The Common-Source Amplifier

Basic Circuit

Fig. 1 shows the circuit diagram of a single stage common-emitter amplifier. The object is to solve for the small-signal voltage gain, input resistance, and output resistance.

\[ V_{GG} = \frac{V^+ R_2 + V^- R_1}{R_1 + R_2} \quad R_{GG} = R_1 || R_2 \]

\[ V_{SS} = V^- \quad R_{SS} = R_S \quad V_{DD} = V^+ \quad R_{DD} = R_D \]

(b) Write the loop equation between the \( V_{GG} \) and the \( V_{SS} \) nodes.

\[ V_{GG} - V_{SS} = V_{GS} + I_S R_{SS} = V_{GS} + I_D R_{SS} \]

(c) Use the equation for the drain current to solve for \( V_{GS} \).

\[ V_{GS} = \sqrt{\frac{I_D}{K}} + V_{TO} \]
(d) Solve the equations simultaneously.

\[ I_D R_{SS} + \sqrt{\frac{I_D}{K}} + [(V_{GG} - V_{SS}) - V_{TO}] = 0 \]

(e) Let \( V_1 = (V_{GG} - V_{SS}) - V_{TO} \). Solve the quadratic for \( I_D \).

\[ I_D = \left( \frac{\sqrt{1 + 4KV_1R_{SS}} - 1}{2\sqrt{KR_{SS}}} \right)^2 \]

(d) Verify that \( V_{DS} > V_{GS} - V_{TO} = \sqrt{I_D/K} \) for the active mode.

\[ V_{DS} = V_D - V_S = (V_{DD} - I_D R_{DD}) - (V^- + I_D R_{SS}) = V_{DD} - V_{SS} - I_D R_{DD} \]

**Small-Signal or AC Solutions**

(a) Redraw the circuit with \( V^+ = V^- = 0 \) and all capacitors replaced with short circuits as shown in Fig. 3.

(b) Calculate \( g_m, r_s, \) and \( r_0 \) from the DC solution.

\[ g_m = 2\sqrt{KI_D} \quad r_s = \frac{1}{g_m} \quad r_0 = \frac{\lambda^{-1} + V_{DS}}{I_D} \]

(c) Replace the circuits looking out of the gate and source with Thévenin equivalent circuits as shown in Fig. 4.

\[ v_{tg} = v_i \frac{R_1 || R_2}{R_1 + R_1 || R_2} \quad R_{tg} = R_1 || R_2 \quad v_{te} = 0 \quad R_{ts} = R_S || R_3 \]
Figure 3: Signal circuit.

Figure 4: Signal circuit with Thévenin gate circuit.
Exact Solution

(a) Replace the circuit seen looking into the drain with its Norton equivalent circuit as shown in Fig. 5. Solve for \( i_{d(sc)} \).

\[
i_{d(sc)} = G_{mg}v_t g = G_{mg}v_t \frac{R_1||R_2}{R_i + R_1||R_2}
\]

\[
G_{mg} = \frac{1}{r_s + R_{ts}} \frac{r_0}{r_0 + R_{ts}}
\]

\[\text{Figure 5: Norton drain circuit.}\]

(b) Solve for \( v_o \).

\[
v_o = -i_{d(sc)}r_{id}||R_D||R_L = -G_{mg}v_t \frac{R_1||R_2}{R_i + R_1||R_2}r_{id}||R_D||R_L
\]

\[
r_{id} = \frac{r_0 + r_s||R_{ts}}{1 - R_{ts}/(r_s + R_{te})} = r_0 \left(1 + \frac{R_{ts}}{r_s'}\right) + R_{ts}
\]

(c) Solve for the voltage gain.

\[
A_v = \frac{v_o}{v_i} = -G_{ms} \frac{R_1||R_2}{R_i + R_1||R_2}r_{id}||R_D||R_L
\]

(d) Solve for \( r_{in} \).

\[
r_{in} = R_1||R_2
\]

(e) Solve for \( r_{out} \).

\[
r_{out} = r_{id}||R_D
\]

(d) Special case for \( R_{ts} = 0 \).

\[
G_{mg} = \frac{1}{r_s} = g_m \quad r_{id} = r_0
\]

Example 1 For the CS amplifier of Fig. ??, it is given that \( R_i = 5 \, \text{k}\Omega, R_1 = 5 \, \text{M}\Omega, R_2 = 1 \, \text{M}\Omega, R_D = 10 \, \text{k}\Omega, R_S = 3 \, \text{k}\Omega, R_3 = 50 \, \Omega, R_L = 20 \, \text{k}\Omega, V^+ = 24 \, \text{V}, V^- = -24 \, \text{V}, K_0 = 0.001 \, \text{A/V}^2, V_{TO} = 1.75 \, \text{V}, \lambda = 0.016 \, \text{V}^{-1}. Solve for the gain \( A_v = v_o/v_i \), the input resistance \( r_{in} \), and the output resistance \( r_{out} \). The capacitors can be assumed to be ac short circuits at the operating frequency.
Solution. For the dc bias solution, replace all capacitors with open circuits. The Thévenin voltage and resistance seen looking out of the gate are

\[ V_{GG} = \frac{V^+ R_2 + V^- R_1}{R_1 + R_2} = -16 \text{ V} \quad R_{BB} = R_1 \parallel R_2 = 833.3 \text{ k} \Omega \]

