The Common-Emitter Amplifier

The common-emitter amplifier is used to obtain a high voltage gain and a high input resistance. The circuit in Fig. 1(a) shows the ac signal circuit. The input source is represented by a Thévenin equivalent connected to the base. The output is taken from the collector. We assume that the dc bias solution is known and that the BJT is biased in its active mode. The small-signal parameters \( r_e, r'_e, \) and \( r_0 \) are given by

\[
\begin{align*}
 r_e &= \frac{V_T}{I_E} \\
 r'_e &= \frac{R_{tb} + r_x}{1 + \beta} + r_e \\
 r_0 &= \frac{V_A + V_{CE}}{I_C}
\end{align*}
\]

![Image of the common-emitter amplifier circuit](image)

Figure 1: (a) Ac signal circuit of the common-emitter amplifier. (b) Equivalent input and output circuits.

The circuit in Fig. 1(b) shows the equivalent input and output circuits. The collector output voltage is given by

\[
v_c = -i_{c(se)} (r_{ic}||R_{tc}) = -G_{mb}v_{tb} (r_{ic}||R_{tc})
\]

It follows that the voltage gain from \( v_{tb} \) to \( v_c \) is given by

\[
A_v = \frac{v_c}{v_{tb}} = -G_{mb} (r_{ic}||R_{tc})
\]

where

\[
G_{mb} = \frac{\alpha}{r'_e + R_{te}} \frac{r_0 - R_{te}/\beta}{r_0 + R_{te}}
\]

\[
r_{ic} = \frac{r_0 + r'_e||R_{te}}{1 - \alpha R_{te}/(r'_e + R_{te})}
\]

Note that the voltage gain is negative. This means that the CE amplifier is an inverting amplifier.

The output resistance seen looking into the \( v_c \) node is

\[
r_{out} = r_{ic}||R_{tc}
\]
The input resistance seen looking into the $v_b$ node is

$$r_{ib} = r_x + (1 + \beta) r_e + R_{te} \frac{(1 + \beta) r_0 + R_{tc}}{r_0 + R_{te} + R_{tc}}$$

When the $r_0$ approximations are used, $G_{mb}$ and $r_{ib}$ are replaced with

$$G_{mb} \simeq \frac{\alpha}{r'_e + R_{te}}$$

$$r_{ib} \simeq r_x + (1 + \beta) (r_e + R_{te})$$

**Example 1** Fig. 2 shows the circuit diagrams of NPN and PNP single-stage CE amplifiers. For each circuit, it is given that $R_S = 5 \, \text{k}\Omega$, $R_1 = 120 \, \text{k}\Omega$, $R_2 = 100 \, \text{k}\Omega$, $R_C = 4.3 \, \text{k}\Omega$, $R_E = 5.6 \, \text{k}\Omega$, $R_3 = 100 \, \Omega$, $R_L = 20 \, \text{k}\Omega$, $V^+ = 15 \, \text{V}$, $V^- = -15 \, \text{V}$, $V_{BE} = 0.65 \, \text{V}$, $\beta = 99$, $\alpha = 0.99$, $r_x = 20 \, \Omega$, $V_A = 100 \, \text{V}$ and $V_T = 0.025 \, \text{V}$. Solve for the gain $A_v = v_o/v_s$, the input resistance $r_{in}$, and the output resistance $r_{out}$. The capacitors can be assumed to be ac short circuits at the operating frequency.

![Figure 2: Single-stage CE amplifiers.](image)

**Solution.** For the dc bias solution, replace all capacitors with open circuits. For the NPN circuit, the Thévenin voltage and resistance seen looking out of the base are

$$V_{BB} = \frac{V^+ R_2 + V^- R_1}{R_1 + R_2} = -1.364 \, \text{V} \quad R_{BB} = R_1 || R_2 = 54.55 \, \text{k}\Omega$$
The Thévenin voltage and resistance seen looking out of the emitter are $V_{EE} = V^-$ and $R_{EE} = R_E$. The bias equation for $I_E$ is

$$I_E = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/(1 + \beta) + R_{EE}} = 2.113 \text{ mA}$$

To test for the active mode, we calculate the collector-base voltage

$$V_{CB} = V_C - V_B = (V^+ - \alpha I_E R_C) - \left( V_{BB} - \frac{I_E}{1 + \beta} R_{BB} \right) = 8.521 \text{ V}$$

Because this is positive, the BJT is biased in its active mode.

