Approximate Analysis of the BJT Diff Amp

Assume $R_0 = \infty$. Use the simplified $T$ model to solve for $i_e$. Then use $i_c = \alpha i_e$ to solve for $i_c$.

Looking into the emitter of the two transistors with $R_0 = \infty$, we see $r_e'$ in series with $v_{eb}$.

\[ r_{e1} = r_{e2} = r_e = \frac{R_B + R_X}{1 + \beta} + r_e \]

\[ r_e = \frac{V_T}{I_E} = \frac{2V_T}{I_Q} \]

\[ i_{e1} = - i_{e2} = \frac{v_{\alpha 1} - v_{\alpha 2}}{2(r_e' + R_E)} \]

\[ i_{c1} = - i_{c2} = \alpha i_{e1} = \frac{\alpha}{2(r_e' + R_E)} (v_{\alpha 1} - v_{\alpha 2}) \]
\[ V_{01} = -V_{02} = -I_{C1} \quad R_C = \frac{-\alpha R_C}{2(R_e' + R_E)} (V_{\lambda 1} - V_{\lambda 2}) \]

With \( V_{\lambda 2} = 0 \),\n\[ R_{\lambda 1} = R_{\lambda 1} + (1 + \beta) (R_e' + R_{\lambda e1}) \]

where \( R_{\lambda e1} = 2R_E + R_{\lambda 2} \)

With \( V_{\lambda 1} = 0 \), \( R_{\lambda 2} = R_{\lambda 2} \).

\[ R_{out1} = R_{out2} = R_C \]
Differential and Common-Mode Analysis of the BJT Diff Amp

Consider the case where the tail supply is not a perfect current source. Let it exhibit a parallel resistance \( R_Q \).

Assume identical transistors.

For the dc analysis, set \( V_{i1} = V_{i2} = 0 \). Divide the tail supply into 2 identical current sources (\( I_Q/2 \) in parallel with \( 2R_Q \)). By symmetry the circuit for either \( Q_1 \) or \( Q_2 \)
The dc bias equation is

\[ 0 - (V_+ - I_Q R_Q) = \frac{I_E}{1+\beta} R_B + V_{BE} + I_E 2R_Q \]

\[ \Rightarrow I_E = \frac{-V_+ + I_Q R_Q - V_{BE}}{\frac{R_B}{1+\beta} + 2R_Q} \]

\[ R_{e1} = R_{e2} = R_e = \frac{V_T}{I_E} \]

For the BJT's to be biased in the active mode, \( V_{C13} > 0 \).
\[ V_{cB} = V_c - V_B = (V^+ - \alpha I_E R_c) - \left(-\frac{I_E}{1+\beta} R_B\right) \]

\[ = V^+ - I_E \left(\alpha R_c - \frac{R_B}{1+\beta}\right) \]

This must be > 0, often it is taken to be \( \frac{1}{2} V^+ \) for a resistively loaded diff amp.

Small-Signal AC Analysis

Let us define the differential and common-mode input voltages as follows:

**Differential:** \( V_{id} = V_{\lambda 1} - V_{\lambda 2} \)

**Common-Mode:** \( V_{\lambda cm} = \frac{1}{2} (V_{\lambda 1} + V_{\lambda 2}) \)

It follows from these definitions that

\[ V_{\lambda 1} = V_{\lambda cm} + \frac{1}{2} V_{id} \]
\[ V_{\lambda 2} = V_{\lambda cm} - \frac{1}{2} V_{id} \]
To solve for $V_{o1}$ and $V_{o2}$, we can use superposition of $V_{id}$ and $V_{cem}$. The ac signal circuit is

![Circuit Diagram]

**Differential Analysis**

Let $V_{\Delta 1} = \frac{1}{2} V_{id}$ and $V_{\Delta 2} = -\frac{1}{2} V_{id}$

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\Rightarrow V_{\Delta 1}$ increases $V_a$ and $V_{\Delta 2}$ decreases $V_a$. The net affect is that $V_a = 0$. Thus we can ground the $V_a$ node. The circuit becomes
Thus, the circuit consists of two CE amplifiers. By symmetry, $V_{02} = -V_{01}$. From our CE amplifier analysis, we have

$$V_{01} = -I_c(s_c) \left[ R_{ic}(d) \parallel R_c \right]$$

$$I_{c1}(s_c) = G_m(b) \left( \frac{V_{id}}{2} \right)$$

$$\Rightarrow V_{01} = \left[ -\frac{1}{2} G_m(b) R_{ic}(d) \parallel R_c \right] V_{id}$$

where $G_m(b)$ and $R_{ic}(d)$ are calculated with $R_{tb} = R_B$ and $R_{te} = R_E$. By symmetry

$$V_{02} = \left[ \frac{1}{2} G_m(b) R_{ic}(d) \parallel R_c \right] V_{id}$$
Common-Mode Analysis

Let \( V_{\Delta 1} = V_{\Delta 2} = V_{\Delta \text{cm}} \).

