The Common-Base Amplifier

Basic Circuit

Fig. 1 shows the circuit diagram of a single stage common-base amplifier. The object is to solve for the small-signal voltage gain, input resistance, and output resistance.

\[ V_{BB} = \frac{V^+ R_2 + V^- R_1}{R_1 + R_2} \quad R_{BB} = R_1 \parallel R_2 \]

\[ V_{EE} = V^- \quad R_{EE} = R_E \quad V_{CC} = V^+ \quad R_{CC} = R_C \]

(b) Make an “educated guess” for \( V_{BE} \). Write the loop equation between the \( V_{BB} \) and the \( V_{EE} \) nodes. To solve for \( I_C \), this equation is

\[ V_{BB} - V_{EE} = I_B R_{BB} + V_{BE} + I_E R_{EE} = \frac{I_C}{\beta} R_{BB} + V_{BE} + \frac{I_C}{\alpha} R_{EE} \]

(c) Solve the loop equation for the currents.

\[ I_C = \alpha I_E = \beta I_B = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/\beta + R_{EE}/\alpha} \]

(d) Verify that \( V_{CB} > 0 \) for the active mode.

\[ V_{CB} = V_C - V_B = (V_{CC} - I_C R_{CC}) - (V_{BB} - I_B R_{BB}) = V_{CC} - V_{BB} - I_C (R_{CC} - R_{BB}/\beta) \]
Small-Signal or AC Solutions

(a) Redraw the circuit with $V^+ = V^- = 0$ and all capacitors replaced with short circuits as shown in Fig. 3.

(b) Calculate $g_m$, $r_\pi$, $r_e$, and $r_0$ from the DC solution.

\[
g_m = \frac{I_C}{V_T} \quad r_\pi = \frac{V_T}{I_B} \quad r_e = \frac{V_T}{I_E} \quad r_0 = \frac{V_A + V_{CE}}{I_C}
\]

(c) Replace the circuits looking out of the base and emitter with Thévenin equivalent circuits as shown in Fig. 4.

\[
v_{bb} = 0 \quad R_{bb} = 0 \quad v_{te} = v_s \frac{R_E}{R_s + R_E} \quad R_{te} = R_s || R_E
\]
Figure 4: Signal circuit with Thévenin emitter circuit.

**Exact Solution**

(a) Replace the BJT in Fig. 4 with the Thévenin emitter circuit and the Norton collector circuit as shown in Fig. 5.

![Figure 5: Emitter and collector equivalent circuits.](image)

(b) Solve for $i_{c(sc)}$.

$$ i_{c(sc)} = -G_{me}v_{te} = -G_{me}v_s \frac{R_E}{R_s + R_E} $$

$$ G_{me} = \frac{1}{R_{te} + r_e^r} \frac{\alpha r_0 + r_e'}{r_0 + r_e'} \quad r_e' = \frac{r_x}{1 + \beta} + r_e $$

c) Solve for $v_o$.

$$ v_o = -i_{c(sc)}r_{ic}R_CR_L = G_{me}v_s \frac{R_E}{R_s + R_E}r_{ic}R_CR_L $$

$$ r_{ic} = \frac{r_0 + r_e^rR_{te}}{1 - \alpha R_{te}/(r_e^r + R_{te})} $$
(d) Solve for the voltage gain.

\[ A_v = \frac{v_o}{v_s} = \frac{R_E}{R_s + R_E} \frac{G_{me} r_{ic}}{R_C} R_L \]

(e) Solve for \( r_{in} \).

\[ r_{in} = R_1 \| R_2 \| r_{ie} \quad r_{ie} = r_e' \frac{r_0 + R_{ic}}{r_e' + r_0 + R_{ic}/(1 + \beta)} \]

(f) Solve for \( r_{out} \).

\[ r_{out} = r_{ic} \| R_C \]

**Example 1** For the CB amplifier in Fig. 1, it is given that \( R_s = 100 \Omega \), \( R_1 = 120 k\Omega \), \( R_2 = 100 k\Omega \), \( R_C = 4.3 k\Omega \), \( R_E = 5.6 k\Omega \), \( R_3 = 100 \Omega \), \( R_L = 20 k\Omega \), \( V^+ = 15 V \), \( V^- = -15 V \), \( V_{BE} = 0.65 V \), \( \beta = 99 \), \( \alpha = 0.99 \), \( r_x = 20 \Omega \), \( V_A = 100 V \) and \( V_T = 0.025 V \). Solve for \( A_v \), \( r_{in} \), and \( r_{out} \).

