The Common-Base Amplifier

Basic Circuit

Fig. 1 shows the circuit diagram of a single stage common-base amplifier. The object is to solve for the small-signal voltage gain, input resistance, and output resistance.

![Common-Base Amplifier Circuit Diagram](image)

Figure 1: Common-base amplifier.

DC Solution

(a) Replace the capacitors with open circuits. Look out of the 3 BJT terminals and make Thévenin equivalent circuits as shown in Fig. 2.

\[
V_{BB} = \frac{V^+ R_2 + V^- R_1}{R_1 + R_2} \quad R_{BB} = R_1 || R_2
\]

\[
V_{EE} = V^- \quad R_{EE} = R_E \quad V_{CC} = V^+ \quad R_{CC} = R_C
\]

(b) Make an “educated guess” for \(V_{BE}\). Write the loop equation between the \(V_{BB}\) and the \(V_{EE}\) nodes. To solve for \(I_C\), this equation is

\[
V_{BB} - V_{EE} = I_B R_{BB} + V_{BE} + I_E R_{EE} = \frac{I_C}{\beta} R_{BB} + V_{BE} + \frac{I_C}{\alpha} R_{EE}
\]

(c) Solve the loop equation for the currents.

\[
I_C = \alpha I_E = \beta I_B = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/\beta + R_{EE}/\alpha}
\]

(d) Verify that \(V_{CB} > 0\) for the active mode.

\[
V_{CB} = V_C - V_B = (V_{CC} - I_C R_{CC}) - (V_{BB} - I_B R_{BB}) = V_{CC} - V_{BB} - I_C (R_{CC} - R_{BB}/\beta)
\]
Small-Signal or AC Solutions

(a) Redraw the circuit with $V^+ = V^- = 0$ and all capacitors replaced with short circuits as shown in Fig. 3.

(b) Calculate $g_m$, $r_\pi$, $r_e$, and $r_0$ from the DC solution.

$$g_m = \frac{I_C}{V_T} \quad r_\pi = \frac{V_T}{I_B} \quad r_e = \frac{V_T}{I_E} \quad r_0 = \frac{V_A + V_{CE}}{I_C}$$

(c) Replace the circuits looking out of the base and emitter with Thévenin equivalent circuits as shown in Fig. 4.

$$v_{ib} = 0 \quad R_{ib} = 0 \quad v_{te} = v_s \frac{R_E}{R_s + R_E} \quad R_{te} = R_s || R_E$$
Figure 4: Signal circuit with Thévenin emitter circuit.

**Exact Solution**

(a) Replace the BJT in Fig. 4 with the Thévenin emitter circuit and the Norton collector circuit as shown in Fig. 5.

(b) Solve for \( i_{c(sc)} \).

\[
    i_{c(sc)} = -G_{me}v_{te} = -G_{me}v_s \frac{R_E}{R_s + R_E}
\]

\[
    G_{me} = \frac{1}{R_{te} + r_e'\|r_0 + r_e} \quad r_e' = \frac{r_x}{1 + \beta} + r_e
\]

(c) Solve for \( v_o \).

\[
    v_o = -i_{c(sc)}r_{ic}\|R_C\|R_L = G_{me}v_s \frac{R_E}{R_s + R_E}r_{ic}\|R_C\|R_L
\]

\[
    r_{ic} = \frac{r_0 + r_e'\|R_{te}}{1 - \alpha R_{te}/(r_e' + R_{te})}
\]
(d) Solve for the voltage gain.

\[ A_v = \frac{v_o}{v_s} = \frac{R_E}{R_s + R_E} G_{me} r_{ic} || R_C || R_L \]

(e) Solve for \( r_{in} \).

\[ r_{in} = R_1 || R_2 || r_{ie} \]

\[ r_{ie} = r'_e \frac{r_0 + R_{ic}}{r'_e + r_0 + R_{ie}/(1 + \beta)} \]

(f) Solve for \( r_{out} \).

\[ r_{out} = r_{ic} || R_C \]

**Example 1** For the CB amplifier in Fig. 1, it is given that \( R_s = 100 \, \Omega \), \( R_1 = 120 \, k\Omega \), \( R_2 = 100 \, k\Omega \), \( R_C = 4.3 \, k\Omega \), \( R_E = 5.6 \, k\Omega \), \( R_3 = 100 \, \Omega \), \( R_L = 20 \, k\Omega \), \( V^+ = 15 \, V \), \( V^- = -15 \, V \), \( V_{BE} = 0.65 \, V \), \( \beta = 99 \), \( \alpha = 0.99 \), \( r_x = 20 \, \Omega \), \( V_A = 100 \, V \) and \( V_T = 0.025 \, V \). Solve for \( A_v \), \( r_{in} \), and \( r_{out} \).

