The MOSFET

Device Symbols

Whereas the JFET has a diode junction between the gate and the channel, the metal-oxide semiconductor FET or MOSFET differs primarily in that it has an oxide insulating layer separating the gate and the channel. The circuit symbols are shown in Fig. 1. Each device has gate (G), drain (D), and source (S) terminals. Four of the symbols show an additional terminal called the body (B) which is not normally used as an input or an output. It connects to the drain-source channel through a diode junction. In discrete MOSFETs, the body lead is connected internally to the source. When this is the case, it is omitted on the symbol as shown in four of the MOSFET symbols. In integrated-circuit MOSFETs, the body usually connects to a dc power supply rail which reverse biases the body-channel junction. In the latter case, the so-called “body effect” must be accounted for when analyzing the circuit.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Depletion MOSFET</th>
<th>Enhancement MOSFET</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td><img src="image1" alt="Depletion MOSFET Symbol" /></td>
<td><img src="image2" alt="Enhancement MOSFET Symbol" /></td>
</tr>
<tr>
<td>P</td>
<td><img src="image1" alt="Depletion MOSFET Symbol" /></td>
<td><img src="image2" alt="Enhancement MOSFET Symbol" /></td>
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</tbody>
</table>

Figure 1: MOSFET symbols.

Device Equations

The discussion here applies to the n-channel MOSFET. The equations apply to the p-channel device if the subscripts for the voltage between any two of the device terminals are reversed, e.g. \( v_{GS} \) becomes \( v_{SG} \). The n-channel MOSFET is biased in the active mode or saturation region for \( v_{DS} \geq v_{GS} - v_{TH} \), where \( v_{TH} \) is the threshold voltage. This voltage is negative for the depletion-mode device and positive for the enhancement-mode device. It is a function of the body-source voltage and is given by

\[
v_{TH} = V_{TO} + \gamma \left[ \sqrt{\phi - v_{BS}} - \sqrt{\phi} \right] \quad (1)
\]

where \( V_{TO} \) is the value of \( v_{TH} \) with \( v_{BS} = 0 \), \( \gamma \) is the body threshold parameter, \( \phi \) is the surface potential, and \( v_{BS} \) is the body-source voltage. The drain current is given by

\[
i_D = \frac{k'}{2} \frac{W}{L} (1 + \lambda v_{DS}) (v_{GS} - v_{TH})^2 \quad (2)
\]

where \( W \) is the channel width, \( L \) is the channel length, \( \lambda \) is the channel-length modulation parameter, and \( k' \) is given by

\[
k' = \mu_0 C_{ox} = \frac{\mu}{t_{ox}}
\]
In this equation, $\mu_0$ is the average carrier mobility, $C_{ox}$ is the gate oxide capacitance per unity area, $\epsilon_{ox}$ is the permittivity of the oxide layer, and $t_{ox}$ is its thickness. It is convenient to define a transconductance coefficient $K$ given by

$$K = \frac{k'W}{2} (1 + \lambda v_{DS}) = K_0 (1 + \lambda v_{DS})$$  \hspace{1cm} (3)

where $K_0$ is given by

$$K_0 = \frac{k'W}{2}$$  \hspace{1cm} (4)

With these definitions, the drain current can be written

$$i_D = K (v_{GS} - v_{TH})^2$$  \hspace{1cm} (5)

Note that $K$ plays the same role in the MOSFET drain current equation as $\beta$ plays in the JFET drain current equation.

Some texts define $K = k' (W/L) (1 + \lambda v_{DS})$ so that $i_D$ is written $i_D = (K/2) (v_{GS} - v_{TH})^2$. In this case, the numerical value of $K$ is twice the value used here. To modify the equations given here to conform to this usage, replace $K$ in any equation given here with $K/2$.

### Transfer and Output Characteristics

The transfer characteristics are a plot of the drain current $i_D$ as a function of the gate-to-source voltage $v_{GS}$ with the drain-to-source voltage $v_{DS}$ held constant. Fig. 2 shows the typical transfer characteristics for a zero body-to-source voltage. In this case, the threshold voltage is a constant, i.e. $v_{TH} = V_{TO}$. For $v_{GS} \leq V_{TO}$, the drain current is zero. For $v_{GS} > V_{TO}$, Eq. (5) shows that the drain current increases as the square of the gate-to-source voltage. The slope of the curve represents the small-signal transconductance $g_m$, which is defined in the following.

![Figure 2: Drain current $i_D$ versus gate-to-source voltage $v_{GS}$ for constant drain-to-source voltage $v_{DS}$.