\( V_{\Delta 1} \) increases \( i_A \) and \( V_{\Delta 2} \) decreases \( i_A \). The net effect is that \( i_A = 0 \). Thus we can open the \( i_A \) branch. The circuit becomes

Thus the circuit again consists of two CE amplifiers. By symmetry, \( V_{01} = V_{02} \). From our CE analysis, we have

\[
V_{01} = -\lambda c_1(\text{sc}) \ R_{\Delta \text{cm}} || R_c
\]

\( \lambda c_1(\text{sc}) = G_{mb}(\text{cm}) \ V_{\Delta \text{cm}} \)
\[ v_{01} = \left[ -Gmb(cm) Ric(cm) \parallel Rc \right] v_{icm} \]

where \( Gmb(cm) \) and \( Ric(cm) \) are calculated with \( R_{tb} = RB \) and \( R_{te} = RE + 2RA \). By symmetry

\[ v_{02} = \left[ -Gmb(cm) Ric(cm) \parallel Rc \right] v_{icm} \]

**Total Solution for \( v_{01} \):**

We add the differential and common-mode solutions to obtain

\[ v_{01} = \left[ -\frac{1}{2} Gmb(d) Ric(d) \parallel Rc \right] (v_{x1} - v_{x2}) \]

\[ + \left[ -Gmb(cm) Ric(cm) \parallel Rc \right] \frac{v_{x1} + v_{x2}}{2} \]

\[ = A_1 v_{x1} + A_2 v_{x2} \]

where \( A_1 \) and \( A_2 \) are given by
\[
A_1 = -\frac{1}{2} \left[ G_{mb(d)} R_{icc(d)} || R_c + G_{mb(cm)} R_{icc(cm)} || R_c \right] \\
A_2 = +\frac{1}{2} \left[ G_{mb(d)} R_{icc(d)} || R_c - G_{mb(cm)} R_{icc(cm)} || R_c \right]
\]

For the case \( R_s \to \infty \), \( G_{mb(cm)} \to 0 \) and we have

\[
A_1 = -A_2 = A = -\frac{1}{2} G_{mb(d)} R_{icc(d)} || R_c
\]

In this case, \( V_{01} \) is given by

\[
V_{01} = A (V_{s1} - V_{s2})
\]

All solutions for \( V_{02} \) can be obtained by interchanging \( V_{s1} \) and \( V_{s2} \) in the solutions for \( V_{01} \).

The Common-Mode Rejection Ratio or CMRR

Let the output be taken from
the $V_{01}$ output. This is given by:

$$V_{01} = -\frac{V_{id}}{2} \left[ G_{mb(d)} R_{ic(d)} \| R_c \right]$$

$$- V_{icm} \left[ G_{mb(cm)} R_{ic(cm)} \| R_c \right]$$

$$= A_d V_{id} + A_{cm} V_{icm}$$

where $A_d$ and $A_{cm}$ are given by:

$$A_d = -\frac{1}{2} G_{mb(d)} R_{ic(d)} \| R_c$$

$$A_{cm} = - G_{mb(cm)} R_{ic(cm)} \| R_c$$

The CMRR is defined by:

$$CMRR = \frac{A_d}{A_{cm}} = \frac{\frac{1}{2} G_{mb(d)} R_{ic(d)} \| R_c}{G_{mb(cm)} R_{ic(cm)} \| R_c}$$

For $R_c \to \infty$, it follows that $CMRR \to \infty$. The larger the CMRR, the lower the common mode gain.
and the closer the diff amp is to an ideal diff amp with the output $V_{01} = -A(V_{x1} - V_{x2})$.

For $\text{CMRR} < \infty$, the gains for the two inputs are not equal. For $\text{CMRR} \to \infty$, the gains are equal.

In practice a $\text{CMRR}$ of 60 to 80 dB ($20 \log (A_{d}/A_{cm})$) can be achieved.