The BJT

BJT Device Equations

Figure 1 shows the circuit symbols for the npn and pnp BJTs. In the active mode, the collector-base junction is reverse biased and the base-emitter junction is forward biased. Because of recombinations of the minority and majority carriers, the equations for the currents can be divided into three regions: low, mid, and high. For the npn device, the currents are given by
Low Level:

$$i_B = I_{SE}\left[\exp\left(\frac{v_{BE}}{nV_T}\right) - 1\right] \tag{1}$$

$$i_C = I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] \tag{2}$$

Mid Level:

$$i_B = \frac{I_S}{\beta_F}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] \tag{3}$$

$$i_C = I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] \tag{4}$$

High Level:

$$i_B = \frac{I_S}{\beta_F}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] \tag{5}$$

$$i_C = I_S\sqrt{\frac{I_K}{I_{SO}}}\left[\exp\left(\frac{v_{BE}}{2V_T}\right) - 1\right] \tag{6}$$

where all leakage currents that are a function of v_{CB} have been neglected. In the current equations, I_S is the saturation current and β_F is the mid-level base-to-collector current gain. These are functions of the collector-base voltage and are given by

$$I_S = I_{SO}\left(1 + \frac{v_{CB}}{V_A}\right) = I_{SO}\left(1 + \frac{v_{CE} - v_{BE}}{V_A}\right) \tag{7}$$

$$\beta_F = \beta_{FO}\left(1 + \frac{v_{CB}}{V_A}\right) = \beta_{FO}\left(1 + \frac{v_{CE} - v_{BE}}{V_A}\right) \tag{8}$$

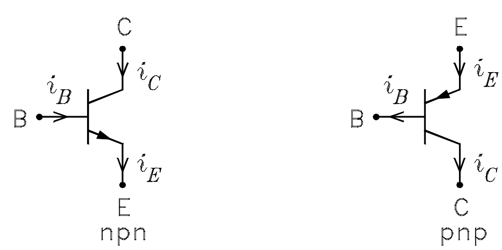

Figure 1: BJT circuit symbols.

In the equations for i_B and i_C, V_A is the Early voltage and I_{SO} and β_{FO}, respectively, are the zero bias values of I_S and β_F. The constant n is the emission coefficient or ideality factor of the base-emitter junction. It accounts for recombinations of holes and electrons in the base-emitter junction at low levels. Its value, typically in the range $1 \leq n \leq 4$, is determined by the slope of the plot of $\ln(i_C)$ versus v_{BE} at low levels. The default value in SPICE is $n = 1.5$. The constant I_{SE} is determined by the value of i_B where transition from the low-level to the mid-level region occurs. The constant I_K is determined by the value of i_C where transition from the mid-level to the high-level region occurs. Note that $I_S/\beta_F = I_{S0}/\beta_{F0}$ so that i_B is not a function of v_{CB} in the mid-level region. The equations apply to the pnp device if the subscripts BE and CB are reversed.

Figure 2 shows a typical plot of i_C versus v_{BE} for v_{CE} constant. The plot is called the transfer characteristics. There is a threshold voltage above which the current appears to increase rapidly. This voltage is typically 0.5 to 0.6 V. In the forward active region, the base-to-emitter voltage is typically 0.6 to 0.7 V. Figure 3 shows typical plots of i_C versus v_{CE} for i_B constant. The plots are called the output characteristics. Note that the slope approaches a constant as v_{CE} is increased. If the straight line portions of the curves are extended back so that they intersect the v_{CE} axis, they would intersect at the voltage $v_{CE} = -V_A + v_{BE} \simeq -V_A$. For v_{CE} small, $v_{BE} > v_{CE}$ and the BJT is in the saturation region.

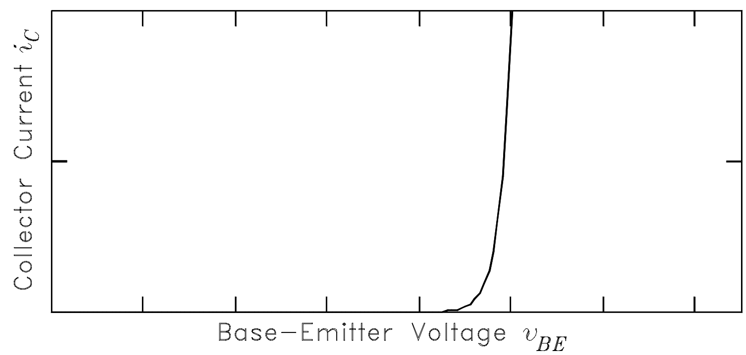

Figure 2: Typical plot of i_C versus v_{BE} for v_{CE} constant.

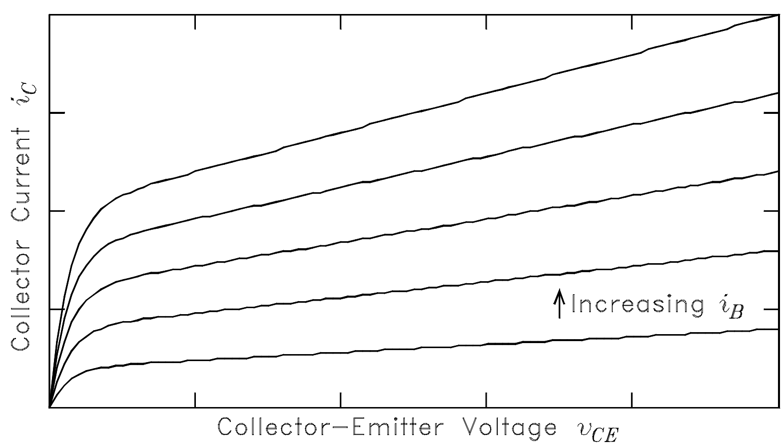

Figure 3: Plots of i_C versus v_{CE} for i_B constant.

In the Gummel-Poon model of the BJT, the current equations are combined to write the general equations for i_B and i_C as follows:

$$i_B = I_{SE}\left[\exp\left(\frac{v_{BE}}{nV_T}\right) - 1\right] + \frac{I_{SO}}{\beta_{FO}}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] \quad (9)$$

$$i_C = \frac{I_S}{K_q}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] \tag{10}$$

where K_q is given by

$$K_q = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{I_S}{I_K}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right]} = 1 + \frac{i_C}{I_K} \tag{11}$$

Fig. 4 illustrates typical plots of $\ln(i_C)$ and $\ln(i_B)$ versus v_{BE}, where it is assumed that v_{CB} is held constant. At low levels, the i_C curve exhibits a slope $m = 1$ while the i_B curve exhibits a slope $m = 1/n$, where the value $n = 1.5$ has been used. At mid levels, both curves exhibit a slope $m = 1$. At high levels, the i_C curve exhibits a slope $m = 1/2$ while the i_B curve exhibits a slope $m = 1$. It follows that the ratio of i_C to i_B is approximately constant at mid levels and decreases at low and high levels.

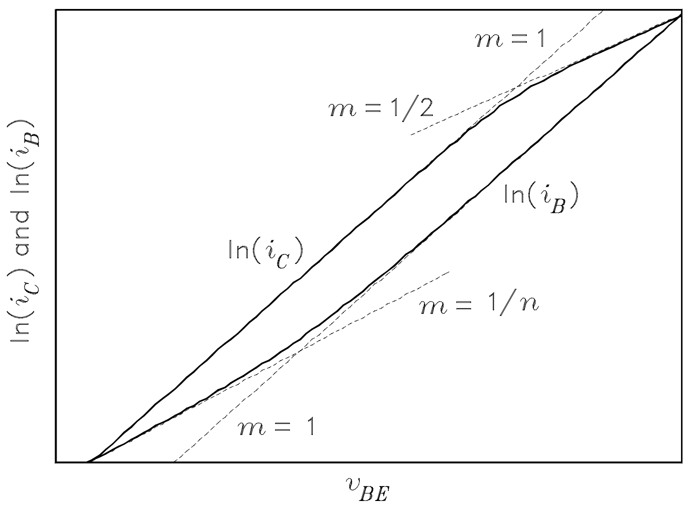

Figure 4: Example plots of $\ln(i_C)$ and $\ln(i_B)$ versus v_{BE}.

