The Common-Collector Amplifier

Basic Circuit

Fig. 1 shows the circuit diagram of a single stage common-collector amplifier. The object is to solve for the small-signal voltage gain, input resistance, and output resistance.

![Common-collector amplifier circuit diagram](image)

Figure 1: Common-collector amplifier.

DC Solution

(a) Replace the capacitors with open circuits. Look out of the 3 BJT terminals and make Thévenin equivalent circuits as shown in Fig. 2.

\[
V_{BB} = \frac{V^+ R_2 + V^- R_1}{R_1 + R_2}, \quad R_{BB} = R_1 \parallel R_2
\]

\[
V_{EE} = V^- \quad R_{EE} = R_E \quad V_{CC} = V^+ \quad R_{CC} = R_C
\]

(b) Make an “educated guess” for \(V_{BE}\). Write the loop equation between the \(V_{BB}\) and the \(V_{EE}\) nodes.

\[
V_{BB} - V_{EE} = I_B R_{BB} + V_{BE} + I_E R_{EE} = I_C R_{BB} + V_{BE} + \frac{I_C}{\alpha} R_{EE}
\]

(c) Solve the loop equation for the currents.

\[
I_C = \alpha I_E = \beta I_B = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/\beta + R_{EE}/\alpha}
\]

(d) Verify that \(V_{CB} > 0\) for the active mode.

\[
V_{CB} = V_C - V_B = (V_{CC} - I_C R_{CC}) - (V_{BB} - I_B R_{BB}) = V_{CC} - V_{BB} - I_C (R_{CC} - R_{BB}/\beta)
\]
Small-Signal or AC Solutions

(a) Redraw the circuit with $V^+ = V^- = 0$ and all capacitors replaced with short circuits as shown in Fig. 3.

\[
g_m = \frac{I_C}{V_T} \quad r_\pi = \frac{V_T}{I_B} \quad r_e = \frac{V_T}{I_E} \quad r_0 = \frac{V_A + V_{CE}}{I_C}
\]

(b) Calculate $g_m$, $r_\pi$, $r_e$, and $r_0$ from the DC solution.

(c) Replace the circuits looking out of the base with a Thévenin equivalent circuit as shown in Fig. 4.

\[
v_{tb} = v_s \frac{R_1 \parallel R_2}{R_s + R_1 \parallel R_2} \quad R_{tb} = R_1 \parallel R_2
\]
Exact Solution

(a) Replace the BJT in Fig. 4 with the Thévenin base emitter circuits as shown in Fig. 5. Solve for $v_{e(oc)}$.

$$v_{e(oc)} = v_{ib} \frac{r_0 + R_{tc}/(1 + \beta)}{r'_e + r_0 + R_{tc}/(1 + \beta)}$$

Note that the Thévenin resistance $R_{tc}$ looking out of the collector is zero in the original circuit. The exact solution gives the correct answer even if $R_{tc} \neq 0$.

(b) Solve for $v_o$.

$$v_o = v_{e(oc)} \frac{R_E R_L}{r_{ie} + R_E R_L} = v_s \frac{R_1 R_2}{R_s + R_1 R_2} \frac{r_0 + R_{tc}/(1 + \beta)}{r'_{ie} + r_0 + R_{tc}/(1 + \beta)} \frac{R_E R_L}{r_{ie} + R_E R_L}$$

$$r_{ie} = r'_e \frac{r_0 + R_{tc}}{r'_{ie} + r_0 + R_{tc}/(1 + \beta)} \quad r'_e = \frac{R_{tb} + r_x + r_\pi}{1 + \beta} = \frac{R_{tb} + r_x}{1 + \beta} + r_e$$

(c) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_s} = \frac{R_1 R_2}{R_s + R_1 R_2} \frac{r_0 + R_{tc}/(1 + \beta)}{r'_{ie} + r_0 + R_{tc}/(1 + \beta)} \frac{R_E R_L}{r_{ie} + R_E R_L}$$

(d) Solve for $r_{in}$.

$$r_{in} = R_1 R_2 r_{ib} \quad r_{ib} = r_x + r_\pi + R_{te} \frac{(1 + \beta) r_0 + R_{tc}}{r_0 + R_{te} + R_{tc}} \quad R_{te} = R_E R_L$$
(e) Solve for $r_{out}$.

