Equivalent Noise Input Voltage

Figure 1 shows the amplifier noise model with a Thévenin input source, where V_s is the source voltage, $Z_s = R_s + jX_s$ is the source impedance, V_{ts} is the thermal noise voltage generated by the source, and V_n and I_n are the noise sources representing the noise generated by the amplifier. The output voltage is given by

$$V_o = AV_i \frac{Z_L}{Z_o + Z_L} = \frac{AZ_i}{Z_s + Z_i} \frac{Z_L}{Z_o + Z_L} \left[V_s + (V_{ts} + V_n + I_n Z_s)\right] \tag{1}$$

where A is the voltage gain and Z_i is the input impedance. The equivalent noise input voltage V_{ni} is defined as the voltage in series with V_s that generates the same noise voltage at the output as all noise sources in the circuit. It consists of the terms in parenthesis in Eq. (1) and is given by

$$V_{ni} = V_{ts} + V_n + I_n Z_s \tag{2}$$

Note that this is independent of both A and Z_i. It is simply the noise voltage across Z_i considering Z_i to be an open circuit.

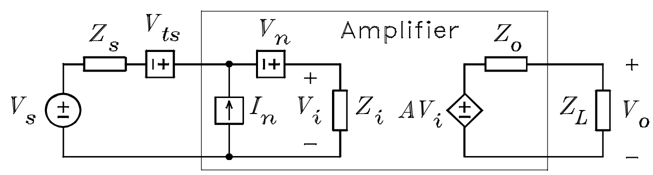

Figure 1: $v_n - i_n$ amplifier model with Thevénin source.

The mean-square value of V_{ni} is solved for as follows:

$$\begin{aligned}
\overline{v_{ni}^2} &= \overline{(V_{ts} + V_n + I_n Z_s)(V_{ts}^* + V_n^* + I_n^* Z_s^*)} \\
&= \overline{V_{ts} V_{ts}^*} + \overline{V_n V_n^*} + 2\,\mathrm{Re}\left[\left(\overline{V_n I_n^*}\right) Z_s^*\right] + \overline{(I_n Z_s)(I_n^* Z_s^*)} \\
&= 4kT\,\mathrm{Re}(Z_s)\,\Delta f + \overline{v_n^2} + 2\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}\,\mathrm{Re}(\gamma Z_s^*) + \overline{i_n^2}\,|Z_s|^2
\end{aligned} \tag{3}$$

where $\gamma = \gamma_r + j\gamma_i$ is the correlation coefficient between V_n and I_n and it is assumed that V_{ts} is independent of both V_n and I_n. The correlation coefficient is given by

$$\gamma = \frac{\overline{V_n I_n^*}}{\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}} \tag{4}$$

Effect of a Series Impedance at the Input

Figure 2 shows the input circuit of an amplifier with an impedance Z_1 added in series with a Thévenin source. The noise source V_{t1} models the thermal noise generated by Z_1. As is shown above, the equivalent noise voltage in series with the source can be solved for by first solving for the open-circuit input voltage, i.e. the input voltage considering Z_i to be an open circuit. It is given by

$$\begin{aligned}
V_{i(oc)} &= V_s + V_{ts} + V_{t1} + V_n + I_n (Z_s + Z_1) \\
&= V_s + V_{ni}
\end{aligned} \tag{5}$$

where V_{ni} is the equivalent noise voltage in series with the source. It is given by

$$\begin{aligned} V_{ni} &= V_{ts} + V_{t1} + V_n + I_n (Z_s + Z_1) \\ &= V_{ts} + V_{ns} + I_{ns} Z_s \end{aligned} \tag{6}$$

where V_{ns} and I_{ns} are the new values of V_n and I_n on the source side of Z_1. It follows from this equation that

$$V_{ns} = V_{t1} + V_n + I_n Z_1 \tag{7}$$

$$I_{ns} = I_n \tag{8}$$

Note that V_{ns} consists of all terms which are not multiplied by Z_s and I_{ns} consists of the coefficient of Z_s in the term that is multiplied by Z_s.

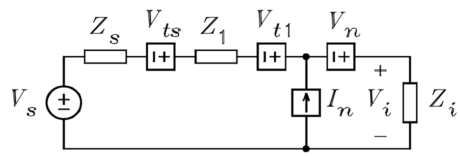

Figure 2: Amplifier input circuit with Thévenin source and series impedance added at input.

It follows that the addition of a series impedance at the input of an amplifier increases the V_n noise but does not change the I_n noise. If Z_1 is not to increase the noise, $|Z_1|$ should be as small as possible. If Z_1 is lossless, it generates no noise itself so that $V_{t1} = 0$. The mean-square values and the correlation coefficient for V_{ns} and I_{ns} are given by

$$\overline{v_{ns}^2} = 4kT \operatorname{Re}(Z_1) \Delta f + \overline{v_n^2} + 2\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} \operatorname{Re}(\gamma Z_1^*) + \overline{i_n^2} |Z_1|^2 \tag{9}$$

$$\overline{i_{ns}^2} = \overline{i_n^2} \tag{10}$$

$$\gamma_s = \frac{\gamma\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + \overline{i_n^2} Z_1}{\sqrt{\overline{v_{ns}^2}}\sqrt{\overline{i_{ns}^2}}} \tag{11}$$