The output characteristics are a plot the drain current $i_D$ as a function of the drain-to-source voltage $v_{DS}$ with the gate-to-source voltage $v_{GS}$ and the body-to-source voltage $v_{BS}$ held constant. Fig. 3 shows the typical output characteristics for several values of gate-to-source voltage $v_{GS}$. The dashed line divides the triode region from the saturation or active region. In the saturation region, the slope of the curves represents the reciprocal of the small-signal drain-source resistance $r_0$, which is defined in the next section.

### Small-Signal Models

There are two small-signal circuit models which are commonly used to analyze MOSFET circuits. These are the hybrid-$\pi$ model and the T model. The two models are equivalent and give identical results. They are described below. In addition, a simplified small-model is derived which is called the source equivalent circuit. The models are first developed for the case of no body effect and then with the body effect. The former
Figure 3: Drain current $i_D$ versus drain-to-source voltage $v_{DS}$ for constant gate-to-source voltage $v_{GS}$.

case assumes that the body-source voltage is zero, i.e. $v_{BS} = 0$. This is the case with discrete MOSFETs in which the source is connected physically to the body. It also applies to small-signal ac analyses for which the body and source leads are connected to the same or different dc voltages. In this case, the small-signal body-source voltage is zero, i.e. $v_{bs} = 0$, and there is no body effect.

**No Body Effect**

The small-signal models in this section assume that the body lead is connected to the source lead. The models also apply when the body and source leads are connected to different dc voltages so that the ac or signal voltage from body to source is zero.

**Hybrid-$\pi$ Model**

Consider the case where the body-source voltage is zero, i.e. $v_{BS} = 0$. In this case, the threshold voltage in Eq. 1 is a constant and given by $v_{TH} = V_{TO}$. Let the drain current and each voltage be written as the sum of a dc component and a small-signal ac component as follows:

$$i_D = I_D + i_d$$

$$v_{GS} = V_{GS} + v_{gs}$$

$$v_{DS} = V_{DS} + v_{ds}$$

If the ac components are sufficiently small, we can write

$$i_d = \frac{\partial I_D}{\partial V_{GS}} v_{gs} + \frac{\partial I_D}{\partial V_{DS}} v_{ds}$$

where the derivatives are evaluated at the dc bias values. Let us define

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = K (V_{GS} - V_{TH}) = 2\sqrt{K I_D}$$

$$r_0 = \left[ \frac{\partial I_D}{\partial V_{DS}} \right]^{-1} = \left[ \frac{k' W}{2 L \lambda (V_{GS} - V_{TH})^2} \right]^{-1} = \frac{1}{\lambda} + \frac{V_{DS}}{I_D}$$

It follows that the small-signal drain current can be written

$$i_d = i'_d + \frac{v_{ds}}{r_0}$$

where

$$i'_d = g_m v_{gs}$$

The small-signal circuit which models these equations is given in Fig. 4. This is called the hybrid-$\pi$ model.
The T model of the MOSFET is shown in Fig. 5. The resistor $r_0$ is given by Eq. (11). The resistor $r_s$ is given by

$$r_s = \frac{1}{g_m}$$

(14)

where $g_m$ is the transconductance defined in Eq. (10). The currents are given by

$$i_d = i_d' + \frac{v_{ds}}{r_0}$$

(15)

$$i_d' = \frac{v_{gs}}{r_s} = g_m v_{gs}$$

(16)

The currents in the T model are the same as for the hybrid-π model. Therefore, the two models are equivalent. Note that the gate and body currents in Fig. 5 are zero because the controlled source supplies the current that flows through $r_s$.

The source equivalent circuit is shown in Fig. 7. Note that there is no $R_{tg}$ in the circuit because there is no current through $R_{tg}$ in the original circuit. Compared to the corresponding circuit for the BJT, the MOSFET circuit replaces $v_{th}$ with $v_{tg}$ and $r'_s$ with $r_s$. Because the gate current is zero, set $\alpha = 1$ and $\beta = \infty$ in converting any BJT formulas to corresponding MOSFET formulas.

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**T Model**

The T model of the MOSFET is shown in Fig. 5. The resistor $r_0$ is given by Eq. (11). The resistor $r_s$ is given by

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**A Source Equivalent Circuit**

Figure 6 shows the MOSFET T model described above with a Thévenin source in series with the gate. We wish to solve for the equivalent circuit in which the source $i_d'$ is replaced by a single source which connects from the drain node to ground having the value $i_d' = i_s'$. We call this the source equivalent circuit. Looking up into the branch labeled $i_s'$, we can write $v_s = v_{tg} - i_s' r_s$. With $v_{tg} = 0$, the resistance $r_s$ seen looking up into the branch labeled $i_s'$ is given by

$$r_s = \frac{1}{g_m}$$

(17)

The source equivalent circuit is shown in Fig. 7. Note that there is no $R_{tg}$ in the circuit because there is no current through $R_{tg}$ in the original circuit. Compared to the corresponding circuit for the BJT, the MOSFET circuit replaces $v_{th}$ with $v_{tg}$ and $r'_s$ with $r_s$. Because the gate current is zero, set $\alpha = 1$ and $\beta = \infty$ in converting any BJT formulas to corresponding MOSFET formulas.
With Body Effect

The small-signal models in this section assume that the body lead is connected to ac signal ground. In integrated circuit design, this ac signal ground is typically a dc power supply rail. In this case, any ac signal voltage on the source lead causes an ac signal voltage between the body and source. The effect of this voltage is called the body effect.