For the small-signal ac analysis, we need $r_0$ and $r_e$. To calculate $r_0$, we first calculate the collector-emitter voltage

$$V_{CE} = V_{CB} + V_{BE} = 9.171 \text{ V}$$

It follows that $r_0$ and $r_e$ have the values

$$r_0 = \frac{V_A + V_{CE}}{\alpha I_E} = 52.18 \text{ k}\Omega \quad r_e = \frac{V_T}{I_E} = 11.83 \text{ \Omega}$$

For the small-signal analysis, $V^+$ and $V^-$ are zeroed and the three capacitors are replaced with ac short circuits. The Thévenin voltage and resistance seen looking out of the base are given by

$$v_{tb} = v_s \frac{R_1 \| R_2}{R_S + R_1 \| R_2} = 0.916 v_s \quad R_{tb} = R_S \| R_1 \| R_2 = 4.58 \text{ k}\Omega$$

The Thévenin resistances seen looking out of the emitter and the collector are

$$R_{te} = R_E \| R_3 = 98.25 \text{ \Omega} \quad R_{tc} = R_C R_L = 3.539 \text{ k}\Omega$$

Next, we calculate $r'_e$, $G_{mb}$, $r_{ic}$, and $r_{ib}$.

$$r'_e = \frac{R_{tb} + r_x}{1 + \beta} + r_e = 57.83 \text{ \Omega}$$

$$G_{mb} = \frac{\alpha}{r'_e + R_{te} r_0} = \frac{r_0 - R_{te}/\beta}{r_0 + R_{te}} = \frac{1}{157.8} \text{ S}$$

$$r_{ic} = \frac{r_0 + r'_e R_{te}}{1 - \alpha R_{te}/(r'_e + R_{te})} = 138.6 \text{ k}\Omega$$

$$r_{ib} = r_x + (1 + \beta) r_e + R_{te} \frac{(1 + \beta) r_0 + R_{te}}{r_0 + R_{te} + R_{tc}} = 10.39 \text{ k}\Omega$$

The output voltage is given by

$$v_o = -G_{mb} (r_{ic} \| R_{tc}) v_{tb} = -G_{mb} (r_{ic} \| R_{tc}) \times 0.916 v_s = -20.04 v_s$$

Thus the voltage gain is $A_v = -20.04$. The input and output resistances are given by

$$r_{in} = R_1 \| R_2 \| r_{ib} = 8.73 \text{ k}\Omega \quad r_{out} = R_{tc} = 3.539 \text{ k}\Omega$$

The solutions for the PNP amplifier are the same as for the NPN circuit.
Example 2 If the $r_0$ approximations are used, calculate the new voltage gain and input resistance for the CE amplifiers.

**Solution.** Only $G_{mb}$ and $r_{ib}$ change. The approximate values are given by

$$G_{mb} \simeq G_m = \frac{\alpha}{r_e + R_{te}} = 157.7 \text{S} \quad r_{ib} \simeq r_x + (1 + \beta)(r_e + R_{te}) = 11.03 \text{k}\Omega$$

The new voltage gain and input resistance are given by

$$A_v = -G_m (r_{ic}||R_{te}) \times 0.916 = -20.05 \quad r_{in} = R_1||R_2||r_{ib} = 9.173 \text{k}\Omega$$

Note that there is very little change in the value of $A_v$.

The Common-Base Amplifier

The common-base amplifier is used to obtain a high voltage gain and a low input resistance. The circuit in Fig. 3(a) shows the ac signal circuit. The input source is represented by a Thévenin equivalent connected to the emitter. The output is taken from the collector. We assume that the dc bias solution is known and that the BJT is biased in its active mode. The small-signal parameters $r_e$, $r'_e$, and $r_0$ are given by

$$r_e = \frac{V_T}{I_E} \quad r'_e = \frac{R_{tb} + r_x}{1 + \beta} + r_e \quad r_0 = \frac{V_A + V_{CE}}{I_C}$$

![Circuit Diagram](image)

Figure 3: (a) Ac signal circuit of the common-base amplifier. (b) Equivalent input and output circuits.