**Solution.** Because the dc bias circuit is the same as for the common-emitter amplifier example, the dc bias values, \( r_e \), \( g_m \), \( r_x \), and \( r_0 \) are the same.

In the signal circuit, the Thévenin voltage and resistance seen looking out of the emitter are given by

\[ v_{te} = \frac{R_E}{R_s + R_E} v_s = 0.9825 v_s \]

\[ R_{te} = R_s || R_E = 98.25 \, \Omega \]

The Thévenin resistances seen looking out of the base and the collector are

\[ R_{tb} = 0 \]

\[ R_{tc} = R_C || R_L = 3.539 \, k\Omega \]

Next, we calculate \( r'_e, G_{me}, r_{ic}, \) and \( r_{ie} \).

\[ r'_e = \frac{R_{db} + r_x}{1 + \beta} + r_e = 12.03 \, \Omega \]

\[ G_{me} = \frac{1}{R_{te} + r'_e} = \frac{1}{1 + \beta} \]

\[ r_{ic} = \frac{r_0 + r'_e || R_{te}}{1 - \alpha R_{te}/(r'_e + R_{te})} = 442.3 \, k\Omega \]

\[ r_{ie} = r'_e \frac{r_0 + R_{tc}}{r'_e + r_0 + R_{ic}/(1 + \beta)} = 12.83 \, \Omega \]

The output voltage is given by

\[ v_o = G_{me} \left( r_{ic} || R_{tc} \right) v_{te} = G_{me} \left( r_{ic} || R_{tc} \right) \frac{R_E}{R_s + R_E} v_s = 30.97 v_s \]

Thus the voltage gain is

\[ A_v = 30.97 \]

The input and output resistances are

\[ r_{in} = R_1 || R_2 || r_{ib} = 12.81 \, \Omega \]

\[ r_{out} = r_{ic} || R_C = 4.259 \, k\Omega \]

**Approximate Solutions**

These solutions assume that \( r_0 = \infty \) except in calculating \( r_{ic} \). In this case, \( i_{c(sc)} = i'_c = \alpha i'_e = \beta i_b \).
Simplified T Model Solution

(a) After making the Thévenin equivalent circuits looking out of the base and emitter, replace the BJT with the simplified T model as shown in Fig. 6.

(b) Solve for $i'_c$ and $r_{ic}$.

$$0 - v_{te} = i'_e (r'_e + R_{te}) = \frac{i'_c}{\alpha} (r'_e + R_{te}) \implies i'_c = \frac{-v_{te}}{r'_e + R_{te}} \cdot \frac{\alpha}{\alpha}$$

$$r_{ic} = \frac{r_0 + r'_e R_{te}}{1 - \alpha R_{te} / (r'_e + R_{te})}$$

(c) Solve for $v_o$.

$$v_o = -i'_c r_{ic} R_C R_L = v_{te} \frac{\alpha}{r'_e + R_{te}} R_C R_L = v_s \frac{R_E}{R_s + R_E r'_e + R_{te}} \frac{\alpha}{r_{ic} R_C R_L}$$

(d) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_s} = \frac{R_s}{R_s + R_E r'_e + R_{te}} \frac{\alpha}{r_{ic} R_C R_L}$$

(e) Solve for $r_{ie}$ and $r_{in}$.

$$0 - v_e = i'_e r'_e \implies i'_e = \frac{-v_e}{r'_e}$$

$$r_{ie} = v_e \frac{1}{-i'_e} = r'_e$$

$$r_{in} = r'_e R_{E}$$

(f) Solve for $r_{out}$.

$$r_{out} = r_{ic} R_C$$

Example 2 For Example 1, use the simplified T-model solutions to calculate the values of $A_v$, $r_{in}$, and $r_{out}$. 
\[ A_v = 0.9825 \times (8.978 \times 10^{-3}) \times (3.511 \times 10^3) = 30.97 \]

\[ r_{in} = 12 \Omega \quad r_{out} = 4.259 \text{k}\Omega \]

**π Model Solution**

(a) After making the Thévenin equivalent circuits looking out of the base and emitter, replace the BJT with the π model as shown in Fig. 7.