**Solution.** Because the dc bias circuit is the same as for the common-emitter amplifier example, the dc bias values, \( r_e \), \( g_m \), \( r_x \), and \( r_0 \) are the same.

In the signal circuit, the Thévenin voltage and resistance seen looking out of the emitter are given by

\[ v_{te} = \frac{R_E}{R_s + R_E} v_s = 0.9825 v_s \quad R_{te} = R_s \| R_E = 98.25 \Omega \]

The Thévenin resistances seen looking out of the base and the collector are

\[ R_{tb} = 0 \quad R_{ic} = R_C \| R_L = 3.539 k\Omega \]

Next, we calculate \( r_e' \), \( G_{me} \), \( r_{ie} \), and \( r_{ie} \).

\[ r_e' = \frac{R_{tb} + r_x}{1 + \beta} + r_e = 12.03 \Omega \quad G_{me} = \frac{1}{R_{te} + r_e'} \frac{\alpha r_0 + r_e'}{r_0 + r_e'} = \frac{1}{111.4} \text{ S} \]

\[ r_{ie} = \frac{r_0 + r_{ie}' R_{te}}{1 - \alpha R_{te}/(r_e' + R_{te})} = 442.3 \text{ k}\Omega \quad r_{ie} = r_e' \frac{r_0 + R_{ic}}{r_e' + r_0 + R_{ic}/(1 + \beta)} = 12.83 \Omega \]

The output voltage is given by

\[ v_o = G_{me} (r_{ie} \| R_{ic}) v_{te} = G_{me} (r_{ie} \| R_{ic}) \frac{R_E}{R_s + R_E} v_s = 30.97 v_s \]

Thus the voltage gain is

\[ A_v = 30.97 \]

The input and output resistances are

\[ r_{in} = R_1 \| R_2 \| r_{ib} = 12.81 \Omega \quad r_{out} = r_{ie} \| R_C = 4.259 k\Omega \]

**Approximate Solutions**

These solutions assume that \( r_0 = \infty \) except in calculating \( r_{ie} \). In this case, \( i_{c(sc)} = i_c' = \alpha i_e' = \beta i_b \).
Emitter Equivalent Circuit Solution

(a) After making the Thévenin equivalent circuits looking out of the base and emitter, replace the BJT with the emitter equivalent circuit as shown in Fig. 6.

(b) Solve for $i'_c$ and $r_{ic}$.

\[ 0 - v_{te} = i'_e \left(r'_e + R_{te}\right) = \frac{i'_c}{\alpha} \left(r'_e + R_{te}\right) \implies i'_c = -\frac{v_{te}}{r'_e + R_{te}} \]

\[ r_{ic} = \frac{r_0 + v_{te} \alpha}{1 - \alpha R_{te} (r'_e + R_{te})} \]

(c) Solve for $v_o$.

\[ v_o = -i'_c r_{ic} R_E R_C = v_{te} \frac{R_E}{r'_e + R_{te}} \alpha R_C R_L = v_s \frac{R_E}{R_s + R_E r'_e + R_{te}} \alpha R_C R_L \]

(d) Solve for the voltage gain.

\[ A_v = \frac{v_o}{v_s} = \frac{R_s}{R_s + R_E} \frac{\alpha \alpha R_C R_L}{R_E r'_e + R_{te}} \]

(e) Solve for $r_{ie}$ and $r_{in}$.

\[ 0 - v_e = i'_e \left(r'_e\right) \implies i'_e = -\frac{v_e}{r'_e} \]

\[ r_{ie} = \frac{v_e}{-i'_e} = r'_e \]

\[ r_{in} = r'_e R_E \]

(f) Solve for $r_{out}$.

\[ r_{out} = r_{ic} R_C \]

Example 2 For Example 1, use the simplified T-model solutions to calculate the values of $A_v$, $r_{in}$, and $r_{out}$.  

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Figure 6: Simplified T model circuit.
\[ A_v = 0.9825 \times \left(8.978 \times 10^{-3}\right) \times \left(3.511 \times 10^3\right) = 30.97 \]

\[ r_{in} = 12 \, \Omega \quad r_{out} = 4.259 \, k\Omega \]

\( \pi \) Model Solution

(a) After making the Thévenin equivalent circuits looking out of the base and emitter, replace the BJT with the \( \pi \) model as shown in Fig. 7.

\[ 0 - v_{te} = i_br_x + v_\pi + i^'_eR_{te} = \frac{i^'_e}{\beta}r_x + \frac{i^'_e}{g_m} + \frac{i^'_e}{\alpha}R_{te} \implies i^'_e = \frac{-v_{te}}{r_x + \frac{1}{g_m} + \frac{R_{te}}{\alpha}} \]

\[ r_{ic} = \frac{r_0 + r'_eR_{te}}{1 - \alpha R_{te}/(r'_e + R_{te})} \]

(b) Solve for \( i^'_e \) and \( r_{ic} \).