Hybrid-π Model

Let the drain current and each voltage be written as the sum of a dc component and a small-signal ac component as follows:

\[ i_D = I_D + i_d \]  \hspace{1cm} (18)
\[ v_{GS} = V_{GS} + v_{gs} \]  \hspace{1cm} (19)
\[ v_{BS} = V_{BS} + v_{bs} \]  \hspace{1cm} (20)
\[ v_{DS} = V_{DS} + v_{ds} \]  \hspace{1cm} (21)

If the ac components are sufficiently small, we can write

\[ i_d = \frac{\partial I_D}{\partial V_{GS}} v_{gs} + \frac{\partial I_D}{\partial V_{BS}} v_{bs} + \frac{\partial I_D}{\partial V_{DS}} v_{ds} \]  \hspace{1cm} (22)

where the derivatives are evaluated at the dc bias values. Let us define

\[ g_m = \frac{\partial I_D}{\partial V_{GS}} = K (V_{GS} - V_{TH}) = 2\sqrt{KI_D} \]  \hspace{1cm} (23)
\[ g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \frac{\gamma \sqrt{KI_D}}{\sqrt{\phi - V_{BS}}} = \chi g_m \]  \hspace{1cm} (24)
\[ \chi = \frac{\gamma}{2\sqrt{\phi - V_{BS}}} \]  \hspace{1cm} (25)
\[ r_0 = \left[ \frac{\partial I_D}{\partial V_{DS}} \right]^{-1} = \left[ \frac{k'}{2} \frac{W}{L} \lambda (V_{GS} - V_{TH}) \right]^{-1} = \frac{V_{DS} + 1/\lambda}{I_D} \] (26)

The small-signal drain current can thus be written
\[ i_d = i_{dg} + i_{db} + \frac{v_{ds}}{r_0} \] (27)

where
\[ i_{dg} = g_m v_{gs} \] (28)
\[ i_{db} = g_{mb} v_{bs} \] (29)

The small-signal circuit which models these equations is given in Fig. 8. This is called the hybrid-\( \pi \) model. If the body (B) lead is connected to the source, then \( v_{bs} = 0 \) and the circuit becomes that given in Fig. 4.

![Figure 8: Hybrid-\( \pi \) model of the MOSFET.](image)

**T Model**

The T model of the MOSFET is shown in Fig. 9. The resistor \( r_0 \) is given by Eq. (26). The resistors \( r_s \) and \( r_{sb} \) are given by
\[ r_s = \frac{1}{g_m} \] (30)
\[ r_{sb} = \frac{1}{g_{mb}} = \frac{1}{\lambda g_m} = \frac{r_s}{\lambda} \] (31)

where \( g_m \) and \( g_{mb} \) are the transconductances defined in Eqs. (23) and (24). The currents are given by
\[ i_d = i_{sg} + i_{sb} + \frac{v_{ds}}{r_0} \] (32)
\[ i_{sg} = \frac{v_{gs}}{r_s} = g_m v_{gs} \] (33)
\[ i_{sb} = \frac{v_{bs}}{r_{sb}} = g_{mb} v_{bs} \] (34)

The currents are the same as for the hybrid-\( \pi \) model. Therefore, the two models are equivalent. Note that the gate and body currents are zero because the two controlled sources supply the currents that flow through \( r_s \) and \( r_{sb} \).

