CE - CC Amplifier Example

For the circuits in the figure, it is given that $V^+ = 10V$, $V^- = -10V$, $R_s = 5\, \text{k}\Omega$, $R_1 = 100\, \text{k}\Omega$, $R_2 = 120\, \text{k}\Omega$, $R_{E1} = 2\, \text{k}\Omega$, $R_3 = 51\, \text{\Omega}$, $R_C = 2.4\, \text{\text{k}\Omega}$, $R_{E2} = 2\, \text{k}\Omega$, $R_L = 1\, \text{k}\Omega$, $V_{BE} = 0.65\, \text{\text{V}}$, $V_T = 0.025\, \text{\text{V}}$, $\alpha = 0.99$, $\beta = 99$, $r_x = 20\, \Omega$, and $r_0 = 50\, \text{k}\Omega$. The capacitors are ac short circuits and dc open circuits.

DC Solution

The dc solution for $Q_1$ is the same as for the CE amplifier and is repeated. To solve for $I_{E1}$, replace the capacitors with open circuits. Look out the base and form a Thévenin equivalent circuit. We have

$$V_{BB1} = \frac{V^+ R_2 + V^- R_1}{R_1 + R_2} = \frac{10 \times 120\, \text{k}\Omega}{100\, \text{k}\Omega + 120\, \text{k}\Omega} - \frac{10 \times 100\, \text{k}\Omega}{100\, \text{k}\Omega + 120\, \text{k}\Omega} = \frac{10}{11}\, \text{\text{V}}$$

$$R_{BB1} = R_1 || R_2 = 100\, \text{k}\Omega || 120\, \text{k}\Omega = 54.55\, \text{k}\Omega$$

$$V_{EE1} = V^- = -10\, \text{\text{V}}$$

$$R_{EE1} = R_E = 2\, \text{k}\Omega$$

The emitter current in $Q_1$ is given by

$$I_{E1} = \frac{V_{BB1} - V_{BE1} - V_{EE1}}{R_{BB1} / (1 + \beta) + R_{EE1}} = \frac{10/11 - 0.65 - (-10)}{54.55\, \text{k}\Omega / (1 + 99) + 2\, \text{k}\Omega} = 4.031\, \text{\text{mA}}$$

The ac emitter intrinsic resistance of $Q_1$ is

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25\, \text{mV}}{4.031\, \text{\text{mA}}} = 6.202\, \Omega$$

Look out of the base and emitter of $Q_2$ and form Thévenin equivalent circuits. We have

$$V_{BB2} = V^+ - \alpha I_{E1} R_C = 10 - 0.99 \times 4.031\, \text{\text{mA}} \times 2.4\, \text{\text{k}\Omega} = 0.4223\, \text{\text{V}}$$

$$R_{BB2} = 2.4\, \text{\text{k}\Omega}$$

$$V_{EE2} = V^- = -10\, \text{\text{V}}$$

$$R_{EE2} = R_{E2} = 2\, \text{k}\Omega$$

The emitter current in $Q_2$ is given by

$$I_{E2} = \frac{V_{BB2} - V_{BE2} - V_{EE2}}{R_{BB2} / (1 + \beta) + R_{EE2}} = \frac{0.4223 - 0.65 - (-10)}{2.4\, \text{k}\Omega / (1 + 99) + 2\, \text{k}\Omega} = 4.828\, \text{\text{mA}}$$
The short circuit collector output current from $Q_2$ is

$$r_{c2} = \frac{V_T}{I_{E2}} = \frac{25 \text{ mV}}{4.828 \text{ mA}} = 5.178 \Omega$$

AC Solution - Method 1

Zero the dc supplies and short the capacitors. Look out the base of $Q_1$ and make a Thévenin equivalent circuit. We have

$$v_{tb1} = v_s \frac{R_1 || R_2}{R_s + R_1 || R_2} = v_s \frac{100 \text{ k}\Omega || 120 \text{ k}\Omega}{5 \text{ k}\Omega + 100 \text{ k}\Omega || 120 \text{ k}\Omega} = \frac{v_s}{1.092} = 0.9160v_s$$

$$R_{tb1} = R_s || R_1 || R_2 = 5 \text{ k}\Omega || 100 \text{ k}\Omega || 120 \text{ k}\Omega = 4.580 \text{ k}\Omega$$