$$r_{out} = r_{ie}||R_E$$

(f) Special case for $r_0 = \infty$.

$$v_{e(oc)} = v_{tb} \quad r_{ib} = r_x + r_\pi + (1 + \beta)R_{te} \quad r_{ie} = r_e'$$

**Example 1** For the CC amplifier in Fig. 1, it is given that $R_S = 5 \, k\Omega$, $R_1 = 120 \, k\Omega$, $R_2 = 100 \, k\Omega$, $R_E = 5.6 \, k\Omega$, $R_L = 20 \, k\Omega$, $V^+ = 15 \, V$, $V^- = -15 \, V$, $V_{BE} = 0.65 \, V$, $\beta = 99$, $\alpha = 0.99$, $r_x = 20 \, \Omega$, $V_A = 100 \, V$ and $V_T = 0.025 \, V$. Solve for $A_v$, $r_{in}$, and $r_{out}$.

**Solution.** Because the dc bias circuits are the same as for the common-emitter amplifier example, the bias values, $r_e$, $g_m$, and $r_\pi$ are the same. Because $V_{CE}$ is different, a new value of $r_0$ must be calculated. The collector-to-emitter voltage is given by

$$V_{CE} = V_C - V_E = V^+ - \left( V_{BB} - \frac{I_E}{1 + \beta} R_{BB} - V_{BE} \right) = 17.01 \, V$$

Thus $r_0$ has the value

$$r_0 = \frac{V_A + V_{CE}}{\alpha I_E} = 55.93 \, k\Omega$$

In the signal circuit, the Thévenin voltage and resistance seen looking out of the base are given by

$$v_{tb} = \frac{R_1||R_2}{R_S + R_1||R_2} v_s = 0.916 v_s \quad R_{tb} = R_S||R_1||R_2 = 4.58 \, k\Omega$$

The Thévenin resistances seen looking out of the emitter and the collector are

$$R_{te} = R_E||R_3 = 4.375 \, k\Omega \quad R_{tc} = 0$$

Next, we calculate $r_e'$, $v_{e(oc)}$, $r_{ie}$, and $r_{ib}$, where $R_{tc} = 0$.

$$r_e' = \frac{R_{tb} + r_x}{1 + \beta} + r_e = 57.83 \, \Omega$$

$$v_{e(oc)} = \frac{r_0 + R_{tc}}{r_e' + r_0 + R_{tc}} v_{tb} = 0.999 v_{tb}$$

$$r_{ie} = r_e' \frac{r_0 + R_{tc}}{r_e' + r_0 + R_{tc}} = 57.77 \, \Omega$$

$$r_{ib} = r_x + (1 + \beta) r_e + R_{te} \frac{(1 + \beta) r_0 + R_{tc}}{r_0 + R_{te} + R_{tc}} = 407 \, k\Omega$$

The output voltage is given by

$$v_o = \frac{R_{te}}{r_{ie} + R_{te}} v_{e(oc)} = 0.987 \times 0.999 \times 0.916 v_s = 0.903 v_s$$

Thus the voltage gain is

$$A_v = 0.903$$

The input and output resistances are given by

$$r_{in} = R_1||R_2||r_{ib} = 48.1 \, k\Omega \quad r_{out} = r_{ie}||R_E||R_L = 57.02 \, \Omega$$
Alternate Solutions

These solutions are exact because the Thévenin resistance $R_{tc}$ looking out of the collector is zero. If $R_{tc} \neq 0$, replace $r_0$ with an open circuit in all formulas, i.e. let $r_0 = \infty$ In this case, the solutions are no longer exact, they are approximate.

Emitter Equivalent Circuit Solution

(a) After making the Thévenin equivalent circuits looking out of the base, replace the BJT with the emitter equivalent circuit as shown in Fig. 6.