Current Gains

Let the collector and base currents be written as the sum of a dc component and a small-signal ac component as follows:

$$i_C = I_C + i_c \tag{12}$$

$$i_B = I_B + i_b \tag{13}$$

The dc current gain $\beta_{F\text{dc}}$ is defined as the ratio of I_C to I_B. It is straightforward to show that it is given by

$$\beta_{F\text{dc}} = \frac{\beta_F}{1 + \dfrac{I_C}{I_K} + \dfrac{\beta_F I_{SE}}{I_C}\left[\left(1 + \dfrac{I_C}{I_S} + \dfrac{I_C^2}{I_S I_K}\right)^{1/n} - 1\right]} \tag{14}$$

Because β_F and I_S are functions of the collector-base voltage V_{CB}, it follows that $\beta_{F\text{dc}}$ is a function of both I_C and V_{CB}. If v_{CB} is held constant so that the change in i_C is due to a change in v_{BE}, the small-signal change in base current can be written

$$i_b = \frac{\partial I_B}{\partial I_C} i_c = \left[\frac{\partial}{\partial I_C}\left(\frac{I_C}{\beta_{F\text{dc}}}\right)\right] i_c = \frac{i_c}{\beta_{F\text{ac}}} \tag{15}$$

where $\beta_{F\mathrm{ac}}$ is the small-signal ac current gain given by

$$\begin{aligned}\beta_{F\mathrm{ac}} &= \left[\frac{\partial}{\partial I_C}\left(\frac{I_C}{\beta_{F\mathrm{dc}}}\right)\right]^{-1} \\ &= \frac{\beta_F}{\left(1+\frac{2I_C}{I_K}\right)\left[1+\frac{\beta_F I_{SE}}{nI_S}\left(1+\frac{I_C}{I_S}+\frac{I_C^2}{I_S I_K}\right)^{\frac{1}{n}-1}\right]}\end{aligned} \tag{16}$$

Note that $\beta_{F\mathrm{ac}}$ is defined for a constant v_{CB}. In the small-signal models, it is common to define the small-signal ac current gain with v_{CE} constant, i.e. $v_{ce} = 0$. This is defined in the next section, where the symbol β is used.

Typical plots of the two current gains as a function of I_C are shown in Fig. 5 where log scales are used. At low levels, the gains decrease with decreasing I_C because the base current decreases at a slower rate than the collector current. At high levels, the gains decrease with increasing I_C because the collector current increases at a slower rate than the base current. At mid levels, both gains are approximately constant and have the same value. In the figure, the mid-level range is approximately two decades wide.

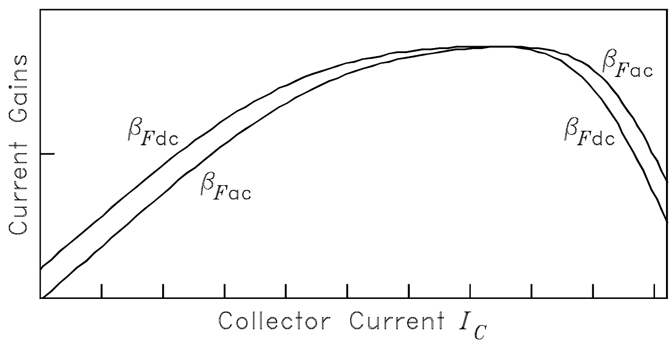

Figure 5: Log-log plots of $\beta_{F\mathrm{dc}}$ and $\beta_{F\mathrm{ac}}$ as functions of I_C.

The emitter-collector dc current gain $\alpha_{F\mathrm{dc}}$ is defined as the ratio of the dc collector current I_C to the dc emitter current I_E. To solve for this, we can write

$$I_E = I_B + I_C = \left(\frac{1}{\beta_{F\mathrm{dc}}}+1\right)I_C = \frac{1+\beta_{F\mathrm{dc}}}{\beta_{F\mathrm{dc}}}I_C \tag{17}$$

It follows that

$$\alpha_{F\mathrm{dc}} = \frac{I_C}{I_E} = \frac{\beta_{F\mathrm{dc}}}{1+\beta_{F\mathrm{dc}}} \tag{18}$$

Thus the dc currents are related by the equations

$$I_C = \beta_{F\mathrm{dc}} I_B = \alpha_{F\mathrm{dc}} I_E \tag{19}$$

Bias Equation

Figure 6(a) shows the BJT with the external circuits represented by Thévenin dc circuits. If the BJT is biased in the active region, we can write

$$\begin{aligned}V_{BB} - V_{EE} &= I_B R_{BB} + V_{BE} + I_E R_{EE} \\ &= \frac{I_C}{\beta_{F\mathrm{dc}}}R_{BB} + V_{BE} + \frac{I_C}{\alpha_{F\mathrm{dc}}}R_{EE}\end{aligned} \tag{20}$$

This equation can be solved for I_C to obtain

$$I_C = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/\beta_{F\mathrm{dc}} + R_{EE}/\alpha_{F\mathrm{dc}}} \tag{21}$$

It can be seen from Fig. 2 that large changes in I_C are associated with small changes in V_{BE}. This makes it possible to calculate I_C by assuming typical values of V_{BE}. Values in the range from 0.6 to 0.7 V are commonly used. In addition, $\beta_{F\mathrm{dc}}$ and $\alpha_{F\mathrm{dc}}$ are functions of I_C and V_{CB}. Mid-level values are commonly assumed for the current gains. Typical values are $\beta_{F\mathrm{dc}} = 100$ and $\alpha_{F\mathrm{dc}} = 1/1.01$.

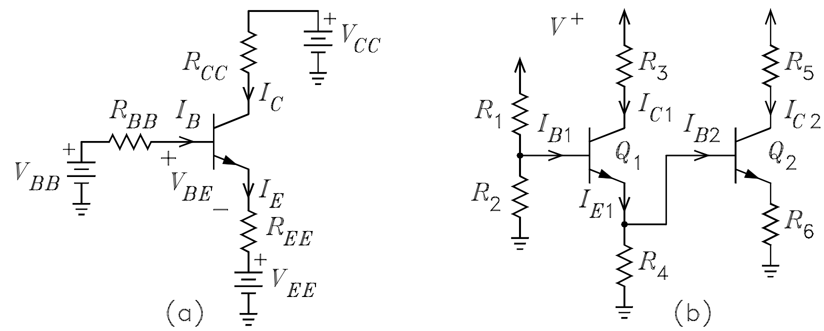

Figure 6: (a) BJT dc bias circuit. (b) Circuit for Example 1.

