The FET Differential Amplifier

Basic Circuit

Fig. 1 shows the circuit diagram of a MOSFET differential amplifier. The tail supply is modeled as a current source I'_Q having a parallel resistance R_Q. In the case of an ideal current source, R_Q is an open circuit. Often a diff amp is designed with a resistive tail supply. In this case, $I'_Q = 0$. The object is to solve for the small-signal output voltages and output resistance.

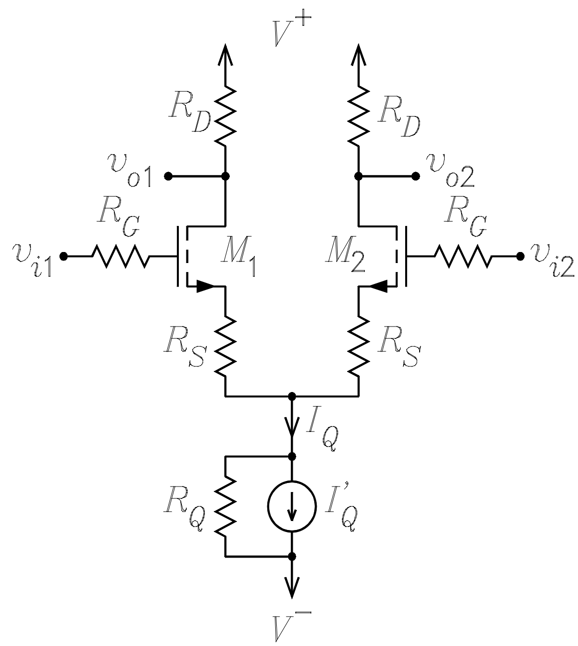

Figure 1: MOSFET differential amplifier.

DC Solutions

(a) Zero both inputs. Divide the tail supply into two equal parallel current sources having a current $I'_Q/2$ in parallel with a resistor $2R_Q$. The circuit obtained for M_1 is shown on the left in Fig. 2. The circuit for M_2 is identical. Now make a Thévenin equivalent as shown in on the right in Fig. 2. This is the basic bias circuit.

(e) With $V_{GG} = 0$, $V_{SS} = V^- - I'_Q R_Q$, and $R_{SS} = R_S + 2R_Q$, the bias equation from the FET bias notes is

$$I_D (R_S + 2R_Q) + \sqrt{\frac{I_D}{K}} - \left[0 - \left(V^- - I'_Q R_Q\right) - V_{TO}\right] = 0$$

(f) Let $a = R_{SS}$, $b = 1/\sqrt{K}$, and $c = -\left[0 - \left(V^- - I'_Q R_Q\right) - V_{TO}\right]$. Use the quadratic equation

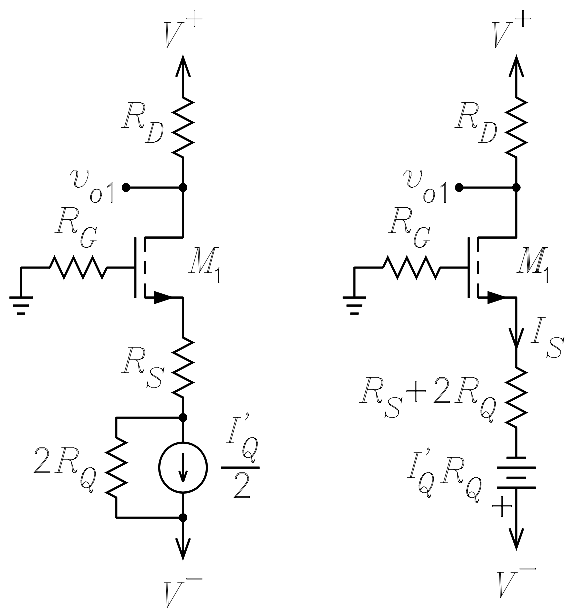

Figure 2: DC bias circuit for M_1.

to solve for $\sqrt{I_D}$, then square the result to obtain

$$I_D = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} \right)^2$$

Note that there is a second solution using the minus sign for the radical. This solution results in $V_{GS} < V_{TO}$, which is a non realizable solution. The desired solution is the one which gives the smaller value of I_D.

(e) Check for the active mode. For the active mode, $V_{DS} > V_{GS} - V_{TO} = \sqrt{I_D/K}$.

$$V_{DS} = V_D - V_S = (V_{DD} - I_D R_{DD}) - V_S = (V_{DD} - I_D R_{DD}) - \left(\sqrt{\frac{I_D}{K}} - V_{TO} \right)$$

(f) If $R_Q = \infty$, it follows that $I_{D1} = I_{D2} = I'_Q/2$.

Small-Signal or AC Solutions

The solutions assume that the two FETs are matched.

