The FET Bias Equation

Basic Bias Equation

(a) Look out of the 3 MOSFET terminals and replace the circuits with Thévenin equivalent circuits as showin in Fig. 1.

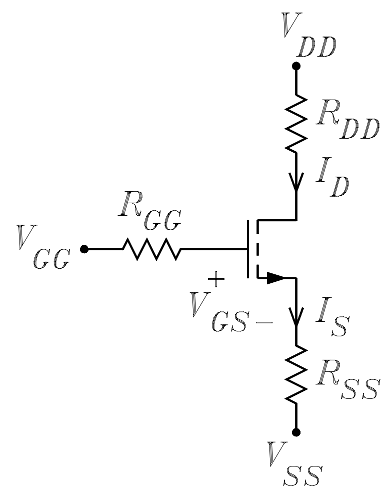

Figure 1: Basic bias circuit.

(b) Solve the FET drain current equation for V_{GS}.

$$V_{GS} = \sqrt{\frac{I_D}{K}} + V_{TO}$$

(c) Write the gate-source loop equation in the gate-source loop and let $I_S = I_D$.

$$V_{GG} - V_{SS} = V_{GS} + I_S R_{SS} = V_{GS} + I_D R_{SS}$$

(d) Solve the loop equation for V_{GS}.

$$V_{GS} = V_{GG} - V_{SS} - I_D R_{SS}$$

(e) Equate the two expressions for V_{GS} and rearrange the terms to obtain a quadratic equation in $\sqrt{I_D}$.

$$I_D R_{SS} + \sqrt{\frac{I_D}{K}} - (V_{GG} - V_{SS} - V_{TO}) = 0$$

(f) Let $a = R_{SS}$, $b = 1/\sqrt{K}$, and $c = -(V_{GG} - V_{SS} - V_{TO})$. In this case, the bias equation becomes

$$aI_D + b\sqrt{I_C} + c = 0$$

Use the quadratic equation to solve for $\sqrt{I_D}$, then square the result to obtain

$$I_D = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} \right)^2$$

Note that there is a second solution using the minus sign for the radical. This solution results in $V_{GS} < V_{TO}$, which is a non realizable solution. The desired solution is the one which gives the smaller value of I_D.

(e) Check for the active mode. For the active mode, $V_{DS} > V_{GS} - V_{TO} = \sqrt{I_D/K}$.

$$V_{DS} = V_D - V_S = (V_{DD} - I_D R_{DD}) - (V_{SS} + I_S R_{SS}) = V_{DD} - V_{SS} - I_D (R_{DD} + R_{SS})$$

Example 1

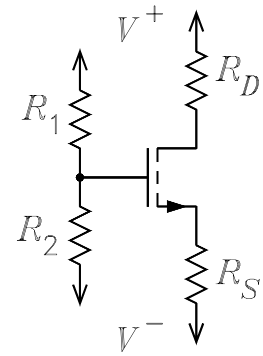

Figure 2: Circuit for Example 1.

$$V_{GG} = \frac{V^+ R_2 + V^- R_1}{R_1 + R_2} \qquad R_{GG} = R_1 \| R_2$$

$$V_{SS} = V^- \qquad R_{SS} = R_S \qquad V_{DD} = V^+ \qquad R_{DD} = R_D$$

Example 2

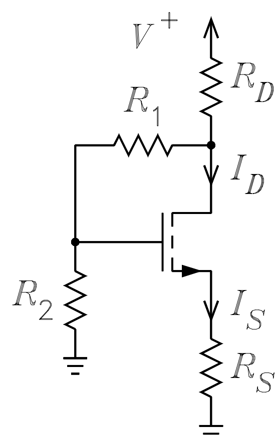

Figure 3: Circuit for Example 2.

$$V_{GG} = V^+ \frac{R_2}{R_D + R_1 + R_2} - I_D \frac{R_D}{R_D + R_1 + R_2} R_2 \qquad R_{GG} = (R_1 + R_D) \| R_2$$

$$V_{DD} = V^+ \frac{R_1 + R_2}{R_D + R_1 + R_2} \qquad R_{DD} = R_D \| (R_1 + R_2)$$

$$V_{SS} = 0 \qquad R_{SS} = R_S$$

The gate-source loop equation is

$$V^+ \frac{R_2}{R_D + R_1 + R_2} - I_D \frac{R_D}{R_D + R_1 + R_2} R_2 = V_{GS} + I_D R_S$$

This can be solved for V_{GS} and equated to $\sqrt{I_D/K} + V_{TO}$ to obtain

$$I_D \left(R_S + \frac{R_D R_2}{R_D + R_1 + R_2} \right) + \sqrt{\frac{I_D}{K}} - \left(\frac{V^+ R_2}{R_D + R_1 + R_2} - V_{TO} \right) = 0$$

The a, b, and c in the bias equation are given by

$$a = R_S + \frac{R_D R_2}{R_D + R_1 + R_2} \qquad b = \frac{1}{\sqrt{K}} \qquad c = -\left(\frac{V^+ R_2}{R_D + R_1 + R_2} - V_{TO} \right)$$

Example 3

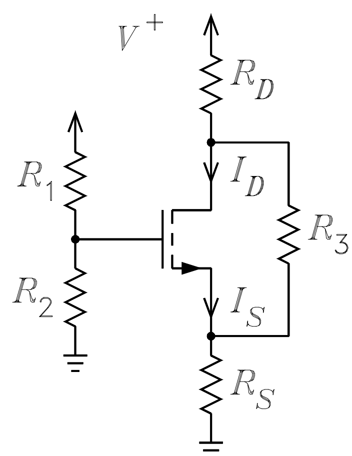

Figure 4: Circuit for Example 3.