Example 1 *Figure 6(b) shows a BJT dc bias circuit. It is given that $V^+ = 15$ V, $R_1 = 20$ kΩ, $R_2 = 10$ kΩ, $R_3 = R_4 = 3$ kΩ, $R_5 = R_6 = 2$ kΩ. Solve for I_{C1} and I_{C2}. Assume $V_{BE} = 0.7$ V and $\beta_{Fdc} = 100$ for each transistor.*

Solution. For Q_1, we have $V_{BB1} = V^+R_2/(R_1 + R_2)$, $R_{BB1} = R_1\|R_2$, $V_{EE1} = -I_{B2}R_4 = -I_{C2}R_4/\beta_{F\mathrm{dc}}$, $V_{EE1} = 0$, and $R_{EE1} = R_4$. For Q_2, we have $V_{BB2} = I_{E1}R_4 = I_{C1}R_4/\alpha_{F\mathrm{dc}}$, $R_{BB2} = R_4$, $V_{EE2} = 0$, $R_{EE2} = R_6$. Thus the bias equations are

$$V^+\frac{R_2}{R_1 + R_2} + \frac{I_{C2}}{\beta_{F\mathrm{dc}}}R_4 = V_{BE} + \frac{I_{C1}}{\beta_{F\mathrm{dc}}}R_1\|R_2 + \frac{I_{C1}}{\alpha_{F\mathrm{dc}}}R_4$$

$$\frac{I_{C1}}{\alpha_{F\mathrm{dc}}}R_4 = V_{BE} + \frac{I_{C2}}{\beta_{F\mathrm{dc}}}R_4 + \frac{I_{C2}}{\alpha_{F\mathrm{dc}}}R_6$$

These equations can be solved simultaneously to obtain $I_{C1} = 1.41$ mA and $I_{C2} = 1.74$ mA.

Small-Signal Models

There are two small-signal circuit models which are commonly used to analyze BJT circuits. These are the hybrid-π model and the T model. The two models are equivalent and give identical results. They are described below.

Hybrid-π Model

Let each current and voltage be written as the sum of a dc component and a small-signal ac component. The currents are given by Eqs. (12) and (13). The voltages can be written

$$v_{BE} = V_{BE} + v_{be} \tag{22}$$

$$v_{CB} = V_{CB} + v_{cb} \tag{23}$$

If the ac components are sufficiently small, i_c can be written

$$\begin{aligned} i_c &= \frac{\partial I_C}{\partial V_{BE}} v_{be} + \frac{\partial I_C}{\partial V_{CB}} v_{cb} = \frac{\partial I_C}{\partial V_{BE}} v_{be} + \frac{\partial I_C}{\partial V_{CB}} (v_{ce} - v_{be}) \\ &= \left(\frac{\partial I_C}{\partial V_{BE}} - \frac{\partial I_C}{\partial V_{CB}} \right) v_{be} + \frac{\partial I_C}{\partial V_{CB}} v_{ce} = g_m v_{be} + \frac{v_{ce}}{r_0} \end{aligned} \tag{24}$$

This equation defines the small-signal transconductance g_m and the collector-emitter resistance r_0. From Eqs. (8) and (10), it follows that r_0 is given by

$$\begin{aligned} r_0 &= \left(\frac{\partial I_C}{\partial V_{CB}} \right)^{-1} = \left\{ \frac{K_q I_{SO}}{V_A} \left[\exp\left(v_{BE}/V_T \right) - 1 \right] \right\}^{-1} \\ &= \frac{V_A + V_{CB}}{I_C} \end{aligned} \tag{25}$$

To solve for g_m, we first solve for $\partial I_C / \partial V_{BE}$. Eqs. (10) and (11) can be combined to write

$$I_C + \frac{I_C^2}{I_K} = I_S \left[\exp\left(\frac{V_{BE}}{V_T} \right) - 1 \right] \tag{26}$$

It follows from this equation that $\partial I_C / \partial V_{BE}$ is given by

$$\frac{\partial I_C}{\partial V_{BE}} = \frac{I_S \exp\left(V_{BE}/V_T \right)}{V_T \left(1 + 2I_C/I_K \right)} = \frac{I_C \left(1 + I_C/I_K \right) + I_S}{V_T \left(1 + 2I_C/I_K \right)} \tag{27}$$

The transconductance is given by

$$g_m = \frac{\partial I_C}{\partial V_{BE}} - \frac{\partial I_C}{\partial V_{CB}} = \frac{I_C \left(1 + I_C/I_K \right) + I_S}{V_T \left(1 + 2I_C/I_K \right)} - \frac{1}{r_0} \tag{28}$$

It is clear from Eq. (9) that i_B is a function of v_{BE} only. We wish to solve for the small-signal ac base current given by $i_b = \left(\partial I_B / \partial V_{BE} \right) v_{be}$. This equation defines the small-signal ac base-emitter resistance $r_\pi = v_b / i_b = \left(\partial I_B / \partial V_{BE} \right)^{-1}$. Although Eq. (9) can be used to solve for this, we use a different approach. The small-signal ac collector current can be written

$$i_c = g_m v_{be} + \frac{v_{ce}}{r_0} = \left(g_m + \frac{1}{r_0} \right) v_{be} + \frac{v_{cb}}{r_0} = \beta_{F\mathrm{ac}} i_b + \frac{v_{cb}}{r_0} \tag{29}$$

It follows from this equation that

$$\left(g_m + \frac{1}{r_0} \right) v_{be} = \beta_{F\mathrm{ac}} i_b \tag{30}$$

Thus r_π is given by

$$r_\pi = \frac{v_{be}}{i_b} = \frac{\beta_{F\mathrm{ac}}}{g_m + 1/r_0} \tag{31}$$

The small-signal ac current gain β is defined as the ratio of i_c to i_b with v_{CE} constant, i.e. $v_{ce} = 0$. To solve for this, we can write for i_c

$$i_c = g_m v_{be} + \frac{v_{ce}}{r_0} = g_m i_b r_\pi + \frac{v_{ce}}{r_0} = \beta i_b + \frac{v_{ce}}{r_0} \tag{32}$$

It follows from this that β is given by

$$\beta = g_m r_\pi = \frac{g_m \beta_{F\text{ac}}}{g_m + 1/r_0} \tag{33}$$

Thus far, we have neglected the base spreading resistance r_x. This is the ohmic resistance of the base contact in the BJT. When it is included in the model, it appears in series with the base lead. Because the base region is very narrow, the connection exhibits a resistance which often cannot be neglected. Fig. 7(a) shows the hybrid-π small-signal model with r_x included. The currents are given by

$$i_c = i_c' + \frac{v_{ce}}{r_0} \tag{34}$$

$$i_c' = g_m v_\pi = \beta i_b \tag{35}$$

$$i_b = \frac{v_{be}}{r_\pi} \tag{36}$$

where r_0, g_m, β, and r_π are given above.

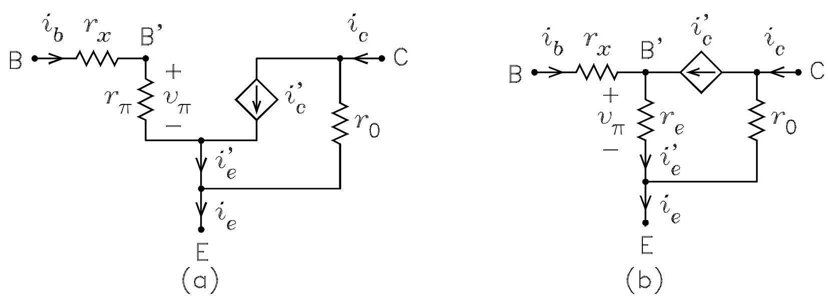

Figure 7: (a) Hybrid-π model. (b) T model.

The equations derived above are based on the Gummel-Poon model of the BJT in the forward active region. The equations are often approximated by assuming that the mid-level current equations hold. In this case, $\beta_{F\text{dc}}$, $\beta_{F\text{ac}}$, r_0, g_m, r_π, and β are given by

$$\beta_{F\text{dc}} = \beta_{F\text{ac}} = \beta_F \tag{37}$$

$$r_0 = \frac{V_{CB} + V_A}{I_C} \tag{38}$$

$$g_m = \frac{I_C + I_S}{V_T} - \frac{1}{r_0} \simeq \frac{I_C}{V_T} \tag{39}$$

$$r_\pi = \frac{\beta_{F\text{ac}}}{g_m + 1/r_0} \simeq \frac{V_T}{I_B} \tag{40}$$

$$\beta = g_m r_\pi \simeq \frac{I_C}{I_B} = \beta_{F\text{dc}} \tag{41}$$

The three approximations in these equations are commonly used for hand calculations.

