The BJT - NPN Device

Modes of Operation

In the active mode, the B-E junction is forward biased. The C-B junction is reverse biased. The labeled current directions are for the active mode. For the PNP device, the current directions are reversed.
In the NPN device, the p type impurity doping in the base is very small compared to the n type impurity doping in the collector and emitter. For this reason, electrons are the majority current carriers. In the PNP device, holes are the majority current carriers.

**NPN BJT in the Active Mode**

![Diagram of NPN BJT in the Active Mode]

Electrons are emitted from the emitter region across the forward biased B-E junction into the base region. The E field across
the reverse biased CB junction attracts these electrons and they are collected by the collector. The fraction of collected electrons is denoted by $\alpha$. Thus we have

$$I_c = \alpha I_E$$

$$I_B = I_E - I_c = (1-\alpha) I_E$$

$$\Rightarrow I_E = \frac{1}{1-\alpha} I_B$$

$$\Rightarrow I_c = \frac{\alpha}{1-\alpha} I_B$$

The current gain $\beta$ is defined by

$$\beta = \frac{\alpha}{1-\alpha}$$

$$\Rightarrow I_c = \beta I_B = \alpha I_E$$
In general, we can write

\[ i_C = \beta i_B = \alpha i_E \]

The Transfer Characteristics

These are plots of \( i_C \) versus \( V_{BE} \) for \( V_{CE} = \text{constant} \)

\[ i_C = I_S e^{V_{BE}/V_T} \]

where \( I_S = I_{S_0} \left( 1 + \frac{V_{CE}}{V_T} \right) = \text{constant} \)

Draw a tangent line at the point \((V_{BE}, i_C)\). The slope of the line can be used to relate changes in \( i_C \) to changes in \( V_{BE} \).
\[ m = \frac{\partial I_C}{\partial V_{BE}} = I_s e^{V_{BE}/V_T} \times \frac{1}{V_T} \]

\[ = \frac{I_C}{V_T} \]

\[ \Rightarrow I_C = \frac{I_C}{V_T} V_{BE} \]

The output characteristics

These are plots of \( I_C \) versus \( V_{CE} \) for \( I_B = \text{constant} \).

\[ I_C = \beta I_B = \beta_0 \left( 1 + \frac{V_{CE}}{V_A} \right) I_B \]

![Diagram showing the relationship between \( I_C \) and \( V_{CE} \) with \( I_B \) increasing.](image)
Draw a tangent line at the point $(V_{CE}, I_C)$. The slope of the line can be used to relate changes in $I_C$ to changes in $V_{CE}$.

$$m = \frac{\Delta I_C}{\Delta V_{CE}} = \beta_0 \frac{1}{V_A} I_B$$

$$= \beta_0 \frac{1}{V_A} \frac{I_C}{\beta}$$

$$= \frac{\beta_0}{V_A} \frac{I_C}{\beta_0 (1 + \frac{V_{CE}}{V_A})}$$

$$= \frac{I_C}{V_A + V_{CE}}$$

$$\Rightarrow \frac{\Delta I_C}{\Delta V_{CE}} = \frac{I_C}{V_A + V_{CE}}$$

Thus, in general, we have

$$I_C = \frac{I_C}{V_T} V_{be} + \frac{I_C}{V_A + V_{CE}} V_{CE}$$
Let us define

\[ q_m = \frac{I_C}{V_T} \]

\[ r_0 = \frac{V_A + V_{CE}}{I_C} \]

\[ \Rightarrow I_C = q_m V_{be} + \frac{V_{CE}}{r_0} \]

Next, we relate the change in \( i_B \) to a change in \( V_{BE} \).

\[ h_B = \delta = \frac{\delta I_C}{\delta} = \frac{I_{so} (1 + \frac{V_{CE}}{V_T}) e^{V_{BE}/V_T}}{\beta_o (1 + \frac{V_{CE}}{V_A})} \]

\[ = \frac{I_{so}}{\beta_o} \cdot e^{V_{BE}/V_T} \]

\[ i_B \]

\[ \Delta i_B \]

\[ i_B + \Delta i_B \]

\[ i_B \]

\[ V_{BE} \]

\[ V_{BE} + V_{ce} \]

