BJT Differential Amplifier Example

\[ R_{p}(x, y) := \frac{x \cdot y}{x + y} \]

Function for calculating parallel resistors.

\[
\begin{align*}
R_C & := 20000 \\
R_B & := 1000 \\
R_E & := 100 \\
I_Q & := 0.001 \\
V_p & := 20 \\
V_m & := -20 \\
V_{BE} & := 0.65 \\
V_T & := 0.025 \\
\beta & := 199 \\
\alpha & := \frac{\beta}{1 + \beta} \\
r_x & := 20 \\
r_0 & := 50000
\end{align*}
\]

There are two ac solutions, one for the second input zeroed and one for the first input zeroed. By superposition, the total solution would be the sum of these two. To keep Mathcad happy, all source voltages are taken to be equal to 1 V so that the output voltage is equal to the voltage gain. In general, the output voltage is equal to the voltage gain multiplied by the source voltage.
DC Bias Solution

Assume the dc value of the sources is zero.

\[ I_{E1} = \frac{I_Q}{2} \]

\[ I_{E1} = 5 \times 10^{-4} \]

\[ I_{E2} = I_{E1} \]

\[ V_{C1} := V_p - a \cdot I_{E1} \cdot R_C \]

\[ V_{C1} = 10.05 \]

\[ V_{B1} := -\frac{I_{E1}}{1 + \beta} \cdot R_B \]

\[ V_{B1} = -2.5 \times 10^{-3} \]

\[ V_{CB1} := V_{C1} - V_{B1} \]

\[ V_{CB1} = 10.0525 \]

Thus active mode. Same for Q2.

\[ r_{e1} := \frac{V_T}{I_{E1}} \]

\[ r_{e1} = 50 \]

\[ r_{e2} = r_{e1} \]

\[ r'_{e1} := \frac{R_B + r_x}{1 + \beta} + r_{e1} \]

\[ r'_{e1} = 55.1 \]

\[ r'_{e2} = r'_{e1} \]
With the input equal to 1, the voltage gain is equal to the output voltage.

\[ v_{e2oc} = v_{i2} \left( \frac{r_0 + \frac{R_C}{1 + \beta}}{r'_{e2} + r_0 + \frac{R_C}{1 + \beta}} \right) \]

\[ v_{e2oc} = 0.9989 \]

\[ r_{ie2} = 76.9015 \]

\[ v_{tb1} := v_{i1} \quad R_{tb1} := R_B \]

\[ v_{te1} := v_{e2oc} \quad R_{te1} := 2 \cdot R_E + r_{ie2} \quad R_{te1} = 276.9015 \]
Voltage gain from first input to first output:

\[ G_{mb1} := \frac{a}{r'_e + R_{te1}} \frac{r_0 - \frac{R_{te1}}{\beta}}{r_0 + R_p(r'_e, R_{te1})} \]

\[ G_{me1} := \frac{a}{r'_e + R_{te1}} \frac{R_{te1} - \frac{r'_e}{a}}{r_0 + R_p(r'_e, R_{te1})} \]

\[ r_{ic1} := \frac{r_0 + R_p(r'_e, R_{te1})}{1 - \frac{a \cdot R_{te1}}{r'_e + R_{te1}}} \]

\[ G_{mb1} = 2.9941 \times 10^{-3} \]

\[ G_{me1} = 2.9975 \times 10^{-3} \]

\[ r_{ic1} = 2.9416 \times 10^5 \]

\[ i_{c1sc} := G_{mb1} \cdot v_{tb1} \]

\[ i_{c1sc} = 2.9941 \times 10^{-3} \]

\[ v_{o1} := -i_{c1sc} R_p(r_{ic1}, R_C) \]

\[ A_v1 := v_{o1} \]

\[ A_v1 = -56.0705 \]

This is the voltage gain from the first input to the first output. The gain from the second input to the second output is the same.
Voltage gain from the second input to the first output.

\[ i_{c1sc} := -G_{me1} \cdot v_{t1} \quad i_{c1sc} = -2.9942 \times 10^{-3} \]

\[ v_{o1} := -i_{c1sc} \cdot R \cdot p_{t1_c} \cdot R_{C} \quad A_{v2} := v_{o1} \]

\[ A_{v2} = 56.0725 \quad \text{This is the voltage gain from the second input to the first output. The gain from the first input to the second output is the same.} \]

\[ v_{o1} := -56.0705 \cdot v_{i1} + 56.0725 \cdot v_{i2} \quad \text{This is the sum ac output from Q1.} \]

\[ v_{o2} := -56.0705 \cdot v_{i2} + 56.0725 \cdot v_{i1} \quad \text{This is the sum ac output from Q2.} \]