The circuit in Fig. 1(b) shows the equivalent input and output circuits. The collector output voltage is given by

$$v_c = -i_{c(sc)} (r_{ic}||R_{te}) = G_{me}v_{te} (r_{ic}||R_{te})$$

It follows that the voltage gain is given by

$$A_v = \frac{v_c}{v_{te}} = G_{me} (r_{ic}||R_{te})$$
where
\[ G_{me} = \frac{1}{R_{te} + r_e'} \frac{\alpha r_0 + r_e'}{r_0 + r_e'} \]
\[ r_{ic} = \frac{r_0 + r_e'}{1 - \alpha R_{te} / (r_e' + R_{te})} \]

Note that the voltage gain is positive. This means that the CB amplifier is a non-inverting amplifier.

The output resistance seen looking into the \( v_c \) node is
\[ r_{out} = r_{ic} R_{te} \]

The input resistance seen looking into the \( v_e \) node is
\[ r_{ie} = \frac{r_e' r_0 + R_{te}}{r_e' + r_0 + R_{te} / (1 + \beta)} \]

When the \( r_0 \) approximations are used, \( G_{me} \) and \( r_{ie} \) are replaced with
\[ G_{me} \approx G_m = \frac{\alpha}{r_e' + R_{te}} \]
\[ r_{ie} \approx r_e' \]

**Example 3** Fig. 4 shows the circuit diagrams of NPN and PNP single-stage CB amplifiers. For each circuit, it is given that \( R_S = 100 \Omega \), \( R_1 = 120 \text{k}\Omega \), \( R_2 = 100 \text{k}\Omega \), \( R_C = 4.3 \text{k}\Omega \), \( R_E = 5.6 \text{k}\Omega \), \( R_3 = 100 \Omega \), \( R_L = 20 \text{k}\Omega \), \( V^+ = 15 \text{V} \), \( V^- = -15 \text{V} \), \( V_{BE} = 0.65 \text{V} \), \( \beta = 99 \), \( \alpha = 0.99 \), \( r_x = 20 \Omega \), \( V_A = 100 \text{V} \) and \( V_T = 0.025 \text{V} \). Solve for the gain \( A_v = v_o / v_s \), the input resistance \( r_{in} \), and the output resistance \( r_{out} \). The capacitors can be assumed to be ac short circuits at the operating frequency.

**Solution.** Because the dc bias circuits are the same as for the common-emitter amplifiers, the bias currents and voltages are the same. In addition, \( r_e \) and \( r_0 \) are the same.

For the small-signal analysis, \( V^+ \) and \( V^- \) are zeroed and the three capacitors are replaced with ac short circuits. The Thévenin voltage and resistance seen looking out of the emitter are given by
\[ v_{te} = v_s \frac{R_E}{R_S + R_E} = 0.9825v_s \quad R_{te} = R_S || R_E = 98.25 \text{\Omega} \]

The Thévenin resistances seen looking out of the base and the collector are
\[ R_{tb} = 0 \quad R_{tc} = R_C R_L = 3.539 \text{\text{k}\Omega} \]

Next, we calculate \( r_e' \), \( G_{me} \), \( r_{ic} \), and \( r_{ie} \).
\[ r_e' = \frac{R_{tb} + r_x}{1 + \beta} + r_e = 12.03 \Omega \]
\[ G_{me} = \frac{1}{R_{te} + r_e'} \frac{\alpha r_0 + r_e'}{r_0 + r_e'} = \frac{1}{111.4} \text{\text{S}} \]
Figure 4: Single-stage common-base amplifiers.
\[ r_{ic} = \frac{r_{0} + r'_{e}||R_{te}}{1 - \alpha R_{te}/(r'_{e} + R_{te})} = 442.3 \, \text{k}\Omega \]

\[ r_{ie} = \frac{r'_{e}}{r'_{e} + r_{0} + R_{te}/(1 + \beta)} = 12.83 \, \Omega \]

The output voltage is given by

\[ v_{o} = G_{me} (r_{ic||R_{tc}} v_{tc} = G_{me} (r_{ic||R_{tc}}) \times 0.9825 v_{s} = 30.97 v_{s} \]

Thus the voltage gain is \( A_{v} = 30.97 \). The input and output resistances are given by

\[ r_{in} = R_{1||R_{2}} r_{ib} = 12.81 \, \Omega \quad r_{out} = R_{tc} = 4.259 \, \text{k}\Omega \]

The solutions for the PNP amplifier are the same as for the NPN circuit.