(b) Solve for $r'_e$.

$$r'_e = \frac{R_{tb} + r_x}{1 + \beta} + r_e \quad R_{tb} = R_S||R_1||R_2$$

(c) Solve for $v_o$.

$$v_o = v_{tb} \frac{r_0||R_E||R_L}{r'_e + r_0||R_E||R_L} = v_s \frac{R_1||R_2}{R_s + R_1||R_2} \frac{r_0||R_E||R_L}{r'_e + r_0||R_E||R_L}$$

(d) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_s} = \frac{R_1||R_2}{R_s + R_1||R_2} \frac{r_0||R_E||R_L}{r'_e + r_0||R_E||R_L}$$

(e) Solve for $r_{in}$. Because the base node is absorbed, use the formula for $r_{ib}$.

$$r_{in} = R_1||R_2||r_{ib} \quad r_{ib} = r_x + (1 + \beta) (r_e + r_0||R_{te}) \quad R_{te} = R_E||R_L$$

(f) Solve for $r_{out}$.

$$r_{out} = r_0||r'_e||R_E$$

Example 2 Use the simplified T-model solutions to calculate the values of $A_v$, $r_{in}$, and $r_{out}$ for Example 1.

$$A_v = 0.916 \times 0.986 = 0.903$$

$$r_{in} = 48.51 \, \text{kΩ} \quad r_{out} = 57.18 \, \text{Ω}$$
**π Model Solution**

(a) After making the Thévenin equivalent circuits looking out of the base and emitter, replace the BJT with the $\pi$ model as shown in Fig. 7.

![Figure 7: Hybrid-$\pi$ model.](image)

(b) Solve for $i'_c$.

$$v_{tb} = i_b (R_{tb} + r_x + r_{\pi}) + i'_c r_0 || R_E || R_L = \frac{i'_c}{1 + \beta} (R_{tb} + r_x + r_{\pi}) + i'_c r_0 || R_E || R_L$$

$$\Rightarrow i'_c = \frac{v_{tb}}{R_{tb} + r_x + r_{\pi} + r_0 || R_E || R_L}$$

(c) Solve for $v_o$.

$$v_o = i'_c r_0 || R_E || R_L = \frac{v_{tb}}{1 + \beta} r_0 || R_E || R_L$$

$$= \frac{v_s R_1 || R_2}{R_s + R_1 || R_2} \frac{R_{tb} + r_x + r_{\pi} + r_0 || R_E || R_L}{1 + \beta}$$

(d) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_s} = \frac{R_1 || R_2}{R_s + R_1 || R_2} \frac{r_0 || R_E || R_L}{R_{tb} + r_x + r_{\pi} + r_0 || R_E || R_L}$$

(e) Solve for $r_{ib}$ and $r_{in}$.

$$v_b = i_b (r_x + r_{\pi}) + i'_c r_0 || R_E || R_L = i_b (r_x + r_{\pi}) + (1 + \beta) i_b r_0 || R_E || R_L$$

$$= i_b [r_x + r_{\pi} + (1 + \beta) r_0 || R_E || R_L]$$

$$r_{ib} = \frac{v_b}{i_b} = r_x + r_{\pi} + (1 + \beta) r_0 || R_E || R_L$$
(f) Solve for $r_{out}.$ First, solve for the open-circuit output voltage. This is the output voltage with $R_L = \infty$.

$$v_{o(oc)} = v_{tb} \frac{r_0 \| R_E}{R_{tb} + r_x + r_\pi + r_0 \| R_E}$$

Next, solve for the short-circuit output current. This is the output current with $R_L = 0$.

The output current is given by

$$i_o = \frac{v_o}{R_L} = \frac{R_{tb} + r_x + r_\pi + r_0 \| R_E}{1 + \beta} \frac{R_0 \| R_L}{R_L} = \frac{v_{tb}}{R_{tb} + r_x + r_\pi} \frac{R_0 \| R_E}{R_L} = \frac{v_{tb} R_{tb} + r_x + r_\pi}{1 + \beta} \frac{1 + \beta}{R_L + 0 \| R_E}$$

Now, let $R_L = 0$ to obtain

$$i_{o(sc)} = \frac{v_{tb}}{R_{tb} + r_x + r_\pi}$$

The output resistance is given by

$$r_{out} = \frac{v_{o(oc)}}{i_{o(sc)}} = \frac{R_{tb} + r_x + r_\pi + r_0 \| R_E}{1 + \beta} \frac{R_0 \| R_L}{R_L} = \frac{v_{tb} R_{tb} + r_x + r_\pi}{1 + \beta} \frac{1 + \beta}{R_L + 0 \| R_E}$$

Note this is simply $r'_e \| 0 \| R_E$, an answer that is obvious using the emitter equivalent circuit.