The Thévenin voltage and resistance seen looking out of the source are \( V_{SS} = V^- \) and \( R_{SS} = R_s \). To calculate \( I_D \), we neglect the Early effect by setting \( K = K_0 \). The bias equation for \( I_D \) is

\[ I_D = \left( \frac{\sqrt{1 + 4K V_1 R_{SS}} - 1}{2\sqrt{KR_{SS}}} \right)^2 = 1.655 \text{ mA} \]

To test for the active mode, we calculate the drain-source voltage

\[ V_{DS} = V_D - V_s = (V^+ - I_D R_D) - (V^- + I_D R_{SS}) = 26.491 \text{ V} \]

This must be greater than \( V_{GS} - V_{TO} = \sqrt{I_D/K} = 1.286 \text{ V} \). It follows that the MOSFET is biased in its active mode.

For the small-signal ac analysis, we need \( g_m \), \( r_s \), and \( r_0 \). When the Early effect is accounted for, the new value of \( K \) is given by

\[ K = K_0 (1 + \lambda V_{DS}) = 1.424 \times 10^{-3} \text{ A/V}^2 \]

Note that this is an approximation because the Early effect was neglected in calculating \( V_{DS} \). However, the approximation should be close to the true value. It follows that \( g_m \), \( r_s \), and \( r_0 \) are given by

\[ g_m = 2\sqrt{KI_D} = 3.07 \times 10^{-3} \text{ A/V} \quad r_s = \frac{1}{g_m} = 325.758 \text{ } \Omega \]

\[ r_0 = \frac{\lambda^{-1} + V_{DS}}{I_D} = 53.78 \text{ k} \Omega \]

For the small-signal analysis, \( V^+ \) and \( V^- \) are zeroed and the three capacitors are replaced with ac short circuits. The Thévenin voltage and resistance seen looking out of the gate are given by

\[ v_{tg} = v_i \frac{R_1 \parallel R_2}{R_i + R_1 \parallel R_2} = 0.994v_i \quad R_{tg} = R_i \parallel R_1 \parallel R_2 = 4.97 \text{ k} \Omega \]

The Thévenin resistances seen looking out of the source and the drain are

\[ R_{ts} = R_5 \parallel R_3 = 49.18 \text{ } \Omega \quad R_{td} = R_D \parallel R_L = 6.667 \text{ k} \Omega \]

Next, we calculate \( G_{mg} \) and \( r_{id} \)

\[ G_{mg} = \frac{1}{r_s + R_{ts} \parallel r_0} = \frac{r_0}{r_0 + R_{ts}} \]

\[ r_{id} = r_0 \left( 1 + \frac{R_{ts}}{r_s} \right) + R_{ts} = 61.95 \text{ k} \Omega \]
The output voltage is given by
\[ v_o = -G_{mg} \times (r_{id}) v_{tg} = -G_{mg} \times (r_{id}) \times 0.916v_i = -15.945v_i \]

Thus the voltage gain is
\[ A_v = \frac{v_o}{v_i} = -15.945 \]

The input and output resistances are given by
\[ r_{in} = R_1 || R_2 = 833.3 \, k\Omega \quad r_{out} = r_{id} || R_D = 8.61 \, k\Omega \]

**Approximate Solutions**

These solutions assume that \( r_0 = \infty \) except in calculating \( r_{id} \). In this case, \( i_{d(sc)} = i_d' = i_s' \).

**Source Equivalent Circuit Solution**

(a) After making the Thévenin equivalent circuits looking out of the gate and source, replace the MOSFET with the source equivalent circuit as shown in Fig. 6.

(b) Solve for \( i_d' = i_s' \) and \( r_{id} \).
\[ v_{tg} = i_s' (r_s' + R_{te}) = i_d' (r_s + R_{ts}) \implies i_d' = v_{tg} \frac{1}{r_s + R_{ts}} \]
\[ r_{id} = \frac{r_0 + r_s || R_{ts}}{1 - R_{ts} / (r_s + R_{te})} = r_0 \left( 1 + \frac{R_{ts}}{r_s'} \right) + R_{ts} \]

(c) Solve for \( v_o \) and \( A_v = v_o/v_i \).
\[ v_o = -i_d' r_{id} || R_D || R_L = v_{tg} \frac{-1}{r_s + R_{ts}} r_{id} || R_D || R_L = -v_i \frac{R_1 || R_2}{R_i + R_1 || R_2} \frac{1}{r_s + R_{ts}} r_{id} || R_D || R_L \]
\[ R_{ts} = R_S || R_3 \]
\[ A_v = \frac{v_o}{v_i} = -\frac{R_1 R_2}{R_i + R_1 R_2 r_s + R_{ts}} \frac{1}{r_{id}} \frac{R_D}{R_L} \]