**Example 4** If the \( r_{0} \) approximations are used, calculate the new voltage gain and input resistance for the CB amplifiers.

**Solution.** Only \( G_{me} \) and \( r_{ie} \) change. The approximate values are given by

\[ G_{me} \simeq G_{m} = \frac{\alpha}{r'_{e} + R_{te}} = \frac{1}{111.4} \, \text{S} \quad r_{ie} \simeq r'_{e} = 12.03 \, \text{k}\Omega \]

The new voltage gain and input resistance are given by

\[ A_{v} = G_{m} (r_{ic||R_{tc}}) \times 0.9825 = 30.97 \quad r_{in} = R_{E||r_{ie}} = 12 \, \Omega \]

To 4 significant places, there is no change in the value of \( A_{v} \).

**The Common-Collector Amplifier**

The common-collector amplifier is used to obtain a voltage gain that is approximately unity and a high input resistance. The circuit in Fig. 5(a) shows the ac signal circuit. The input source is represented by a Thévenin equivalent connected to the base. The output is taken from the emitter. We assume that the dc bias solution is known and that the BJT is biased in its active mode. The small-signal parameters \( r_{e}, r'_{e}, \) and \( r_{0} \) are given by

\[ r_{e} = \frac{V_{T}}{I_{E}} \quad r'_{e} = \frac{R_{tb} + r_{x}}{1 + \beta} + r_{e} \quad r_{0} = \frac{V_{A} + V_{CE}}{I_{C}} \]

The circuit in Fig. 1(b) shows the equivalent input and output circuits. The emitter output voltage is given by

\[ v_{e} = v_{e(oc)} \frac{R_{te}}{r_{ie} + R_{te}} \]

where

\[ v_{e(oc)} = v_{tb} \frac{r_{0} + R_{tc}/(1 + \beta)}{r'_{e} + r_{0} + R_{tc}/(1 + \beta)} \]

It follows that the voltage gain from \( v_{tb} \) to \( v_{e} \) is given by

\[ A_{v} = \frac{v_{e}}{v_{tb}} = \frac{R_{te}}{r_{ie} + R_{te}} \frac{r_{0} + R_{tc}/(1 + \beta)}{r'_{e} + r_{0} + R_{tc}/(1 + \beta)} \]
where \[ r_{ie} = \frac{r_e' r_0 + R_{te}}{r_e' + r_0 + R_{te} (1 + \beta)} \]

Note that the voltage gain is positive. This means that the CC amplifier is a non-inverting amplifier.

The output resistance seen looking into the \( v_e \) node is

\[ r_{out} = r_{ie} || R_{te} \]

The input resistance seen looking into the \( v_b \) node is

\[ r_{ib} = r_x + (1 + \beta) r_e + \frac{(1 + \beta) r_0 + R_{te}}{r_0 + R_{te} + R_{te}} \]

When the \( r_0 \) approximations are used, \( v_{e(oc)}, r_{ie}, \) and \( r_{ib} \) are replaced with

\[ v_{e(oc)} \approx v_{ib} \]
\[ r_{ie} \approx r_e' \]
\[ r_{ib} \approx r_x + (1 + \beta) (r_e + R_{te}) \]

**Example 5** Fig. 6 shows the circuit diagrams of NPN and PNP single-stage CC amplifiers. For each circuit, it is given that \( R_S = 5 \, \text{k}\Omega, R_1 = 120 \, \text{k}\Omega, R_2 = 100 \, \text{k}\Omega, R_E = 5.6 \, \text{k}\Omega, R_3 = 100 \, \Omega, R_L = 20 \, \text{k}\Omega, V^+ = 15 \, \text{V}, V^- = -15 \, \text{V}, V_{BE} = 0.65 \, \text{V}, \beta = 99, \alpha = 0.99, r_x = 20 \, \Omega, V_A = 100 \, \text{V} \) and \( V_T = 0.025 \, \text{V} \). Solve for the gain \( A_v = v_o/v_s \), the input resistance \( r_{in} \), and the output resistance \( r_{out} \). The capacitors can be assumed to be ac short circuits at the operating frequency.