T Model

The T model replaces the resistor r_π in series with the base with a resistor r_e in series with the emitter. This resistor is called the emitter intrinsic resistance. To solve for r_e, we first solve for the small-signal ac emitter-to-collector current gain α. In Fig. 7, the current i'_e can be written

$$i'_e = i_b + i'_c = \left(\frac{1}{\beta} + 1\right) i'_c = \frac{1+\beta}{\beta} i'_c = \frac{i'_c}{\alpha} \tag{42}$$

where α is given by

$$\alpha = \frac{i'_c}{i'_e} = \frac{\beta}{1+\beta} \tag{43}$$

Thus the current i'_c can be written

$$i'_c = \alpha i'_e \tag{44}$$

The voltage v_π can be related to i'_e as follows:

$$v_\pi = i_b r_\pi = \frac{i'_c}{\beta} r_\pi = \frac{\alpha i'_e}{\beta} r_\pi = i'_e \frac{r_\pi}{1+\beta} \tag{45}$$

It follows that the intrinsic emitter resistance is given by

$$r_e = \frac{v_\pi}{i'_e} = \frac{r_\pi}{1+\beta} \simeq \frac{V_T}{(1+\beta_{F\text{dc}}) I_B} = \frac{V_T}{I_E} \tag{46}$$

where the approximation is based on Eqs. (40) and (41). It is often used for hand calculations. The T model of the BJT is shown in Fig. 7(b). The currents in both the π and T models are related by the equations

$$i'_c = g_m v_\pi = \beta i_b = \alpha i'_e \tag{47}$$

Small-Signal Equivalent Circuits

Several equivalent circuits are derived below which facilitate writing small-signal low-frequency equations for the BJT. We assume that the circuits external to the device can be represented by Thévenin equivalent circuits. The Norton equivalent circuit seen looking into the collector and the Thévenin equivalent circuits seen looking into the base and the emitter are derived. Although the T model is used for the derivation, identical results are obtained with the hybrid-π model. Several examples are given which illustrate use of the equivalent circuits.

Simplified T Model

Figure 8 shows the T model with a Thévenin source in series with the base. We wish to solve for an equivalent circuit in which the source i'_c connects from the collector node to ground rather than from the collector node to the B' node. The first step is to replace the source i'_c with two identical series sources with the common node grounded. The circuit is shown in Fig. 9(a).

For the circuit in Fig. 9(a), we can write

$$v_e = v_{tb} - \frac{i'_e}{1+\beta}(R_{tb} + r_x) - i'_e r_e = v_{tb} - i'_e \left(\frac{R_{tb} + r_x}{1+\beta} + r_e\right) \tag{48}$$

Let us define the resistance r'_e by

$$r'_e = \frac{R_{tb} + r_x}{1+\beta} + r_e = \frac{R_{tb} + r_x + r_\pi}{1+\beta} \tag{49}$$

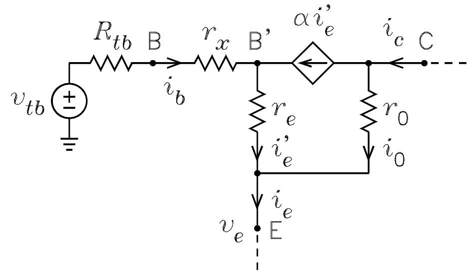

Figure 8: T model with Thévenin source connected to the base.

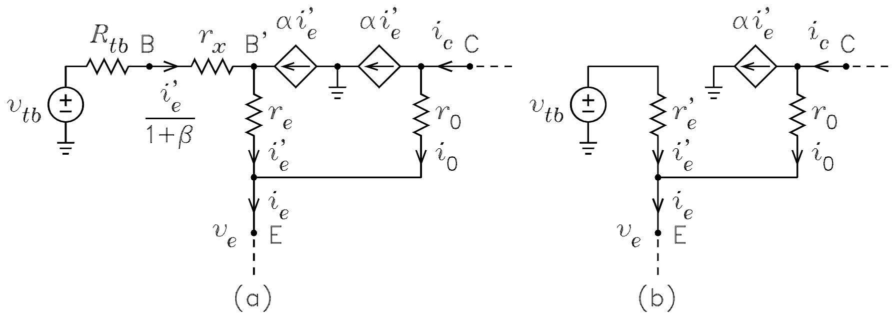

Figure 9: (a) Circuit with the i_c' source replaced by identical series sources. (b) Simplified T model.

With this definition, v_e is given by

$$v_e = v_{tb} - i_e' r_e' \tag{50}$$

The circuit which models this equation is shown in Fig. 9(b). This will be called the simplified T model. It predicts the same emitter and collector currents as the circuit in Fig. 8. Note that the resistors R_{tb} and r_x do not appear in this circuit. They are part of the resistor r_e'.

Norton Collector Circuit

The Norton equivalent circuit seen looking into the collector can be used to solve for the response of the common-emitter and common-base stages. It consists of a parallel current source $i_{c(sc)}$ and resistor r_{ic} from the collector to signal ground. Fig. 10(a) shows the BJT with Thévenin sources connected to its base and emitter. With the collector grounded, the collector current is the short-circuit or Norton collector current. To solve for this, we use the simplified T model in Fig. 10(b). We use superposition of v_{tb} and v_{te} to solve for $i_{c(sc)}$.

With $v_{te} = 0$, it follows from Fig. 10(b) that

$$\begin{aligned} i_{c(sc)} &= \alpha i_e' + i_0 = \alpha i_e' - i_e' \frac{R_{te}}{r_0 + R_{te}} \\ &= \frac{v_{tb}}{r_e' + R_{te} \| r_0} \left(\alpha - \frac{R_{te}}{r_0 + R_{te}} \right) \end{aligned} \tag{51}$$

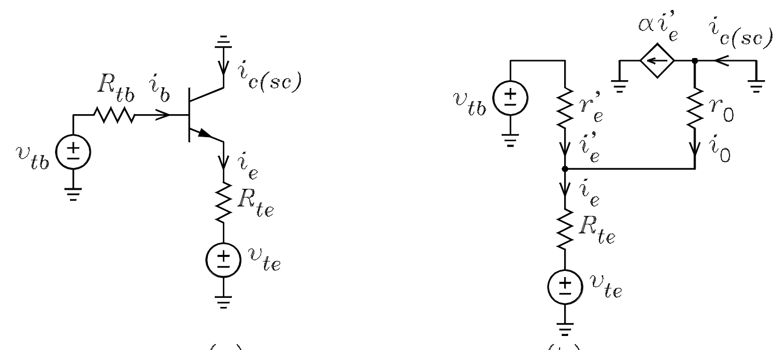

Figure 10: (a) BJT with Thevenin sources connected to the base and the emitter. (b) Simplified T model.