The Thévenin equivalent circuit looking into the $v_e'$ branch is $v_{tb1}$ in series with $r_{e1}'$, where

$$r_{e1}' = \frac{R_{tb1} + r_x1}{1 + \beta_1} + r_{e1} = \frac{4.580 \text{ k}\Omega + 20}{1 + 99} + 6.202 = 52.20 \Omega$$

The resistance looking out of the emitter of $Q_1$ is

$$R_{te1} = R_E || R_3 = 2 \text{ k}\Omega || 51 = 49.73 \Omega$$

The resistance looking into the collector of $Q_1$ is

$$r_{ic1} = \frac{r_{01} + r_{e1}'}{1 - \alpha_1 R_{te1} / (r_{e1}' + R_{te1})} = \frac{50 \text{ k}\Omega + 52.20 || 49.73}{1 - 0.99 \times 49.73 / (49.73 + 52.20)} = 97.76 \text{ k}\Omega$$

The short circuit collector output current from $Q_1$ is

$$i_{c1(sc)} = G_{mb1} v_{tb1} = \frac{\alpha_1}{r_{e1}' + R_{te1}} \frac{r_{01} - R_{te1} / \beta_1}{r_{e1}' + R_{te1}} v_{tb1}$$

$$= \frac{0.99}{52.20 + 49.73} \frac{50 \text{ k}\Omega - 49.73/99}{52.20 + 49.73} \frac{v_{tb1}}{v_{tb1}} = \frac{v_{tb1}}{103.0} = \frac{v_s}{112.4}$$

Look out of the base of $Q_2$ and make a Thévenin equivalent circuit. We have

$$v_{tb2} = -i_{c2(sc)} R_C || r_{ic1} = -\frac{v_s}{112.4} \times 2.4 \text{ k}\Omega || 97.76 \text{ k}\Omega = -20.84v_s$$

$$R_{tb2} = R_C || r_{ic1} = 2.4 \text{ k}\Omega || 97.76 \text{ k}\Omega = 2.342 \text{ k}\Omega$$

Replace $Q_2$ with its simplified T model. Looking into the $r_{e2}'$ branch, we see $v_{tb2}$ in series with $r_{e2}'$ given by

$$r_{e2}' = \frac{R_{tb2} + r_x2}{1 + \beta_2} + r_{e2} = \frac{2.342 \text{ k}\Omega + 20}{1 + 99} + 5.178 = 28.80$$

The resistance seen looking out of the emitter of $Q_2$ is

$$R_{te2} = R_{E2} || R_L = 666.7 \Omega$$

By voltage division, $v_o$ is given by

$$v_o = v_{tb2} \frac{r_{02} || R_{te2}}{r_{e2}' + r_{02} || R_{te2}} = -20.84v_s \frac{50 \text{ k}\Omega || 666.7}{28.80 + 50 \text{ k}\Omega || 666.7} = -19.97v_s$$

Thus the voltage gain is

$$\frac{v_o}{v_s} = -19.97$$
The output resistance is
\[ r_{out} = R_{E2} || r_{e2}' = 2 \, k\Omega || 50 \, k\Omega \approx 28.80 = 28.38 \, \Omega \]

To solve for the input resistance, we need \( r_{ib1} \). To calculate this, we need \( R_{tc1} \), which requires us to know \( r_{ib2} \). For the latter, we have
\[
\begin{align*}
  r_{ib2} &= r_{x2} + (1 + \beta_2) r_{e2} + R_{te2} \frac{(1 + \beta_2) r_{02} + R_{tc2}}{r_{02} + R_{te2} + R_{tc2}} \\
  &= 20 + (1 + 99) 5.178 + 666.7 \frac{(1 + 99) 50 \, k\Omega}{50 \, k\Omega + 666.7} \\
  &= 66.33 \, k\Omega
\end{align*}
\]

