times \frac{1}{r_{ie} + R_{te}} \times \alpha \times -\left(r_{ic}||R_{tc}\right)$$

Example 1 For the CE amplifier of Fig. 1, it is given that $R_s = 5 \, k\Omega$, $R_1 = 120 \, k\Omega$, $R_2 = 100 \, k\Omega$, $R_C = 4.3 \, k\Omega$, $R_E = 5.6 \, k\Omega$, $R_3 = 100 \, \Omega$, $R_L = 20 \, k\Omega$, $V^+ = 15 \, V$, $V^- = -15 \, V$, $V_{BE} = 0.65 \, V$, $r_x = 20 \, \Omega$, $\beta = 99$, $\alpha = 0.99$, $V_A = 100 \, V$ and $V_T = 0.025 \, V$. Solve for the gain $A_v = v_o/v_s$, the input resistance $r_{in}$, and the output resistance $r_{out}$. The capacitors can be assumed to be ac short circuits at the operating frequency.

Solution. For the dc bias solution, replace all capacitors with open circuits. The Thévenin voltage and resistance seen looking out of the base are

$$V_{BB} = \frac{V^+ R_2 + V^- R_1}{R_1 + R_2} = -1.364 \, V \quad R_{BB} = R_1||R_2 = 54.55 \, k\Omega$$
The Thévenin voltage and resistance seen looking out of the emitter are \( V_{EE} = V^− \) and \( R_{EE} = R_E \).

The bias equation for \( I_E \) is

\[
I_E = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/(1 + \beta) + R_{EE}} = 2.113 \text{mA} \quad I_C = \alpha I_E = 2.092 \text{mA} \quad I_B = \frac{I_E}{1 + \beta} = 21.13 \mu\text{A}
\]

To test for the active mode, we calculate the collector-base voltage

\[
V_{CB} = V_C - V_B = (V^+ - \alpha I_C R_C) - \left( V_{BB} - \frac{I_E}{1 + \beta} R_{BB} \right) = 8.521 \text{V}
\]

Because this is positive, the BJT is biased in its active mode.

For the small-signal ac analysis, we need \( r_0 \) and \( r_e \). To calculate \( r_0 \), we first calculate the collector-emitter voltage

\[
V_{CE} = V_{CB} + V_{BE} = 9.171 \text{V}
\]

It follows that \( r_0 \), \( g_m \), \( r_\pi \), and \( r_e \) have the values

\[
 r_0 = \frac{V_A + V_{CE}}{\alpha I_E} = 52.18 \text{k}\Omega \quad g_m = \frac{I_C}{V_T} = \frac{2.092}{25} = \frac{1}{11.95} \text{S}
\]

\[
r_\pi = \frac{V_T}{I_B} = \frac{\beta V_T}{I_C} = \frac{99 \times 25}{2.113} = 1.183 \text{k}\Omega \quad r_e = \frac{V_T}{I_E} = 11.83 \text{\Omega}
\]

For the small-signal analysis, \( V^+ \) and \( V^- \) are zeroed and the three capacitors are replaced with ac short circuits. The Thévenin voltage and resistance seen looking out of the base are given by

\[
v_{tb} = v_s \frac{R_1 \| R_2}{R_s + R_1 \| R_2} = 0.916 v_s \quad R_{tb} = R_s \| R_1 \| R_2 = 4.58 \text{k}\Omega
\]

The Thévenin resistances seen looking out of the emitter and the collector are

\[
R_{te} = R_E \| R_3 = 98.25 \text{\Omega} \quad R_{tc} = R_C \| R_L = 3.539 \text{\k}\Omega
\]

Note that the base spreading resistance \( r_x \) is not zero for this problem. Next, we calculate \( G_m \), \( r_{ic} \), and \( r_{ib} \).

\[
G_m = \frac{R_{tb} + r_x}{\beta} + \frac{1}{g_m} + \frac{R_{te}}{\alpha} = \frac{4580 + 20}{99} + 11.83 + \frac{98.25}{0.99} = \frac{1}{157.7} \text{S}
\]

\[
r_{ic} = r_0 \left( 1 + \frac{\beta R_{te}}{R_{tb} + r_x + r_\pi + R_{te}} \right) + (R_{tb} + r_x + r_\pi) \| R_{te} = 138.6 \text{k}\Omega
\]

\[
r_{ib} = r_x + r_\pi + (1 + \beta) R_{te} = 11.03 \text{k}\Omega
\]

The voltage is

\[
A_v = \frac{v_o}{v_s} = \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \times G_m \times (r_{ic} \| R_{tc}) = 0.916 \times \frac{1}{157.7} \times (138.6 \text{k} \| 3.539 \text{k}) = -20.05
\]

The input and output resistances are

\[
r_{in} = R_1 \| R_2 \| r_{ib} = 9.173 \text{k}\Omega \quad r_{out} = r_{ic} \| R_C = 4.17 \text{k}\Omega
\]