With $v_{tb} = 0$, we have

$$\begin{aligned} i_{c(sc)} &= \alpha i'_e + i_0 = \alpha i_e \frac{r_0}{r_0 + r'_e} + i_e \frac{r'_e}{r_0 + r'_e} \\ &= -\frac{v_{te}}{R_{te} + r'_e \| r_0} \frac{\alpha r_0 + r'_e}{r_0 + r'_e} \end{aligned} \tag{52}$$

These equations can be combined to obtain

$$i_{c(sc)} = \frac{v_{tb}}{r'_e + R_{te} \| r_0} \left(\alpha - \frac{R_{te}}{r_0 + R_{te}} \right) - \frac{v_{te}}{R_{te} + r'_e \| r_0} \frac{\alpha r_0 + r'_e}{r_0 + r'_e} \tag{53}$$

This equation is of the form

$$i_{c(sc)} = G_{mb} v_{tb} - G_{me} v_{te} \tag{54}$$

where

$$G_{mb} = \frac{1}{r'_e + R_{te} \| r_0} \left(\alpha - \frac{R_{te}}{r_0 + R_{te}} \right) = \frac{\alpha}{r'_e + R_{te} \| r_0} \frac{r_0 - R_{te}/\beta}{r_0 + R_{te}} \tag{55}$$

$$G_{me} = \frac{1}{R_{te} + r'_e \| r_0} \frac{\alpha r_0 + r'_e}{r_0 + r'_e} = \frac{\alpha}{r'_e + R_{te} \| r_0} \frac{r_0 + r'_e/\alpha}{r_0 + R_{te}} \tag{56}$$

The next step is to solve for the resistance seen looking into the collector with $v_{tb} = v_{te} = 0$. Figure 11(a) shows the simplified T model with a test source connected to the collector. The resistance seen looking into the collector is given by $r_{ic} = v_t / i_c$. To solve for r_{ic}, we can write

$$\begin{aligned} i_c &= \alpha i'_e + i_0 = -\alpha i_0 \frac{R_{te}}{r'_e + R_{te}} + i_0 \\ &= \frac{v_t}{r_0 + r'_e \| R_{te}} \left(1 - \frac{\alpha R_{te}}{r'_e + R_{te}} \right) \end{aligned} \tag{57}$$

It follows that r_{ic} is given by

$$r_{ic} = \frac{v_t}{i_c} = \frac{r_0 + r'_e \| R_{te}}{1 - \alpha R_{te} / \left(r'_e + R_{te} \right)} \tag{58}$$

The Norton equivalent circuit seen looking into the collector is shown in Fig. 11(b).

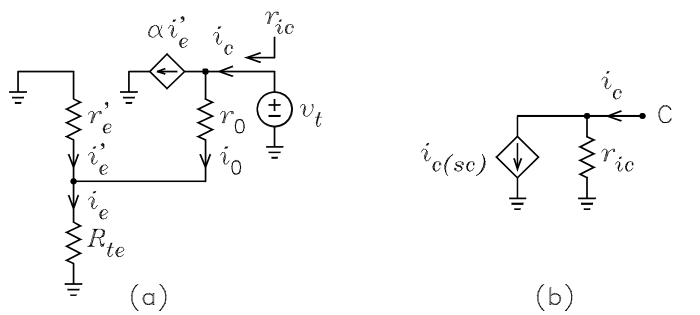

Figure 11: (a) Circuit for calculating r_{ic}. (b) Norton collector circuit.

For the case $r_0 \gg R_{te}$ and $r_0 \gg r'_e$, we can write

$$i_{c(sc)} = G_m \left(v_{tb} - v_{te} \right) \tag{59}$$

where

$$G_m = \frac{\alpha}{r'_e + R_{te}} \tag{60}$$

The value of $i_{c(sc)}$ calculated with this approximation is simply the value of $\alpha i'_e$, where i'_e is calculated with r_0 considered to be an open circuit. The term "r_0 approximations" is used in the following when r_0 is neglected in calculating $i_{c(sc)}$ but not neglected in calculating r_{ic}.

Thévenin Emitter Circuit

The Thévenin equivalent circuit seen looking into the emitter is useful in calculating the response of common-collector stages. It consists of a voltage source $v_{e(oc)}$ in series with a resistor r_{ie} from the emitter node to signal ground. Fig. 12(a) shows the BJT symbol with a Thévenin source connected to the base. The resistor R_{tc} represents the external load resistance in series with the collector. With the emitter open circuited, we denote the emitter voltage by $v_{e(oc)}$. The voltage source in the Thévenin emitter circuit has this value. To solve for it, we use the simplified T model in Fig. 12(b).

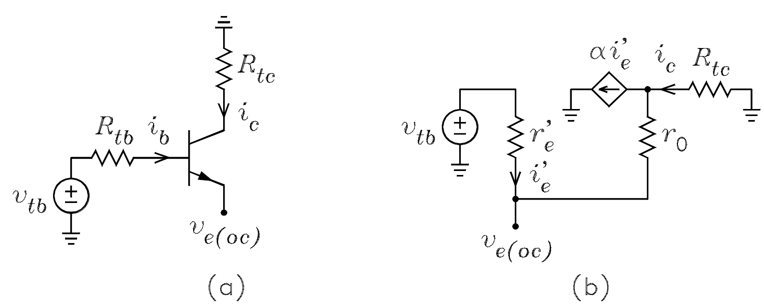

Figure 12: (a) BJT with Thévenin source connected to the base. (b) Simplified T model circuit for calculating $v_{e(oc)}$.

The current i'_e can be solved for by superposition of the sources v_{tb} and $\alpha i'_e$. It is given by

$$i'_e = \frac{v_{tb}}{r'_e + r_0 + R_{tc}} + \alpha i'_e \frac{R_{tc}}{r'_e + r_0 + R_{tc}} \tag{61}$$

This can be solved for i'_e to obtain

$$i'_e = \frac{v_{tb}}{r'_e + r_0 + (1-\alpha) R_{tc}} = \frac{v_{tb}}{r'_e + r_0 + R_{tc}/(1+\beta)} \tag{62}$$

The open-circuit emitter voltage is given by

$$v_{e(oc)} = v_{tb} - i'_e r'_e = v_{tb} \frac{r_0 + R_{tc}/(1+\beta)}{r'_e + r_0 + R_{tc}/(1+\beta)} \tag{63}$$

We next solve for the resistance seen looking into the emitter node. It can be solved for as the ratio of the open-circuit emitter voltage $v_{e(oc)}$ to the short-circuit emitter current. The circuit for calculating the short-circuit current is shown in Fig. 13(a). By superposition of i'_e and $\alpha i'_e$, we can write

$$\begin{aligned} i_{e(sc)} &= i'_e - \alpha i'_e \frac{R_{tc}}{r_0 + R_{tc}} = i'_e \frac{r_0 + (1-\alpha) R_{tc}}{r_0 + R_{tc}} \\ &= \frac{v_{tb}}{r'_e} \frac{r_0 + R_{tc}/(1+\beta)}{r_0 + R_{tc}} \end{aligned} \tag{64}$$

The resistance seen looking into the emitter is given by

$$r_{ie} = \frac{v_{e(oc)}}{i_{e(sc)}} = r'_e \frac{r_0 + R_{tc}}{r'_e + r_0 + R_{tc}/(1+\beta)} \tag{65}$$

The Thévenin equivalent circuit seen looking into the emitter is shown in Fig. 13(b).

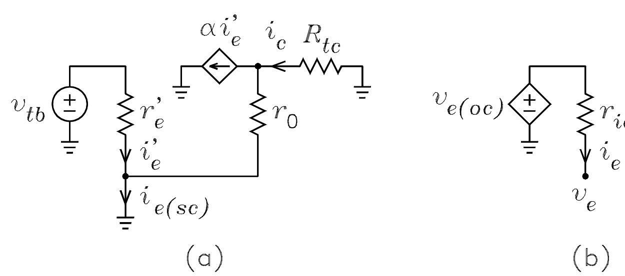

Figure 13: (a) Circuit for calculating $i_{e(sc)}$. (b) Thévenin emitter circuit.