(a) Calculate g_m and r_s.

$$g_m = 2\sqrt{KI_D} \qquad r_s = \frac{1}{g_m}$$

(b) Redraw the circuit with $V^+ = V^- = 0$ and $I'_Q = 0$. Replace the two FETs with the simple T model. The source part of the circuit obtained is shown in 3.

(c) Solve for i'_{s1} and i'_{s2}.

$$i'_{s1} = \frac{v_{i1}}{r_s + R_S + R_Q \| (r_s + R_S)} - \frac{v_{i2}}{r_s + R_S + R_Q \| (r_s + R_S)} \frac{R_Q}{R_Q + r_s + R_S}$$

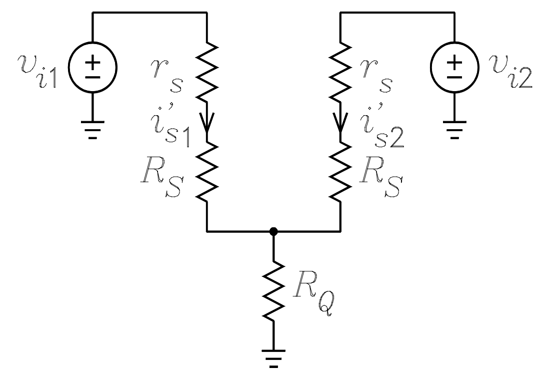

Figure 3: Source equivalent circuit for $r_0 = \infty$.

$$i'_{s2} = \frac{v_{i2}}{r_s + R_S + R_Q \| (r_s + R_S)} - \frac{v_{i1}}{r_s + R_S + R_Q \| (r_s + R_S)} \frac{R_Q}{R_Q + r_s + R_S}$$

(d) Solve for v_{o1}, v_{o2}, r_{out1}, and r_{out2} from the drain equivalent circuits.

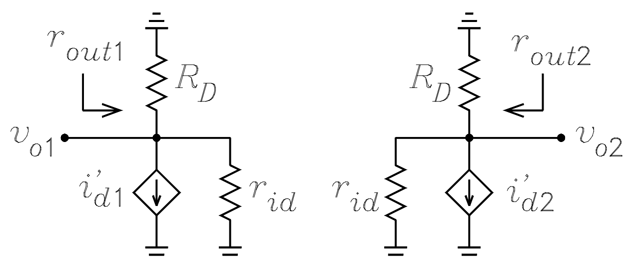

Figure 4: Drain equivalent circuits.

$$v_{o1} = -i'_{d1} r_{id} \| R_D = -i'_{s1} r_{id} \| R_D = \frac{-r_{id} \| R_D}{r_s + R_S + R_Q \| (r_s + R_S)} \left(v_{i1} - v_{i2} \frac{R_Q}{R_Q + r_s + R_S} \right)$$

$$v_{o2} = -i'_{d2} r_{id} \| R_D = -i'_{s1} r_{id} \| R_D = \frac{-r_{id} \| R_D}{r_s + R_S + R_Q \| (r_s + R_S)} \left(v_{i2} - v_{i1} \frac{R_Q}{R_Q + r_s + R_S} \right)$$

$$r_{out1} = r_{out2} = r_{id} \| R_D$$

(e) Special case for $R_Q = \infty$.

$$v_{o1} = \frac{-r_{id} \| R_D}{2 (r_s + R_S)} (v_{i1} - v_{i2}) \qquad v_{o2} = \frac{-r_{id} \| R_D}{2 (r_s + R_S)} (v_{i2} - v_{i1})$$

(f) To include the body effect in these equations, divide all input voltages by $1 + \chi$ and replace r_s with $r'_s = r_s / (1 + \chi)$, where $\chi = g_{mb}/g_m$ is the transconductance ratio. If $R_S = 0$ and $R_Q = \infty$, the factor $1 + \chi$ cancels out in the equations for v_{o1} and v_{o2} and the body effect goes away.

Common-Mode Rejection Ratio

The $CMRR$ for the BJT differential amplifier was defined with the output taken from only one side of the diff amp. To illustrate another way of defining the $CMRR$, it will be assumed that the output is taken differentially between the two outputs. In this case, the $CMRR$ is doubled.