![Figure 7: Hybrid-π model circuit.](image)

(b) Solve for \( i'_c \) and \( r_{ic} \).

\[
0 - v_{te} = i_b r_x + v_{\pi} + i'_c R_{te} = i'_c r_x + \frac{i'_c}{g_m} + \frac{i'_e R_{te}}{\alpha} \implies i'_c = \frac{-v_{te}}{r_x + \frac{1}{g_m} + \frac{R_{te}}{\alpha}}
\]

\[ r_{ic} = \frac{r_0 + r'_e || R_{te}}{1 - \alpha R_{te}/(r'_e + R_{te})} \]

(c) Solve for \( v_o \).

\[
v_o = -i'_e r_{ic} || R_C || R_L = \frac{v_{te}}{r_x + \frac{1}{g_m} + \frac{R_{te}}{\alpha}} r_{ic} || R_C || R_L = v_s \frac{R_E}{R_s + R_E} \frac{1}{r_x + \frac{1}{g_m} + \frac{R_{te}}{\alpha}} r_{ic} || R_C || R_L
\]

(d) Solve for the voltage gain.

\[
A_v = \frac{v_o}{v_s} = \frac{R_E}{R_s + R_E} \frac{1}{r_x + \frac{1}{g_m} + \frac{R_{te}}{\alpha}} r_{ic} || R_C || R_L
\]

(e) Solve for \( r_{out} \).

\[ r_{out} = r_{ic} || R_C \]
(f) Solve for \( r_{ie} \) and \( r_{in} \).

\[
0 - v_e = i_b (r_x + r_\pi) = \frac{v'_e}{1 + \beta} (r_x + r_\pi) \implies v'_e = -v_e \frac{1 + \beta}{r_x + r_\pi} \\
\]

\[
r_{ie} = \frac{v_e}{i'_e} = \frac{r_x + r_\pi}{1 + \beta} \\
r_{in} = r_{ie} || R_E
\]

**Example 3** For Example 1, use the \( \pi \)-model solutions to calculate the values of \( A_v, r_{in}, \) and \( r_{out} \).

\[
A_v = 0.9825 \times (8.978 \times 10^{-3}) \times (3.539 \times 10^3) = 30.97 \\
r_{in} = 12 \Omega \quad r_{out} = 4.259 \, k\Omega
\]

**T Model Solution**

(a) After making the Thévenin equivalent circuits looking out of the base and emitter, replace the BJT with the T model as shown in Fig. ??.

\[
\]

\[
\]

(b) Solve for \( i'_c \) and \( r_{ic} \).

\[
0 - v_{te} = i_b r_x + i'_c (r_e + R_{te}) = \frac{i'_c}{\beta} r_x + \frac{i'_c}{\alpha} (r_e + R_{te}) \implies i'_c = \frac{-v_{te}}{\beta} + \frac{r_x}{\alpha} \\
\]

\[
r_{ic} = \frac{r_0 + r'_e R_{te}}{1 - \alpha R_{te} / (r'_e + R_{te})}
\]

(c) Solve for \( v_o \).

\[
v_o = -i'_c r_{ic} || R_C || R_L = \frac{v_{te}}{\beta} + \frac{r_e + R_{te}}{\alpha} \frac{r_{ic} || R_C || R_L}{\beta} = v_s \frac{R_E}{R_s + R_E} \frac{r_x}{\beta} + \frac{1}{\alpha} \frac{r_e + R_{te}}{r_{ic} || R_C || R_L}
\]
(d) Solve for the voltage gain.

\[ A_v = \frac{v_o}{v_s} = \frac{R_E}{R_s + R_E} \frac{1}{\frac{r_x}{\beta} + \frac{r_e + R_{ie}}{\alpha}} R_C R_L \]

(e) Solve for \( r_{ie} \) and \( r_{in} \).

\[
0 - v_e = i_b r_x + i_e' r_e = \frac{i'_e r_x}{1 + \beta} + i'_e r_e = i'_e \left( \frac{r_x}{1 + \beta} + r_e \right) \Rightarrow i'_e = \frac{-v_e}{1 + \beta + r_e}
\]

\[
r_{ie} = \frac{v_e}{i'_e} = \frac{r_x}{1 + \beta} + r_e
\]

\[
r_{in} = R_E r_{ie}
\]

(f) Solve for \( r_{out} \).

\[
r_{out} = r_{ic} R_C
\]

**Example 4** For Example 1, use the T-model solutions to calculate the values of \( A_v \), \( r_{in} \), and \( r_{out} \).

\[
A_v = 0.9825 \times (8.978 \times 10^{-3}) \times (3.539 \times 10^3) = 30.97
\]

\[
r_{in} = 12 \Omega \quad r_{out} = 4.259 \, k\Omega
\]