Thévenin Base Circuit

Although the base is not an output terminal, the Thévenin equivalent circuit seen looking into the base is useful in calculating the base current. It consists of a voltage source $v_{b(oc)}$ in series with a resistor r_{ib} from the base node to signal ground. Fig. 14(a) shows the BJT symbol with a Thévenin source connected to its emitter. Fig. 14(b) shows the T model for calculating the open-circuit base voltage. Because $i_b = 0$, it follows that $i'_e = 0$. Thus there is no drop across r_x and r_e so that $v_{b(oc)}$ is given by

$$v_{b(oc)} = v_e = v_{te} \frac{r_0 + R_{tc}}{R_{te} + r_0 + R_{tc}} \tag{66}$$

The next step is to solve for the resistance seen looking into the base. It can be calculated by setting $v_{te} = 0$ and connecting a test current source i_t to the base. It is given by $r_{ib} = v_b / i_t$. Fig. 15(a) shows the T circuit for calculating v_b, where the current source βi_t has been divided

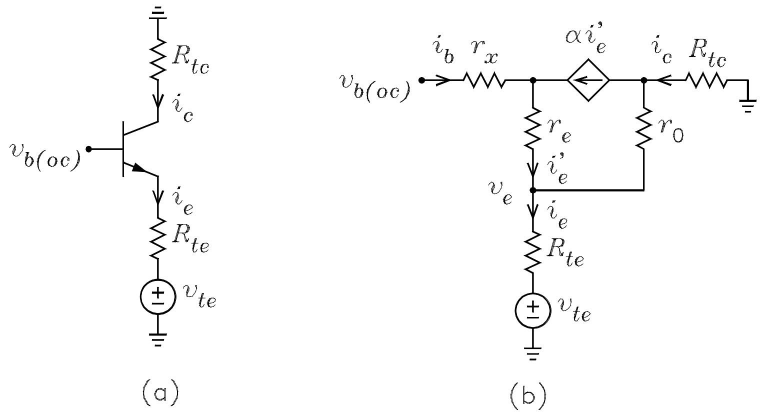

Figure 14: (a) BJT with Thevenin source connected to the emitter. (b) T model for calculating $v_{b(oc)}$.

into identical series sources with their common node grounded to simplify use of superposition. By superposition of i_t and the two βi_t sources, we can write

$$v_b = i_t r_x + (i_t + \beta i_t) [r_x + r_e + R_{te} \| (r_0 + R_{tc})] - \beta i_t \frac{R_{tc} R_{te}}{R_{tc} + r_0} \tag{67}$$

This can be solved for r_{ib} to obtain

$$r_{ib} = \frac{v_b}{i_t} = r_x + (1 + \beta) [r_e + R_{te} \| (r_0 + R_{tc})] - \frac{\beta R_{tc} R_{te}}{R_{tc} + r_0 + R_{te}} \tag{68}$$

The Thévenin base circuit is shown in Fig. 15(b).

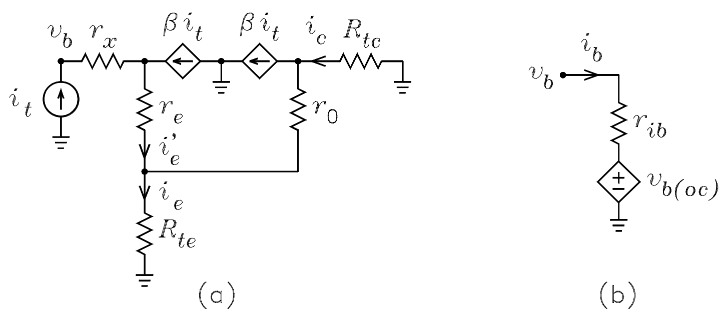

Figure 15: (a) Circuit for calculating v_b. (b) Thévenin base circuit.

Summary of Models

Figure 16 summarizes the four equivalent circuits derived above.

Example Amplifier Circuits

This section describes several examples which illustrate the use of the small-signal equivalent circuits derived above to write by inspection the voltage gain, the input resistance, and the output resistance of both single-stage and two-stage amplifiers.

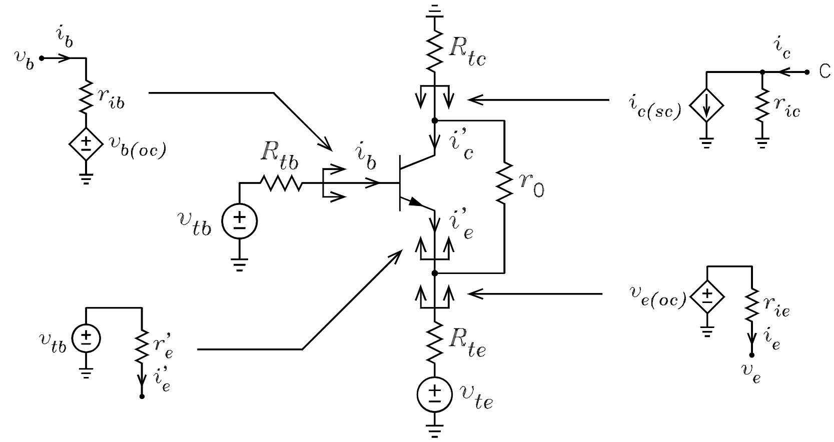

Figure 16: Summary of the small-signal equivalent circuits.

The Common-Emitter Amplifier

Figure 17(a) shows the ac signal circuit of a common-emitter amplifier. We assume that the bias solution and the small-signal resistances r'_e and r_0 are known. The output voltage and output resistance can be calculated by replacing the circuit seen looking into the collector by the Norton equivalent circuit of Fig. 11(b). With the aid of this circuit, we can write

$$v_o = -i_{c(sc)} \left(r_{ic} \| R_{tc} \right) = -G_{mb} \left(r_{ic} \| R_{tc} \right) v_{tb} \tag{69}$$

$$r_{\text{out}} = r_{ic} \| R_{tc} \tag{70}$$

where G_{mb} and r_{ic}, respectively, are given by Eqs. (55) and (58). The input resistance is given by

$$r_{\text{in}} = R_{tb} + r_{ib} \tag{71}$$

where r_{ib} is given by Eq. (68).

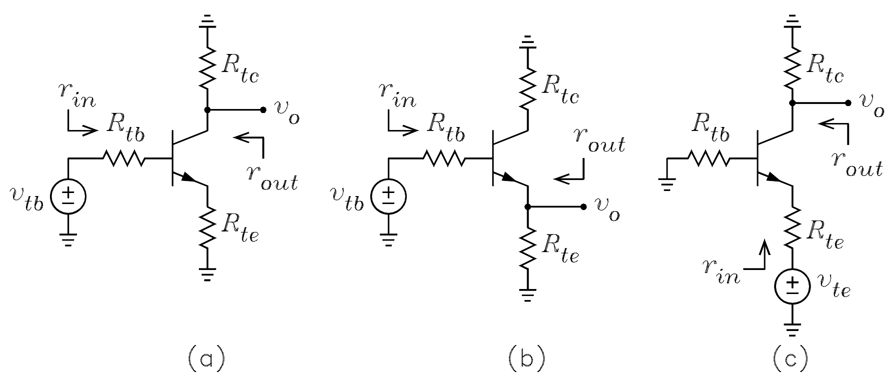

Figure 17: (a) Common-emitter amplifier. (b) Common-collector amplifier. (c) Common-base amplifier.