(a) Define the differential input and output voltages

$$v_{id} = v_{i1} - v_{i2} \qquad v_{od} = v_{o1} - v_{o2}$$

(b) With $v_{i1} = v_{id}/2$ and $v_{i2} = -v_{id}/2$, use the solutions from above to solve for the differential output voltage v_{od}.

$$\begin{aligned} v_{od} &= v_{o1} - v_{o2} = \frac{-r_{id} \| R_D}{r_s + R_S + R_Q \| \left(r_s + R_S\right)} \left(1 + \frac{R_Q}{R_Q + r_s + R_S}\right) \frac{v_{id}}{2} \\ &= \frac{-r_{id} \| R_D}{r_s + R_S + R_Q \| \left(r_s + R_S\right)} \frac{2R_Q + r_s + R_S}{R_Q + r_s + R_S} \frac{v_{id}}{2} \end{aligned}$$

Solve for the differential gain A_v.

$$A_{vd} = \frac{v_{od}}{v_{id}} = \frac{1}{2} \frac{-r_{id} \| R_D}{r_s + R_S + R_Q \| \left(r_s + R_S\right)} \frac{2R_Q + r_s + R_S}{R_Q + r_s + R_S}$$

(c) Define the common-mode input and output voltages v_{icm} and v_{ocm}.

$$v_{icm} = \frac{v_{i1} + v_{i2}}{2} \qquad v_{ocm} = \frac{v_{o1} + v_{o2}}{2}$$

(d) With $v_{i1} = v_{i2} = v_{icm}$, use the solutions from above to solve for the common-mode output voltage v_{ocm}.

$$\begin{aligned} v_{ocm} &= \frac{v_{o1} + v_{o2}}{2} = \frac{1}{2} \frac{-r_{id} \| R_D}{r_s + R_S + R_Q \| \left(r_s + R_S\right)} \left(1 - \frac{R_Q}{R_Q + r_s + R_S}\right) v_{icm} \\ &= \frac{1}{2} \frac{-r_{id} \| R_D}{r_s + R_S + R_Q \| \left(r_s + R_S\right)} \frac{r_s + R_S}{R_Q + r_s + R_S} v_{icm} \end{aligned}$$

Solve for the common-mode voltage gain A_{vcm}.

$$A_{vcm} = \frac{v_{ocm}}{v_{icm}} = \frac{1}{2} \frac{-r_{id} \| R_D}{r_s + R_S + R_Q \| \left(r_s + R_S\right)} \frac{r_s + R_S}{R_Q + r_s + R_S}$$

(e) Solve for the common-mode rejection ratio.

$$CMRR = \left| \frac{A_{vd}}{A_{vcm}} \right| = \frac{2R_Q + r_s + R_S}{r_s + R_S} = 1 + \frac{2R_Q}{r_s + R_S}$$

Express this in dB.

$$CMRR_{dB} = 20 \log \left(1 + \frac{2R_Q}{r_s + R_S}\right)$$

(f) To include the body effect in the equation for $CMRR$, replace r_s with $r_s' = r_s / \left(1 + \chi\right)$.

Example 1 *For* $I_Q = 2\,\text{mA}$, $R_Q = 50\,\text{k}\Omega$, $R_G = 1\,\text{k}\Omega$, $R_S = 100\,\Omega$, $R_D = 10\,\text{k}\Omega$, $V^+ = 20\,\text{V}$, $V^- = -20\,\text{V}$, $K = 2.5 \times 10^{-3}\,\text{A}/\,\text{V}^2$, $V_{TO} = 1.5\,\text{V}$, *and* $\lambda = 0.01$, *calculate* v_{o1}, v_{o2}, v_{od}, r_{out}, *and* $CMRR$.

Solution.

$$I_{D1} = I_{D2} = \frac{I_Q}{2} = 1\,\text{mA} \qquad V_{GS} = V_{TO} + \sqrt{\frac{I_D}{K}} = 2.132\,\text{V}$$

$$V_{DS} = V_D - V_S = \left(V^+ - \frac{I_Q}{2} R_D\right) - \left(-V_{GS}\right) = 12.13\,\text{V}$$

$$g_m = 2\sqrt{KI_D} = 3.162\,\text{mS} \qquad r_s = \frac{1}{g_m} = 316.23\,\Omega \qquad r_0 = \frac{\lambda^{-1} + V_{DS}}{I_D} = 112.1\,\text{k}\Omega$$

$$R_{ts} = R_S + R_Q \| (r_s + R_S) = 512.79\,\Omega \qquad r_{id} = r_0\left(1 + \frac{R_{ts}}{r_s}\right) + R_{ts} = 294.5\,\text{k}\Omega$$

$$v_{o1} = \frac{-r_{id}\|R_D}{r_s + R_S + R_Q\|(r_s + R_S)}\left(v_{i1} - v_{i2}\frac{R_Q}{R_Q + r_s + R_S}\right) = -11.67v_{i1} + 11.57v_{i2}$$

$$v_{o2} = -11.67v_{i2} + 11.57v_{i1}$$

$$v_{od} = v_{o1} - v_{o2} = -23.24\,(v_{i1} - v_{i2}) \qquad r_{out} = r_{id}\|R_D = 9.672\,\text{k}\Omega$$