$$V_{GG} = \frac{V^+ R_2}{R_1 + R_2} \qquad R_{GG} = R_1 \| R_2$$

$$V_{SS} = V^+ \frac{R_S}{R_D + R_3 + R_S} - I_D \frac{R_D}{R_D + R_3 + R_S} R_S \qquad R_{SS} = R_S \| \left(R_D + R_3 \right)$$

$$V_{DD} = V^+ \frac{R_3 + R_S}{R_D + R_3 + R_S} + I_S \frac{R_S}{R_D + R_3 + R_S} R_D \qquad R_{DD} = R_D \| \left(R_3 + R_S \right)$$

Let $I_S = I_D$. The bias equation for I_D is

$$\frac{V^+ R_2}{R_1 + R_2} - \left(V^+ \frac{R_S}{R_D + R_3 + R_S} - I_D \frac{R_D}{R_D + R_3 + R_S} R_S \right) = \sqrt{\frac{I_D}{K}} + V_{TO} + I_D \left[R_S \| \left(R_D + R_3 \right) \right]$$

which gives

$$I_D \left(R_S \| \left(R_D + R_3 \right) - \frac{R_D R_S}{R_D + R_3 + R_S} \right) + \sqrt{\frac{I_D}{K}} - \left(\frac{V^+ R_2}{R_1 + R_2} - \frac{V^+ R_S}{R_D + R_3 + R_S} - V_{TO} \right) = 0$$

The a, b, and c in the bias equation are given by

$$a = R_S \| \left(R_D + R_3 \right) - \frac{R_D R_S}{R_D + R_3 + R_S} \qquad b = \sqrt{\frac{1}{K}}$$

$$c = -\left(\frac{V^+ R_2}{R_1 + R_2} - \frac{V^+ R_S}{R_D + R_3 + R_S} - V_{TO} \right)$$

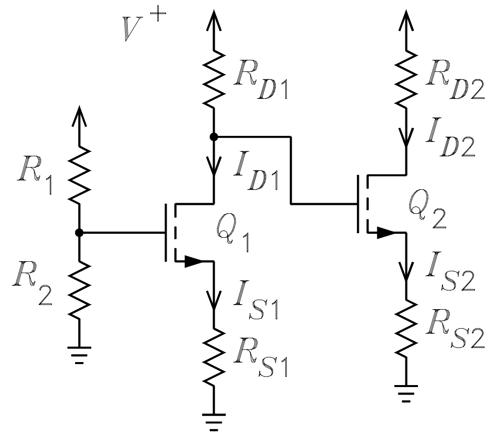

Figure 5: Circuit for Example 4.

Example 4

For M_1

$$V_{GG1} = V^+ \frac{R_2}{R_1 + R_2} \qquad R_{GG1} = R_1 \| R_2 \qquad V_{SS1} = 0 \qquad R_{SS1} = R_{S1}$$

$$V_{DD1} = V^+ \qquad R_{DD1} = R_{D1}$$

The loop equation for I_{D1} is

$$V^+ \frac{R_2}{R_1 + R_2} = V_{GS1} + I_{D1} R_S$$

This and the equation for V_{GS1} can be solved for I_{D1}.

For M_2

$$V_{GG2} = V^+ - I_{D1} R_{D1} \qquad R_{GG2} = R_{D1}$$

$$V_{SS2} = 0 \qquad R_{SS2} = R_{S2} \qquad V_{DD2} = V^+ \qquad R_{DD2} = R_{D2}$$

The loop equation for I_{D2} is

$$V^+ - I_{D1} R_{D1} = V_{GS2} + I_{D2} R_{S2}$$

This and the equation for V_{GS2} can be solve for I_{D2}.

Given I_{D1} and I_{D2}, it can be determined if the two MOSFETs are in the active mode.

Example 5

$$V_{GG1} = V^+ \frac{R_2}{R_1 + R_2} \qquad R_{GG1} = R_1 \| R_2 \qquad V_{SS1} = 0 \qquad R_{SS1} = R_{S1}$$

$$V_{GG2} = I_{S1} R_{S1} \qquad R_{GG2} = R_{S1} \qquad V_{SS2} = 0 \qquad R_{SS2} = R_{S2} \qquad V_{DD2} = V^+ \qquad R_{DD2} = R_{D2}$$

Let the currents to be solved for be I_{D1} and I_{D2} and let $I_{S1} = I_{D1}$ and $I_{S2} = I_{D2}$.

The gate-source loop equation for I_{D1} is

$$V^+ \frac{R_2}{R_1 + R_2} = V_{GS1} + I_{D1} R_{S1}$$

This and the equation for V_{GS1} can be solved for I_{D1}.

The gate-source loop equation for I_{D2} is

$$I_{D1} R_{S1} = V_{GS2} + I_{D2} R_{S2}$$

Given I_{D1} and I_{D2}, it can be determined if the two MOSFETs are in the active mode.

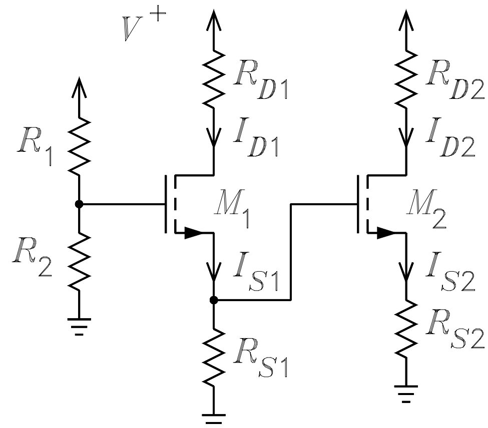

Figure 6: Circuit for Example 5.