(c) Solve for \( v_o \).

\[ v_o = -i^'_e r_{ic} \| R_C \| R_L = \frac{v_{te}}{r_x + \frac{1}{g_m} + \frac{R_{te}}{\alpha}} r_{ic} \| R_C \| R_L = v_s \frac{R_E}{R_s + R_E} r_x + \frac{1}{g_m} + \frac{R_{te}}{\alpha} r_{ic} \| R_C \| R_L \]

(d) Solve for the voltage gain.

\[ A_v = \frac{v_o}{v_s} = \frac{R_E}{R_s + R_E} r_x + \frac{1}{g_m} + \frac{R_{te}}{\alpha} r_{ic} \| R_C \| R_L \]

(e) Solve for \( r_{out} \).

\[ r_{out} = r_{ic} \| R_C \]
(f) Solve for $r_{ie}$ and $r_{in}$.

$$0 - v_e = i_b (r_x + r_\pi) = \frac{i'_e}{1 + \beta} (r_x + r_\pi) \Rightarrow i'_e = v_e \frac{1 + \beta}{r_x + r_\pi}$$

$$r_{ie} = \frac{v_e}{-i'_e} = \frac{r_x + r_\pi}{1 + \beta}$$

$$r_{in} = r_{ie} || R_E$$

**Example 3** For Example 1, use the $\pi$-model solutions to calculate the values of $A_v$, $r_{in}$, and $r_{out}$.

$$A_v = 0.9825 \times (8.978 \times 10^{-3}) \times (3.539 \times 10^3) = 30.97$$

$$r_{in} = 12 \Omega \quad r_{out} = 4.259 \text{k}\Omega$$

**T Model Solution**

(a) After making the Thévenin equivalent circuits looking out of the base and emitter, replace the BJT with the T model as shown in Fig. ??.

![T model diagram](image)

Figure 8: T model circuit.

(b) Solve for $i'_c$ and $r_{ic}$.

$$0 - v_{te} = i_b r_x + i'_e (r_e + R_{te}) = \frac{i'_c}{\beta} r_x + \frac{i'_c}{\alpha} (r_e + R_{te}) \Rightarrow i'_c = v_{te} \frac{r_x}{\beta} + \frac{r_e + R_{te}}{\alpha}$$

$$r_{ic} = \frac{r_0 + r'_e \| R_{te}}{1 - \alpha R_{te} / (r'_e + R_{te})}$$

(c) Solve for $v_o$.

$$v_o = -i'_c r_{ic} || R_L = \frac{v_{te}}{\beta} + \frac{r_e + R_{te}}{\alpha}$$

$$v_o = \frac{v_{te}}{R_s + R_E} \frac{r_e + R_{te}}{r_x} \frac{1}{\beta} + \frac{r_e + R_{te}}{\alpha} r_{ic} || R_C || R_L$$
(d) Solve for the voltage gain.

\[ A_v = \frac{v_o}{v_s} = \frac{R_E}{R_s + R_E} \frac{1}{r_x + \frac{r_e + r_{ie}}{\alpha}} R_C \| R_L \]

(e) Solve for \( r_{ie} \) and \( r_{in} \).

\[ 0 - v_e = i_br_x + i_e^l r_e = \frac{i_e'}{1 + \beta} r_x + i_e^l r_e = i_e' \left( \frac{r_x}{1 + \beta} + r_e \right) \Rightarrow i_e' = \frac{-v_e}{1 + \beta + r_e} \]

\[ r_{ie} = \frac{v_e}{i_e'} = \frac{r_x}{1 + \beta} + r_e \]

\[ r_{in} = R_E \| r_{ie} \]

(f) Solve for \( r_{out} \).

\[ r_{out} = r_{ic} \| R_C \]

Example 4 For Example 1, use the T-model solutions to calculate the values of \( A_v \), \( r_{in} \), and \( r_{out} \).

\[ A_v = 0.9825 \times (8.978 \times 10^{-3}) \times (3.539 \times 10^3) = 30.97 \]

\[ r_{in} = 12 \Omega \quad r_{out} = 4.259 \text{ k}\Omega \]