$$v_{ocm} = \frac{v_{o1} + v_{o2}}{2} = -0.096v_{icm}$$

$$A_{vd} = \frac{-r_{id}\|R_D}{r_s + R_S + R_Q\|(r_s + R_S)}\frac{2R_Q + r_s + R_S}{R_Q + r_s + R_S} = -23.24$$

$$A_{vcm} = \frac{-r_{id}\|R_D}{r_s + R_S + R_Q\|(r_s + R_S)}\frac{r_s + R_S}{R_Q + r_s + R_S} = -0.096$$

$$CMRR_{dB} = 20\log\left|\frac{A_{vd}}{A_{vcm}}\right| = 47.65\,\text{dB}$$

The MOSFET Diff Amp with Current-Mirror Load

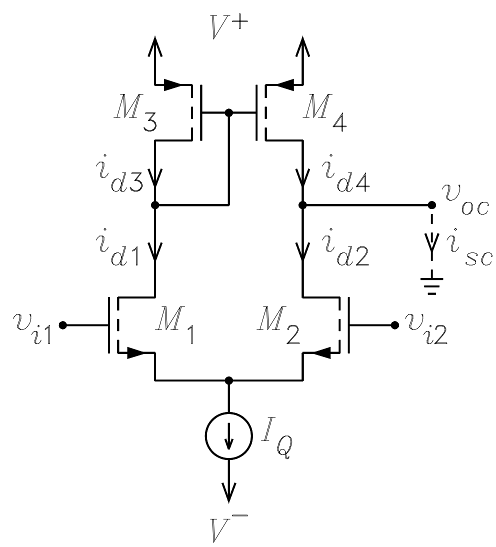

Figure 5: MOSFET diff amp with current-mirror load.

Fig. 5 shows a MOSFET differential amplifier with a current mirror load. If we assume that M_1 and M_2 are matched, the dc current I_Q divides equally between the devices so that $I_{D1} = I_{D2} = I_Q/2$. Thus we can write $g_{m1} = g_{m2} = g_m$. To solve for the open-circuit small-signal output voltage v_{oc}, we first solve for the short-circuit small-signal output current i_{sc}. First, we resolve the inputs into common-mode and differential components. Because a current source is used to bias the sources, the common-mode signal currents in the devices are zero. Therefore, we only need to consider the differential solution. To obtain this, replace v_{i1} with the voltage

$v_{id} = (v_{i1} - v_{i2})/2$ and v_{i2} with $-v_{id}$. For the differential inputs, the signal voltage at the sources is zero. Because both the drain and source of M_2 are connected to signal ground, the Early effect is absent in M_2. Similarly, it is absent for M_4. Although the drains of M_1 and M_3 are not at signal ground, it would be expected that the small-signal voltage across them is small because M_3 is connected as a diode. Thus it would be expected that the Early effect can be neglected for M_1 and M_3. In this case, the current i_{sc} can be written by inspection.

When the Early effect is neglected, i_{sc} can be written

$$i_{sc} = i_{d4} - i_{d2} = i_{d3} - i_{d2} = i_{d1} - i_{d2} = 2i_{d1}$$

The latter is because $i_{d2} = -i_{d1}$. To solve for i_{d1}, replace the circuits seen looking into the sources of M_1 and M_2 by small-signal Thévenin equivalent circuits. The circuit is shown in Fig. 3 with $R_S = 0$ and $R_Q = \infty$. It follows that i_{d1} and i_{sc} are given by

$$i_{d1} = \frac{v_{i1} - v_{i2}}{2r_s} = \frac{g_m}{2}(v_{i1} - v_{i2}) \qquad i_{sc} = 2i_{d1} = g_m(v_{i1} - v_{i2})$$

To solve for the small-signal output voltage v_{oc}, we must know the small-signal resistance seen looking into the output terminal. This is calculated with $v_{i1} = v_{i2} = 0$. The resistance seen r_{id4} looking into the drain of M_4 is r_{04}. To determine the resistance r_{id2} seen looking into the drain of M_2, the Thévenin resistance R_{ts2} seen looking out of its source is required. This is the Thévenin resistance r_{is1} seen looking into the source of M_1. If r_{01} is neglected, this is given by $r_{is1} = r_{s1} = g_m^{-1}$. Thus r_{id2} is given by

$$r_{id2} = r_{02}\left(1 + \frac{R_{ts2}}{r_{is1}}\right) + R_{ts2} = 2r_{02} + \frac{1}{g_{m2}}$$

It follows that r_{out} and the open-circuit output voltage v_{oc} are given by

$$r_{out} = r_{id3} \| r_{id2} = r_{03} \| \left(2r_{02} + \frac{1}{g_{m2}}\right)$$

$$v_{oc} = i_{sc} r_{out} = g_m \left[r_{03} \| \left(2r_{02} + \frac{1}{g_{m2}}\right)\right](v_{i1} - v_{i2})$$