**Hybrid-π Model with Body Effect**

Let the drain current and each voltage be written as the sum of a dc component and a small-signal ac component as follows:

\[ i_D = I_D + i_d \]  
\[ v_{GS} = V_{GS} + v_{gs} \]  
\[ v_{BS} = V_{BS} + v_{bs} \]  
\[ v_{DS} = V_{DS} + v_{ds} \]  

(1) (2) (3) (4)

If the ac components are sufficiently small, we can write

\[ i_d = \frac{\partial I_D}{\partial V_{GS}} v_{gs} + \frac{\partial I_D}{\partial V_{BS}} v_{bs} + \frac{\partial I_D}{\partial V_{DS}} v_{ds} \]  

(5)

where the derivatives are evaluated at the dc bias values. Let us define

\[ g_m = \frac{\partial I_D}{\partial V_{GS}} = K (V_{GS} - V_{TH}) = 2\sqrt{KI_D} \]  

(6)

\[ g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \frac{\gamma \sqrt{KI_D}}{\sqrt{\phi - V_{BS}}} = \chi g_m \]  

(7)

\[ \chi = \frac{\gamma}{2\sqrt{\phi - V_{BS}}} \]  

(8)

\[ r_0 = \left[ \frac{\partial I_D}{\partial V_{DS}} \right]^{-1} = \left[ \frac{k'}{2} \frac{W}{L} \lambda (V_{GS} - V_{TH})^2 \right]^{-1} = \frac{V_{DS} + 1/\lambda}{I_D} \]  

(9)

The small-signal drain current can thus be written

\[ i_d = i_{dg} + i_{db} + \frac{v_{ds}}{r_0} \]  

(10)

where

\[ i_{dg} = g_m v_{gs} \]  

(11)

\[ i_{db} = g_{mb} v_{bs} \]  

(12)

The small-signal circuit which models these equations is given in Fig. 1. This is called the hybrid-π model.

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**Figure 1: Hybrid-π model of the MOSFET.**