Norton Collector Circuit

The Norton equivalent circuit seen looking into the collector can be used to solve for the response of the common-emitter and common-base stages. Fig. 1(a) shows the bjt with Thévenin sources connected to its base and emitter. With the collector grounded, the collector current is called the short-circuit output current or $i_{c(sc)}$. The current source in the Norton collector circuit has this value. To solve for this current, we use the simplified T model in Fig. 1(b). The current $i_{c(sc)}$ can be solved for by superposition of $v_{tb}$ and $v_{te}$.

![Diagram](image)

Figure 1: (a) BJT with Thevenin sources connected to the base and the emitter. (b) Simplified T model.

With $v_{te}$ and $\alpha v'_e$ sources set to zero, it follows from Fig. 1(b) that

$$i_{c(sc)} = -\frac{v_{tb}}{r'_{e} + R_{te}||r_{0}} \frac{R_{te}}{R_{te} + r_{0}} \quad i'_e = \frac{v_{tb}}{r'_{e} + R_{te}||r_{0}}$$

With the $v_{tb}$ and $\alpha v'_e$ sources set to zero, we have

$$i_{c(sc)} = -\frac{v_{te}}{R_{te} + r'_e||r_{0}} \frac{r'_e}{r_{0} + r'_e} \quad i'_e = -\frac{v_{te}}{R_{te} + r'_e||r_{0}} \frac{r_{0}}{r_{0} + r'_e}$$

With the $v_{tb}$ and $v_{te}$ sources set to zero, we have

$$i_{c(sc)} = \alpha i'_e \quad i'_e = 0$$

By the principle of superposition, these equations can be combined to obtain the total solution given by

$$i_{c(sc)} = -\frac{v_{tb}}{r'_{e} + R_{te}||r_{0}} \frac{R_{te}}{R_{te} + r_{0}} - \frac{v_{te}}{R_{te} + r'_e||r_{0}} \frac{r'_e}{r_{0} + r'_e} + \alpha i'_e$$

$$= \frac{v_{tb}}{r'_{e} + R_{te}||r_{0}} \left( \alpha - \frac{R_{te}}{R_{te} + r_{0}} \right) - \frac{v_{te}}{R_{te} + r'_e||r_{0}} \frac{\alpha r_{0} + r'_e}{r_{0} + r'_e}$$

This equation is of the form

$$i_{c(sc)} = G_{mb}v_{tb} - G_{me}v_{te}$$
where
\[ G_{mb} = \frac{1}{r_e' + R_{te}} \left( \alpha - \frac{R_{te}}{r_0 + R_{te}} \right) = \frac{\alpha}{r_e' + R_{te}} \frac{r_0 - R_{te}/\beta}{r_0 + R_{te}} \] (6)
\[ G_{me} = \frac{1}{R_{te} + r_e'} \left( \frac{\alpha r_0 + r_e'}{r_0 + r_e'} \right) = \frac{\alpha}{R_{te} + r_e'} \frac{r_0 + r_e'/\alpha}{r_0 + r_e'} \] (7)

Figure 2(a) shows the simplified T model with \( v_{tb} = v_{te} = 0 \) and a test source connected to the collector. The resistance seen looking into the collector is given by \( r_{ic} = v_t/i_c \). The resistor in the collector Norton equivalent circuit has this value. To solve for \( r_{ic} \), we can write
\[ i_c = \alpha i_e' + i_0 = -\alpha i_0 \frac{R_{te}}{r_e' + R_{te}} + i_0 = \frac{v_t}{r_0 + r_e'} \left( 1 - \frac{\alpha R_{te}}{r_e' + R_{te}} \right) \] (8)
where current division has been used to express \( i_e' \) as a function of \( i_0 \). It follows that \( r_{ic} \) is given by
\[ r_{ic} = \frac{v_t}{i_c} = \frac{r_0 + r_e' R_{te}}{1 - \alpha R_{te}/(r_e' + R_{te})} \] (9)
The Norton equivalent circuit seen looking into the collector is shown in Fig. 2(b).

Figure 2: (a) Circuit for calculating \( r_{ic} \). (b) Norton collector circuit.

For the case \( r_0 \gg R_{te} \) and \( r_0 \gg r_e' \), we can write
\[ i_{c(sc)} = G_m (v_{tb} - v_{te}) \] (10)
where
\[ G_m = \frac{\alpha}{r_e' + R_{te}} \] (11)
The value of \( i_{c(sc)} \) calculated with this approximation is simply the value of \( \alpha i_e' \), where \( i_e' \) is calculated with \( r_0 \) considered to be an open circuit. The term “\( r_0 \) approximation” is used in the following when \( r_0 \) is neglected in calculating \( i_{c(sc)} \) but not neglected in calculating \( r_{ic} \).