The Common-Collector Amplifier

Figure 17(b) shows the ac signal circuit of a common-collector amplifier. We assume that the bias solution and the small-signal resistances r_e' and r_0 are known. The output voltage and output resistance can be calculated by replacing the circuit seen looking into the emitter by the Thévenin equivalent circuit of Fig. 13(b). With the aid of this circuit, we can write

$$v_o = v_{e(oc)} \frac{R_{te}}{r_{ie} + R_{te}} = \frac{r_0 + R_{tc}/(1+\beta)}{r_e' + r_0 + R_{tc}/(1+\beta)} \frac{R_{te}}{r_{ie} + R_{te}} v_{tb} \tag{72}$$

$$r_{\text{out}} = r_{ie} \| R_{te} \tag{73}$$

where r_{ie} is given by Eq. (65). The input resistance is given by

$$r_{\text{in}} = R_{tb} + r_{ib} \tag{74}$$

where r_{ib} is given by Eq. (68).

The Common-Base Amplifier

Figure 17(c) shows the ac signal circuit of a common-base amplifier. We assume that the bias solution and the small-signal parameters r_e' and r_0 are known. The output voltage and output resistance can be calculated by replacing the circuit seen looking into the collector by the Norton equivalent circuit of Fig. 11(b). The input resistance can be calculated by replacing the circuit seen looking into the emitter by the Thévenin equivalent circuit of Fig. 13 with $v_{e(oc)} = 0$. With the aid of these circuits, we can write

$$v_o = -i_{c(sc)} (r_{ic} \| R_{tc}) = G_{me} (r_{ic} \| R_{tc}) v_{te} \tag{75}$$

$$r_{\text{out}} = r_{ic} \| R_{tc} \tag{76}$$

$$r_{\text{in}} = R_{te} + r_{ie} \tag{77}$$

where G_{me}, r_{ic}, and r_{ie}, respectively, are given by Eqs. (56), (58), and (65).

The CE/CC Amplifier

Figure 18(a) shows the ac signal circuit of a two-stage amplifier consisting of a CE stage followed by a CC stage. Such a circuit is used to obtain a high voltage gain and a low output resistance. The voltage gain can be written

$$\begin{aligned} \frac{v_o}{v_{tb1}} &= \frac{i_{c1(sc)}}{v_{tb1}} \times \frac{v_{tb2}}{i_{c1(sc)}} \times \frac{v_{e2(oc)}}{v_{tb2}} \times \frac{v_o}{v_{e2(oc)}} \\ &= G_{mb1} \left[-(r_{ic1} \| R_{C1}) \right] \frac{r_0}{r_{e2}' + r_0} \frac{R_{te2}}{r_{ie2} + R_{te2}} \end{aligned} \tag{78}$$

where r_{e2}' is calculated with $R_{tb2} = r_{ic1} \| R_{C1}$. The input and output resistances are given by

$$r_{\text{in}} = R_{tb1} + r_{ib1} \tag{79}$$

$$r_{\text{out}} = r_{ie2} \| R_{te2} \tag{80}$$

Although not a part of the solution, the resistance seen looking out of the collector of Q_1 is $R_{tc1} = R_{C1} \| r_{ib2}$.

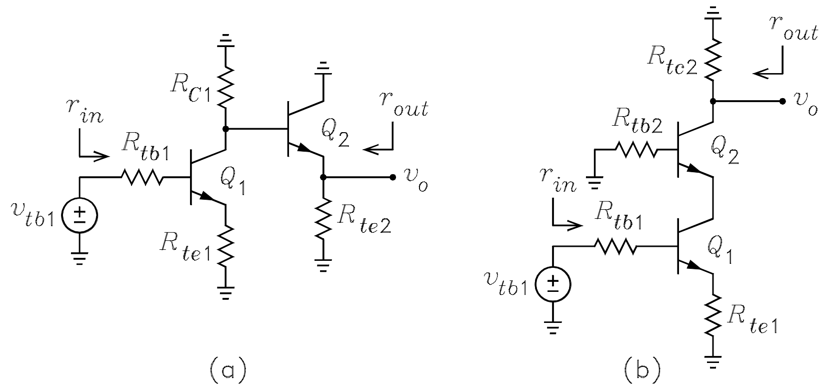

Figure 18: (a) CE-CC amplifier. (b) Cascode amplifier.

The Cascode Amplifier

Figure 18(b) shows the ac signal circuit of a cascode amplifier. The voltage gain can be written

$$\begin{aligned}\frac{v_o}{v_{tb1}} &= \frac{i_{c1(sc)}}{v_{tb1}} \times \frac{v_{te2}}{i_{c1(sc)}} \times \frac{i_{c2(sc)}}{v_{te2}} \times \frac{v_o}{i_{c2(sc)}} \\ &= G_{m1}\left(-r_{ic1}\right)\left(-G_{me2}\right)\left(-r_{ic2}\|R_{tc2}\right)\end{aligned}$$

where G_{me2} and r_{ic2} are calculated with $R_{te2} = r_{ic1}$. The input and output resistances are given by

$$r_{\text{in}} = R_{tb1} + r_{ib1}$$

$$r_{\text{out}} = R_{tc2}\|r_{ic2}$$

The resistance seen looking out of the collector of Q_1 is $R_{tc1} = r_{ie2}$.

A second cascode amplifier is shown in Fig. 19(a) where a pnp transistor is used for the second stage. The voltage gain is given by

$$\begin{aligned}\frac{v_o}{v_{tb1}} &= \frac{i_{c1(sc)}}{v_{tb1}} \times \frac{v_{te2}}{i_{c1(sc)}} \times \frac{i_{c2(sc)}}{v_{te2}} \times \frac{v_o}{i_{c2(sc)}} \\ &= G_{m1}\left(-r_{ic1}\|R_{C1}\right)\left(-G_{me2}\right)\left(-r_{ic2}\|R_{tc2}\right)\end{aligned}$$

The expressions for r_{in} and r_{out} are the same as for the cascode amplifier in Fig. 18(b). The resistance seen looking out of the collector of Q_1 is $R_{tc1} = R_{C1}\|r_{ie2}$.

The Differential Amplifier

Figure 19(b) shows the ac signal circuit of a differential amplifier. For the case of an active tail bias supply, the resistor R_Q represents its small-signal ac resistance. We assume that the transistors are identical, biased at the same currents and voltages, and have identical small-signal parameters. Looking out of the emitter of Q_1, the Thévenin voltage and resistance are given by

$$\begin{aligned}v_{te1} &= v_{e2(oc)}\frac{R_Q}{R_Q + R_E + r_{ie}} \\ &= v_{tb2}\frac{r_0 + R_{tc}/\left(1+\beta\right)}{r'_e + r_0 + R_{tc}/\left(1+\beta\right)}\,\frac{R_Q}{R_Q + R_E + r_{ie}} \qquad (81)\end{aligned}$$

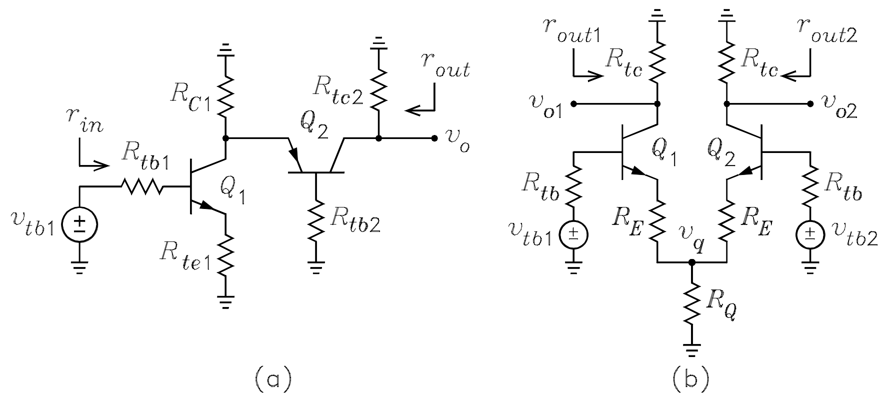

Figure 19: (a) Second cascode amplifier. (b) Differential amplifier.