**Example 3** Use the $\pi$-model solutions to calculate the values of $A_v$, $r_{in}$, and $r_{out}$ for Example 1.

$$A_v = 0.916 \times 0.986 = 0.903$$

$$r_{in} = 48.51 \, k\Omega \quad r_{out} = 57.18 \, \Omega$$

**T Model Solution**

(a) After making the Thévenin equivalent circuits looking out of the base and emitter, replace the BJT with the T model as shown in Fig. 8.

(b) Solve for $i'_e$.

$$v_{tb} = i_b (R_{tb} + r_x) + i'_e (r_e + r_0 \| R_E \| R_L) = \frac{i'_e}{1 + \beta} (R_{tb} + r_x) + i'_e (r_e + r_0 \| R_E \| R_L)$$

$$\Rightarrow \quad i'_e = \frac{v_{tb}}{R_{tb} + r_x + r_e + r_0 \| R_E \| R_L}$$

(c) Solve for $v_o$.

$$v_o = i'_e r_0 \| R_E \| R_L = \frac{v_{tb}}{R_{tb} + r_x + r_e + r_0 \| R_E \| R_L}$$

$$= \frac{v_s R_1 \| R_2}{R_s + R_1 \| R_2} \frac{R_{tb} + r_x + r_e + r_0 \| R_E \| R_L}{1 + \beta \frac{R_0 \| R_E}{R_L}}$$
(d) Solve for the voltage gain.
\[
A_v = \frac{v_o}{v_s} = \frac{R_1 || R_2}{R_s + R_1 || R_2} \frac{r_0 || R_E || R_L}{R_{tb} + r_x + r_e + r_0 || R_E || R_L}
\]

(e) Solve for \(r_{ib}\) and \(r_{in}\).
\[
v_b = i_{ib}r_x + i'_e (r_e + r_0 || R_E || R_L) = i_{ib}r_x + (1 + \beta) i_{ib} (r_e + r_0 || R_E || R_L)
\]
\[
= i_{ib} [r_x + (1 + \beta) (r_e + R_0 || R_E || R_L)]
\]
\[
r_{ib} = \frac{v_b}{i_b} = r_x + (1 + \beta) (r_e + r_0 || R_E || R_L)
\]
\[
r_{in} = R_1 || R_2 || r_{ib}
\]

(f) Solve for \(r_{out}\). First, solve for the open-circuit output voltage. This is the output voltage with \(R_L = \infty\).
\[
v_{o(oc)} = v_{tb} \frac{r_0 || R_E}{R_{tb} + r_x + r_e + r_0 || R_E}
\]
Next, solve for the short-circuit output current. This is the output current with \(R_L = 0\). The output current is given by
\[
i_{o} = \frac{v_o}{R_L} = \frac{v_{tb}}{R_{tb} + r_x + r_e + r_0 || R_E || R_L} \frac{r_0 || R_E || R_L}{R_L} = \frac{v_{tb}}{R_{tb} + r_x + r_e} \frac{r_0 || R_E}{R_L + r_0 || R_E}
\]
Now, let \(R_L = 0\) to obtain
\[
i_{o(sc)} = \frac{v_{tb}}{R_{tb} + r_x} \frac{1}{1 + \beta} + r_e
\]

The output resistance is given by
\[
r_{out} = \frac{v_{o(oc)}}{i_{o(sc)}} = \frac{r_0 || R_E}{R_{tb} + r_x} \frac{1}{1 + \beta} + r_e + r_0 || R_E \left( \frac{R_{tb} + r_x}{1 + \beta} + r_e \right) = \left( \frac{R_{tb} + r_x}{1 + \beta} + r_e \right) || r_0 || R_E
\]
This is the same answer obtained from the emitter equivalent circuit.

**Example 4** Use the T-model solutions to calculate the values of $A_v$, $r_{in}$, and $r_{out}$ for Example 1.

\[
A_v = 0.916 \times 0.986 = 0.903 \\
r_{in} = 48.51 \text{ k}\Omega \\
r_{out} = 57.18 \Omega
\]