**Source Equivalent Circuit**

Figure 10 shows the MOSFET T model with a Thévenin source in series with the gate and the body connected to signal ground. We wish to solve for the equivalent circuit in which the sources \( i_{sg} \) and \( i_{sb} \) are replaced by a single source which connects from the drain node to ground having the value \( i_0' = i_s' \). We call this
the source equivalent circuit. The first step is to look up into the branch labeled $i'_s$ and form a Thévenin equivalent circuit. With $i'_s = 0$, we can use voltage division to write

$$v_{s(oc)} = v_{tg} \frac{r_{sb}}{r_s + r_{sb}} = v_{tg} \frac{r_s/\chi}{r_s + r_s/\chi} = \frac{v_{tg}}{1 + \chi} \quad (35)$$

With $v_{tg} = 0$, the resistance $r'_s$ seen looking up into the branch labeled $i'_s$ is

$$r'_s = r_s || r_{sb} = \frac{r_s}{1 + \chi} \frac{1}{(1 + \chi) g_m} \quad (36)$$

The source equivalent circuit is shown in Fig.11. Compared to the corresponding circuit without the body effect, the circuit replaces $v_{tg}$ with $v_{tg} / (1 + \chi)$ and $r_s$ with $r'_s = r_s / (1 + \chi)$. To convert the source equivalent circuit with the body effect to one without the body effect, simply set $\chi = 0$.

**The $r_0$ Approximations**

**No Body Effect**

The $r_0$ approximations approximate $r_0$ as an open circuit except when calculating the resistance seen looking into the drain. Fig. 7 shows the source equivalent circuit for calculating $r_{id} = v_l / i_d$. The resistor $r_s$ is given by $r_s = 1/g_m$. We can write

$$i_d = i_0 + i_s = i_0 \left( 1 - \frac{R_{ls}}{r_s + R_{ts}} \right) = \frac{v_l}{r_0 + r_s || R_{ts}} \left( 1 - \frac{R_{ts}}{r_c + R_{ts}} \right) \quad (37)$$

Figure 9: T model of the MOSFET.

Figure 10: T model with Thévenin source connected to the gate and the body connected to signal ground.

The source equivalent circuit with the body effect to one without the body effect, simply set $\chi = 0$. 

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Figure 9: T model of the MOSFET.
It follows that $r_{id}$ is given by

$$r_{id} = \frac{v_t}{i_d} = \frac{r_0 + r_s R_{ts}}{1 - R_{ts}/(r_s + R_{ts})} = r_0 \left( 1 + \frac{R_{ts}}{r_s} \right) + R_{ts}$$

(38)

The $r_0$ approximations for the source equivalent circuit, the hybrid π model, and the T model, respectively, are given Figs. 13 through 15. Because $r_0$ no longer connects to the source, there is only one source current and $i_s = i_s'$. If $r_0 = \infty$, then $r_{ic}$ is an open circuit in each.

$$i_d = i_0 + i_s' = i_0 \left( 1 - \frac{R_{ts}}{r_s' + R_{ts}} \right) = \frac{v_t}{r_0 + r_s' R_{ts}} \left( 1 - \frac{R_{ts}}{r_s' + R_{ts}} \right)$$

(39)
It follows that $r_{id}$ is given by

$$
r_{id} = \frac{v_t}{i_d} = \frac{r_0 + r'_s R_{ts}}{1 - R_{ts} / (r'_s + R_{ts})} = r_0 \left(1 + \frac{R_{ts}}{r'_s}ight) + R_{ts}
$$

The $r_0$ approximations for the source equivalent circuit, the hybrid π model, and the T model, respectively, are given Figs. 17 through 19. Because $r_0$ no longer connects to the source, there is only one source current and $i_s = i'_s$. If $r_0 = \infty$, then $r_{ic}$ is an open circuit in each.

**Small-Signal High-Frequency Models**

Figures 20 and 21 show the hybrid-π and T models for the MOSFET with the gate-source capacitance $c_{gs}$, the source-body capacitance $c_{sb}$, the drain-body capacitance $c_{db}$, the drain-gate capacitance $c_{dg}$, and the gate-body capacitance $c_{gb}$ added. These capacitors model charge storage in the device which affects its high-frequency performance. The first three capacitors are given by

$$
c_{gs} = \frac{2}{3} W L C_{ox}
$$

$$
c_{sb} = \frac{c_{sb0}}{(1 + V_{SB}/\psi_0)^{1/2}}
$$
Figure 17: Source equivalent circuit with $r_0$ approximations.

Figure 18: Hybrid π model with the $r_0$ approximations.

Figure 19: T model with the $r_0$ approximations.
\[ c_{db} = \frac{c_{db0}}{(1 + V_{DB}/\psi_0)^{1/2}} \]  

where \( V_{SB} \) and \( V_{DB} \) are dc bias voltages; \( c_{sb0} \) and \( c_{db0} \) are zero-bias values; and \( \psi_0 \) is the built-in potential. Capacitors \( c_{gd} \) and \( c_{gb} \) model parasitic capacitances. For IC devices, \( c_{gd} \) is typically in the range of 1 to 10 fF for small devices and \( c_{gb} \) is in the range of 0.04 to 0.15 fF per square micron of interconnect.

Figure 20: High-frequency hybrid-\( \pi \) model.

Figure 21: High-frequency T model.