Differential input resistance.

\[ r_{id} : = \frac{v_{id}}{2} + \frac{r_{id}}{2} \quad R_{B} \quad r_{ib1} \quad r_{ib2} \quad r_{id} \]

\[ r_{ib1} : = r_{x} + (1 + \beta) \cdot \left( r_{e1} + R \cdot p_{t1_c} \cdot r_{0} \cdot R_{C} \right) - \frac{\beta \cdot R_{t1_c} \cdot R_{C}}{R_{t1_c} + r_{0} + R_{C}} \]

\[ r_{ib1} = 4.95 \times 10^{4} \quad \text{r}_{ib2} := r_{ib1} \]

\[ r_{id} : = 2 \cdot R_{B} + r_{ib1} + r_{ib2} \quad r_{id} = 1.01 \times 10^{5} \]
Common-Mode Rejection Ratio

\[ A_{v1} = 56.0705 \quad A_{v2} = 56.0725 \]

Let us take the output from the collector of the first transistor. Because neither \( \beta \) nor \( r_0 \) is infinity, the two voltage gains are not equal. This causes the CMRR to be non infinite. We calculate it below.

\[
v_{id} := 1 \quad v_{i1} := \frac{v_{id}}{2} \quad v_{i2} := -\frac{v_{id}}{2}
\]

\[
v_{o1} := A_{v1} \cdot v_{i1} + A_{v2} \cdot v_{i2} \quad A_{d} := v_{o1}
\]

\[ A_{d} = 56.0715 \quad \text{This is the differential voltage gain.} \]

\[
v_{icm} := 1 \quad v_{i1} := v_{icm} \quad v_{i2} := v_{icm}
\]

\[
v_{o1} := A_{v1} \cdot v_{i1} + A_{v2} \cdot v_{i2} \quad A_{cm} := v_{o1}
\]

\[ A_{cm} = 1.9938 \times 10^{-3} \quad \text{This is the common mode voltage gain.} \]

\[
\text{CMRR} := \left| \frac{A_{d}}{A_{cm}} \right| \quad \text{CMRR} = 2.8123 \times 10^4
\]

\[
\text{CMRR}_{dB} := 20 \cdot \log(\text{CMRR}) \quad \text{CMRR}_{dB} = 88.9811
\]

If \( R_Q \) (the ac resistance of the current source) is not infinity, the CMRR would be lower.
Solution with the $r_0$ approximations. We neglect $r_0$ except in calculating $r_{ic}$. Thus we can use the emitter equivalent circuit to solve for $i_{c1}$ and $i_{c2}$, then multiply by $\alpha$ to solve for the collector currents. Because the common mode gain is zero if we neglect $r_0$, we will assume a differential input signal.

\[ i_{c1} = \frac{v_{i1} - v_{i2}}{r'_{c1} + 2R_E + r_{ie2}} \]
\[ i_{c2} = -i_{c1} \]

\[ v_{o1} = -\alpha i_{c1} R_P \left( R_C + r_{ic1} \right) \]
\[ v_{o1} = -56.1236 \] This is the differential voltage gain to the first output.

\[ v_{o2} = -v_{o1} \]
\[ v_{o2} = 56.1236 \] This is the differential voltage gain to the second output.

\[ r_{ib1} = r_X + (1 + \beta) \left( r_{c1} + R_{te1} \right) \]
\[ r_{ib1} = 6.54 \times 10^4 \]
\[ r_{ib2} = r_{ib1} \]

\[ r_{id} = 2R_B + r_{ib1} + r_{ib2} \]
\[ r_{id} = 1.328 \times 10^5 \] This is the differential input resistance.

There is more error using the $r_0$ approximations than I had expected for this problem. Usually the answers are much closer. The major cause of the error here is the effect of $r_0$ on $r_{ie}$. If $r_0$ is infinity, then $r_{ie}$ is equal to $r'_{c}$. There is a fairly big difference between these two resistances in this problem.