$$R_{te1} = R_E + R_Q \| \left(R_E + r_{ie} \right) \tag{82}$$

The small-signal collector voltage of Q_1 is given by

$$\begin{aligned} v_{o1} &= -i_{c1(sc)} \left(r_{ic} \| R_{tc} \right) = -\left(G_{mb} v_{tb1} - G_{me} v_{te1} \right) \left(r_{ic} \| R_{tc} \right) \\ &= -G_{mb} \left(r_{ic} \| R_{tc} \right) v_{tb1} \\ &\quad + G_{me} \frac{r_0 + R_{tc} / \left(1+\beta\right)}{r'_e + r_0 + R_{tc} / \left(1+\beta\right)} \frac{R_Q}{r_{ie} + R_E + R_Q} v_{tb2} \end{aligned} \tag{83}$$

By symmetry, v_{o2} is obtained by interchanging the subscripts 1 and 2 in this equation. The small-signal resistance seen looking into either output is

$$r_{\text{out}} = R_{tc} \| r_{ic} \tag{84}$$

where r_{ic} calculated from Eq. (58) with $R_{te} = R_E + R_Q \| \left(R_E + r_{ie} \right)$. Although not labeled on the circuit, the input resistance seen by both v_{tb1} and v_{tb2} is $r_{\text{in}} = r_{ib}$.

A second solution of the diff amp can be obtained by replacing v_{tb1} and v_{tb2} with differential and common-mode components as follows:

$$v_{tb1} = v_{i(cm)} + \frac{v_{i(d)}}{2} \tag{85}$$

$$v_{tb2} = v_{i(cm)} - \frac{v_{i(d)}}{2} \tag{86}$$

where $v_{i(d)} = v_{tb1} - v_{tb2}$ and $v_{i(cm)} = \left(v_{tb1} + v_{tb2} \right) / 2$. Superposition of $v_{i(d)}$ and $v_{i(cm)}$ can be used to solve for v_{o1} and v_{o2}. With $v_{i(cm)} = 0$, the effects of $v_{tb1} = v_{i(d)}/2$ and $v_{tb2} = -v_{i(d)}/2$ are to cause $v_q = 0$. Thus the v_q node can be grounded and the circuit can be divided into two common-emitter stages in which $R_{te(d)} = R_E$ for each transistor. In this case, $v_{o1(d)}$ can be written

$$\begin{aligned} v_{o1(d)} &= \frac{i_{c1(sc)}}{v_{tb1(d)}} \times \frac{v_{o1(d)}}{i_{c1(sc)}} v_{tb1(d)} = G_{m(d)} \left(-r_{ic(d)} \| R_{tc} \right) \frac{v_{i(d)}}{2} \\ &= G_{m(d)} \left(-r_{ic(d)} \| R_{tc} \right) \frac{v_{tb1} - v_{tb2}}{2} \end{aligned} \tag{87}$$

By symmetry $v_{o2(d)} = -v_{o1(d)}$.

With $v_{i(d)} = 0$, the effects of $v_{tb1} = v_{tb2} = v_{i(cm)}$ are to cause the emitter currents in Q_1 and Q_2 to change by the same amounts. If R_Q is replaced by two parallel resistors of value $2R_Q$, it follows by symmetry that the circuit can be separated into two common-emitter stages each with $R_{te(cm)} = R_E + 2R_Q$. In this case, $v_{o1(cm)}$ can be written

$$
\begin{aligned}
v_{o1(cm)} &= \frac{i_{c1(sc)}}{v_{tb1(cm)}} \times \frac{v_{o1(cm)}}{i_{c1(sc)}} v_{tb1(cm)} = G_{m(cm)} \left(-r_{ic(cm)} \| R_{tc}\right) v_{i(cm)} \\
&= G_{m(cm)} \left(-r_{ic(cm)} \| R_{tc}\right) \frac{v_{tb1} + v_{tb2}}{2} \qquad (88)
\end{aligned}
$$

By symmetry $v_{o2(cm)} = v_{o1(cm)}$.

Because R_{te} is different for the differential and common-mode circuits, G_m, r_{ic}, and r_{ib} are different. However, the total solution $v_{o1} = v_{o1(d)} + v_{o1(cm)}$ is the same as that given by Eq. (83), and similarly for v_{o2}. The small-signal base currents can be written $i_{b1} = v_{i(cm)}/r_{ib(cm)} + v_{i(d)}/r_{ib(d)}$ and $i_{b2} = v_{i(cm)}/r_{ib(cm)} - v_{i(d)}/r_{ib(d)}$. If $R_Q \to \infty$, the common-mode solutions are zero. In this case, the differential solutions can be used for the total solutions. If $R_Q \gg R_E + r_{ie}$, the common-mode solutions are often approximated by zero.

Small-Signal High-Frequency Models

Figure 20 shows the hybrid-π and T models for the BJT with the base-emitter capacitance c_π and the base-collector capacitance c_μ added. The capacitor c_{cs} is the collector-substrate capacitance which in present in monolithic integrated-circuit devices but is omitted in discrete devices. These capacitors model charge storage in the device which affects its high-frequency performance. The capacitors are given by

$$c_\pi = c_{je} + \frac{\tau_F I_C}{V_T} \qquad (89)$$

$$c_\mu = \frac{c_{jc}}{\left[1 + V_{CB}/\phi_C\right]^{m_c}} \qquad (90)$$

$$c_{cs} = \frac{c_{jcs}}{\left[1 + V_{CS}/\phi_C\right]^{m_c}} \qquad (91)$$

where I_C is the dc collector current, V_{CB} is the dc collector-base voltage, V_{CS} is the dc collector-substrate voltage, c_{je} is the zero-bias junction capacitance of the base-emitter junction, τ_F is the forward transit time of the base-emitter junction, c_{jc} is the zero-bias junction capacitance of the base-collector junction, c_{jcs} is the zero-bias collector-substrate capacitance, ϕ_C is the built-in potential, and m_c is the junction exponential factor. For integrated circuit lateral pnp transistors, c_{cs} is replaced with a capacitor c_{bs} from base to substrate, i.e. from the B node to ground.

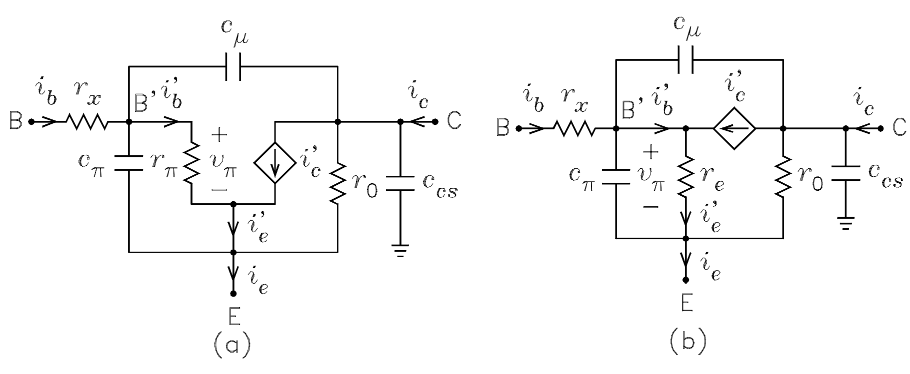

Figure 20: High-frequency small-signal models of the BJT. (a) Hybrid-π model. (b) T model.

In these models, the currents are related by

$$i_c' = g_m v_\pi = \beta i_b' = \alpha i_e' \tag{92}$$

These relations are the same as those in Eq. (47) with i_b replaced with i_b'.