**Solution.** Because the dc bias circuits are the same as for the common-emitter amplifiers, the bias currents and voltages are the same. In addition, \( r_e \) is the same. Because \( V_{CE} \) is different, a new value of \( r_0 \) must be calculated. The collector-to-emitter voltage is given by

\[ V_{CE} = V_C - V_E = V^+ - \left( V_{BB} - \frac{I_E}{1 + \beta} R_{BB} - V_{BE} \right) = 17.01 \, \text{V} \]
Thus $r_0$ has the value

$$r_0 = \frac{V_A + V_{CE}}{\alpha I_E} = 55.93 \text{ k}\Omega$$

For the small-signal analysis, $V^+$ and $V^-$ are zeroed and the three capacitors are replaced with ac short circuits. The Thévenin voltage and resistance seen looking out of the base are given by

$$v_{tb} = v_s \frac{R_1 \parallel R_2}{R_S + R_1 \parallel R_2} = 0.916 v_s \quad R_{tb} = R_S \parallel R_1 \parallel R_2 = 4.58 \text{ k}\Omega$$

The Thévenin resistances seen looking out of the emitter and the collector are

$$R_{te} = R_E \parallel R_3 = 4.375 \text{ k}\Omega \quad R_{tc} = 0$$

Next, we calculate $r'_e$, $v_{e(oc)}$, $r_{ie}$, and $r_{ib}$.

$$r'_e = \frac{R_{tb} + r_x}{1 + \beta} + r_e = 57.83 \text{ } \Omega$$

$$v_{e(oc)} = v_{tb} \frac{r_0 + R_{te}/(1 + \beta)}{r'_e + r_0 + R_{te}/(1 + \beta)} = 0.999 v_{tb}$$

$$r_{ie} = \frac{r'_e \frac{r_0 + R_{tc}}{r'_e + r_0 + R_{tc}/(1 + \beta)}}{r_0 + R_{te} + R_{tc}} = 57.77 \text{ } \Omega$$

$$r_{ib} = r_x + (1 + \beta) r_e + R_{te} \frac{(1 + \beta) r_0 + R_{tc}}{r_0 + R_{te} + R_{tc}} = 407 \text{ k}\Omega$$
The output voltage is given by
\[ v_o = v_{e(oc)} \frac{R_{te}}{r_{ie} + R_{te}} = \frac{R_{te}}{r_{ie} + R_{te}} \times 0.999 \times 0.916v_s = 0.903v_s \]

Thus the voltage gain is \( A_v = 0.903 \). The input and output resistances are given by
\[ r_{in} = R_1||R_2||r_{ib} = 48.1 \, k\Omega \quad r_{out} = r_{ie}||R_E||R_L = 57.02 \, \Omega \]

The solutions for the PNP amplifier are the same as for the NPN circuit.

**Example 6** If the \( r_0 \) approximations are used, calculate the new voltage gain and input resistance for the CB amplifiers.

**Solution.** In this case, \( v_{e(oc)} \), \( r_{ie} \), and \( r_{ib} \) change. The approximate values are given by
\[ v_{e(oc)} \simeq v tb = 0.916v_s \quad r_{ie} \simeq r'_e = 57.83 \, \Omega \]
\[ r_{ib} \simeq r_x + (1 + \beta)(r_e + R_{te}) = 438.7 \, k\Omega \]

The new voltage gain, output resistance, and input resistance are given by
\[ A_v = \frac{R_{te}}{r_{ie} + R_{te}} \times 0.916 = 0.904 \quad r_{out} = r_{ie}||R_E||R_L = 57.08 \, \Omega \]
\[ r_{in} = R_E||r_{ie} = 48.51 \, k\Omega \]

These answers are close to those for the exact solution.