Thus the resistance seen looking out of the collector of \( Q_1 \) is
\[ R_{tc1} = R_C || r_{ib2} = 2.4 \, k\Omega || 66.33 \, k\Omega = 2.316 \, k\Omega \]

The resistance looking into the base of \( Q_1 \) is
\[
\begin{align*}
  r_{ib1} &= r_{x1} + (1 + \beta_1) r_{e1} + R_{te1} \frac{(1 + \beta_1) r_{01} + R_{tc1}}{r_{01} + R_{te1} + R_{tc1}} \\
  &= 5.391 \, k\Omega
\end{align*}
\]

The input resistance is
\[ r_{in} = R_1 || R_2 || r_{ib1} = 100 \, k\Omega || 120 \, k\Omega || 5.613 \, k\Omega = 5.089 \, k\Omega \]

If \( Q_2 \) and \( R_{E2} \) are omitted from the circuit and the left node of \( C_2 \) is connected to the collector of \( Q_1 \), we have a common-emitter amplifier. In this case, the output voltage is
\[
\begin{align*}
  v_o &= -i_{c1(s)c} R_C || r_{ic1} || R_L = -\frac{v_s}{112.4} \times 2.4 \, k\Omega || 97.76 \, k\Omega || 1 \, k\Omega = -6.235 v_o
\end{align*}
\]

Thus the voltage gain drops to
\[ \frac{v_o}{v_s} = -6.235 \]

This is lower than with the CC stage by a factor of 3.25 or by 10.2 dB. This illustrates how a stage that has a gain less than unity can increase the gain of a circuit when it is used to drive the load resistor.

**AC Solution - Method 2**

For this solution, we use the \( r_0 \) approximations for \( Q_1 \). That is, we neglect the current through \( r_{01} \) in calculating \( i_{c1(s)c} \) but not in calculating \( r_{ic1} \). The short circuit collector output current of \( Q_1 \) is
\[
\begin{align*}
  i_{c1(s)c} &= G_{m1} v_{ib1} = \frac{\alpha}{r_{e1}' + R_{te1}} v_{ib1} = \frac{0.99 v_{ib1}}{52.20 + 49.73} = \frac{v_{ib1}}{103.0} = \frac{v_{s1}}{111.3}
\end{align*}
\]

Look out of the base of \( Q_2 \) and make a Thévenin equivalent circuit. We have
\[
\begin{align*}
  v_{ib2} &= -i_{c1(s)c} R_C || r_{ic1} = -\frac{v_s}{111.3} \times 2.4 \, k\Omega || 97.76 \, k\Omega = -21.05 v_s \\
  R_{ib2} &= R_C || r_{ic1} = 2.4 \, k\Omega || 97.76 \, k\Omega = 2.342 \, k\Omega
\end{align*}
\]

Replace \( Q_2 \) with its simplified T model. Looking into the \( r_{e2}' \) branch, we see \( v_{ib2} \) in series with \( r_{e2}' \) given by
\[
\begin{align*}
  r_{e2}' &= \frac{R_{ib2} + r_{x2}}{1 + \beta_2} + r_{e2} = \frac{2.342 \, k\Omega + 20}{1 + 99} + 5.178 = 28.80
\end{align*}
\]
By voltage division, \( v_o \) is given by

\[
v_o = v_{ib2} \frac{r_{02} R_{te2}}{r_{e2} + r_{02} R_{te2}} = -21.05v_s \frac{50 \text{k}\Omega \| 666.7}{28.80 + 50 \text{k}\Omega \| 666.7} = -20.17v_s
\]

Thus the voltage gain is

\[
\frac{v_o}{v_s} = -20.17
\]

This differs from the answer by Method 1 by 0.99%.

The solutions for \( r_{out} \) and \( r_{in} \) are the same as for Method 1.