Note that this is of the form
\[ A_v = \frac{v_tg}{v_i} \times \frac{i'_s}{v_tg} \times \frac{i'_d}{i'_s} \times \frac{v_o}{v_tg} \]

(d) Solve for \( r_{out} \).
\[ r_{out} = r_{id} || R_D \]

**Example 2** Use the simplified T-model solutions to calculate the values of \( A_v, r_{in}, \) and \( r_{out} \) for Example 1.

\[ A_v = 0.994 \times (2.667 \times 10^{-3}) \times (-6.019 \times 10^{3}) = -15.957 \]
\[ r_{in} = 833.3 \, k\Omega \quad r_{id} = 61.95 \, k\Omega \quad r_{out} = 8.61 \, k\Omega \]

**π Model Solution**

(a) After making the Thévenin equivalent circuits looking out of the gate and source, replace the MOSFET with the π model as shown in Fig. 7.

![Figure 7: Hybrid π model circuit.](image)

(b) Solve for \( i'_d \) and \( r_{id} \).
\[ v_{tg} = v_{\pi} + i'_s R_{ts} = \frac{i'_d}{g_m} + i'_d R_{ts} \implies i'_d = \frac{v_{tg}}{\frac{1}{g_m} + R_{ts}} \]
\[ r_{id} = \frac{r_0 + r_s R_{ts}}{1 - R_{ts} / (r_s + R_{te})} = r_0 \left( 1 + \frac{R_{ts}}{r'_s} \right) + R_{ts} \]

(c) Solve for \( v_o \).
\[ v_o = -i'_d r_{id} || R_D || R_L = -\frac{v_{tg}}{\frac{1}{g_m} + R_{ts}} r_{id} || R_D || R_L = v_i \frac{R_1 || R_2}{R_i + R_1 || R_2} \frac{1}{\frac{1}{g_m} + R_{ts}} \]
(d) Solve for the voltage gain.

\[ A_v = \frac{v_o}{v_i} = \frac{R_1||R_2}{R_i + R_1||R_2} \frac{1}{g_{m}} (-r_{id}||R_D||R_L) \]

This is of the form

\[ A_v = \frac{v_{tg}}{v_i} \times \frac{i'_d}{v_{tg}} \times \frac{v_o}{i'_d} \]

(e) Solve for \( r_{in} \).

\[ r_{in} = R_1||R_2 \]

(f) Solve for \( r_{out} \).

\[ r_{out} = r_{id}||R_D \]

**Example 3** Use the \( \pi \)-model solutions to calculate the values of \( A_v, r_{in}, \) and \( r_{out} \) for Example 1.

\[ A_v = 0.994 \times (2.667 \times 10^{-3}) \times (-6.019 \times 10^3) = -15.957 \]

\[ r_{in} = 833.3 \text{k\Omega} \quad r_{id} = 61.95 \text{k\Omega} \quad r_{out} = 8.61 \text{k\Omega} \]

**T Model Solution**

(a) After making the Thévenin equivalent circuits looking out of the gate and source, replace the MOSFET with the T model as shown in Fig. 8.

![T model circuit](image)

Figure 8: T model circuit.

(b) Solve for \( i'_d \).

\[ v_{tg} = i'_s (r_s + R_{ts}) = i'_d (r_s + R_{ts}) \implies i'_d = \frac{v_{tg}}{r_s + R_{ts}} \]

(c) Solve for \( v_o \).

\[ v_o = -i'_d r_{id}||R_D||R_L = \frac{v_{tg}}{r_s + R_{ts}} r_{id}||R_D||R_L = v_i \frac{R_1||R_2}{R_i + R_1||R_2} \frac{-r_{id}||R_D||R_L}{r_s + R_{ts}} \]

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(d) Solve for the voltage gain.

\[ A_v = \frac{v_o}{v_i} = \frac{R_1 \parallel R_2}{R_i + R_1 \parallel R_2} \frac{1}{r_s + R_{ts}} (-r_{id} \parallel R_D \parallel R_L) \]

(e) Solve for \( r_{in} \).

\[ r_{in} = R_1 \parallel R_2 \]

(f) Solve for \( r_{out} \).

\[ r_{out} = r_{id} \parallel R_D \]

Example 4 Use the T-model solutions to calculate the values of \( A_v, r_{in}, \) and \( r_{out} \) for Example 1.

\[ A_v = 0.994 \times (2.667 \times 10^{-3}) \times (-6.019 \times 10^3) = -15.957 \]

\[ r_{in} = 833.3 \text{k}\Omega \quad r_{id} = 61.95 \text{k}\Omega \quad r_{out} = 8.61 \text{k}\Omega \]