Draw a tangent line at the point \((V_{BE}, I_B)\). The slope of the line...
can be used to relate changes in $i_B$ to changes in $V_{BE}$.

$$m = \frac{dI_B}{dV_{BE}} = \frac{I_{so} e^{V_{BE}/nT}}{nT} \times \frac{1}{V_T}$$

$$= \frac{I_B}{V_T}$$

$$\Rightarrow i_B = \frac{I_B}{V_T} V_{be}$$

Let us define $R_{\pi} = \frac{V_T}{I_B}$

$$\Rightarrow i_B = \frac{V_{be}}{R_{\pi}}$$

The Hybrid-$\pi$ Model

The basic equations are

$$i_C = q_m V_{be} + \frac{V_{ce}}{R_0}$$

$$i_B = \frac{V_{be}}{R_{\pi}}$$

We can draw the model as follows:
We seek the relationships between $i_c'$, $i_b'$, and $i_e'$. 

\[ i_c' = g_m v_{be} = g_m (\lambda_b R_{\pi}) \]
\[ = g_m R_{\pi} \lambda_b = \frac{I_c}{V_t} \frac{V_t}{I_b} \lambda_b \]
\[ = \frac{I_c}{I_b} \lambda_b = \beta \lambda_b \]

\[ i_c' = i_c' + i_b' = \lambda_c' + \frac{1}{\beta} \lambda_c' \]
\[ = \lambda_c' \left( 1 + \frac{1}{\beta} \right) = \lambda_c' \frac{1 + \beta}{\beta} \]
\[ = \frac{\lambda_c'}{2} \]
Thus we have

\[ i_c' = g_m v_{be} = \beta i_b = \lambda i_e' \]

If \( R_o = \infty \) (open circuit), the primes can be dropped.

The Base Spreading Resistance

The base region is narrow and its ohmic contact is small. Its resistance is denoted by \( R_x \).
In this case, we write

\[ I_c' = q_m \nu = \beta I_B = \alpha I_c' \]

**DC Current Relations**

\[ I_B = \frac{I_c}{\beta} \]

\[ I_E = I_B + I_c \]

\[ = I_c \left( \frac{1}{\beta} + 1 \right) \]

\[ = I_c \left( \frac{1+\beta}{\beta} \right) \]

\[ = \frac{I_c}{\alpha} \]

\[ \Rightarrow I_c = \beta I_B = \alpha I_E \]

\[ I_B = I_E - I_c = I_E - \alpha I_E \]

\[ = I_E \left( 1 - \alpha \right) = I_E \left( 1 - \frac{\beta}{1+\beta} \right) \]

\[ = \frac{I_E}{1+\beta} \]
Summary

\[ I_C = \beta I_B = \alpha I_E \]
\[ I_E = \frac{I_C}{\alpha} = (1+\beta) I_B \]
\[ I_B = \frac{I_C}{\beta} = \frac{I_E}{1+\beta} \]
\[ \alpha = \frac{\beta}{1+\beta} \quad \beta = \frac{\alpha}{1-\alpha} \]

The BJT T Model

The T model replaces \( R_n \) through which \( i_b \) flows with \( R_e \) through which \( i_c' \) flows. The voltage \( V_n \) must be the same for the two.

\[ V_n = i_b R_n = \frac{i_c'}{\beta} R_n = \frac{\alpha i_e'}{\beta} R_n \]
\[ = i_e' \frac{\alpha}{\beta} R_n = i_e' \frac{\alpha}{\beta} \frac{V_T}{I_B} = i_e' \frac{\alpha V_T}{I_c} \]
\[ = i_e' \frac{V_T}{I_E} \]
Let \( R_e = \frac{V_T}{I_e} \)

\[ \Rightarrow V_{\pi} = \beta e R_e \]

The resistor \( R_e \) is called the intrinsic emitter resistance. The \( T \) model is

\[ i_c' = g_m V_{\pi} = \beta i_b = \alpha i_e' \]

Both the \( T \) model and the hybrid-\( \pi \) models give identical answers when numbers are substituted into the equations.