The mean-square equivalent noise input voltage is given by

$$\begin{aligned} \overline{v_{ni}^2} &= 4kT \operatorname{Re}(Z_s + Z_1) \Delta f + \overline{v_n^2} \\ &\quad + 2\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} \operatorname{Re}[\gamma (Z_s^* + Z_1^*)] + \overline{i_n^2} |Z_s + Z_1|^2 \end{aligned} \tag{12}$$

Effect of a Shunt Impedance at the Input

Figure 3 shows the input circuit of an amplifier with an impedance Z_2 added in parallel with the source. The noise source I_{t2} models the thermal noise generated by Z_2. The open-circuit input voltage is given by

$$\begin{aligned} V_{i(oc)} &= (V_s + V_{ts}) \frac{Z_2}{Z_s + Z_2} + V_n + (I_{t2} + I_n) Z_s \| Z_2 \\ &= (V_s + V_{ni}) \frac{Z_2}{Z_s + Z_2} \end{aligned} \tag{13}$$

where V_{ni} is the equivalent noise voltage in series with the source. It is given by

$$\begin{aligned} V_{ni} &= V_{ts} + V_n \left(1 + \frac{Z_s}{Z_2}\right) + (I_{t2} + I_n) Z_s \\ &= V_{ts} + V_{ns} + I_{ns} Z_s \end{aligned} \tag{14}$$

where V_{ns} and I_{ns} are the new values of V_n and I_n on the source side of Z_2. It follows from this equation that

$$V_{ns} = V_n \tag{15}$$

$$I_{ns} = I_{t2} + \frac{V_n}{Z_2} + I_n \tag{16}$$

Note that V_{ns} consists of the term which is not multiplied by Z_s and I_{ns} consists of the sum of the coefficients of Z_s in the terms that are multiplied by Z_s.

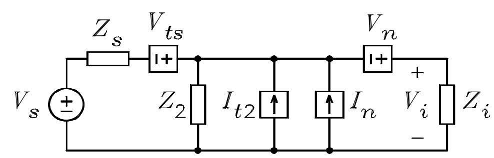

Figure 3: Amplifier input circuit with Thévenin source and shunt impedance added at input.

It follows that the addition of a parallel impedance at the input of an amplifier increases the I_n noise but does not change the V_n noise. If Z_2 is not to increase the noise, $|Z_2|$ should be as large as possible. If Z_2 is lossless, it generates no noise itself so that $I_{t2} = 0$. The mean-square values and the correlation coefficient for V_{ns} and I_{ns} are given by

$$\overline{v_{ns}^2} = \overline{v_n^2} \tag{17}$$

$$\overline{i_{ns}^2} = 4kT \operatorname{Re}\left(\frac{1}{Z_2}\right) \Delta f + \frac{\overline{v_n^2}}{|Z_2|^2} + 2\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} \operatorname{Re}\left(\frac{\gamma}{Z_2}\right) + \overline{i_n^2} \tag{18}$$

$$\gamma_s = \frac{\gamma \sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + \dfrac{\overline{v_n^2}}{Z_2^*}}{\sqrt{\overline{v_{ns}^2}}\sqrt{\overline{i_{ns}^2}}} \tag{19}$$

The mean-square equivalent noise input voltage is given by

$$\begin{aligned} \overline{v_{nis}^2} &= 4kT \operatorname{Re}\left(Z_s + \frac{|Z_s|^2}{Z_2}\right) \Delta f + \overline{v_n^2} \left|1 + \frac{Z_s}{Z_2}\right|^2 \\ &\quad + 2\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} \operatorname{Re}\left[\gamma \left(1 + \frac{Z_s}{Z_2}\right) Z_s^*\right] + \overline{i_n^2} |Z_s|^2 \end{aligned} \tag{20}$$

Dc bias networks and rf matching networks usually consist of series and parallel elements at the input to an amplifier. One method of analyzing the effect of these elements on the amplifier noise is by transforming the V_n and I_n sources from the amplifier input back to the source by use of the above relations. This is illustrated in the following example.

Example 1 *Figure 4shows the input circuit of an amplifier. It is given that* $R_s = 75\ \Omega$*,* $R_1 = 1$ *k*Ω*,* $C = 10$ *nF,* $R_2 = 100\ \Omega$*,* $\sqrt{\overline{v_n^2}} = 2$ *nV,* $\sqrt{\overline{i_n^2}} = 1.5$ *pA,* $\gamma = 0.2 + j0.1$*. The noise specifications are for a frequency* $f = 100$*kHz and a bandwidth* $\Delta f = 1$ *Hz. Calculate* $\sqrt{\overline{v_{ni}^2}}$ *in series with the source by transforming* V_n *and* I_n *back to the source with Eqs. (9) through (11) and (17) through (19).*

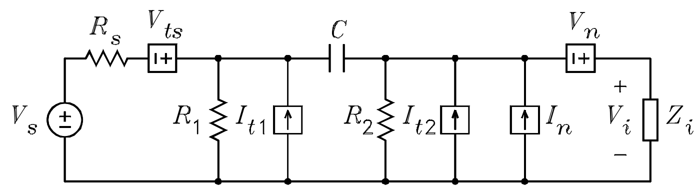

Figure 4: Amplifier input circuit.

Solution. To the left of R_2

$$\sqrt{\overline{v_{na}^2}} = \sqrt{\overline{v_n^2}} = 2 \text{ nV}$$

$$\sqrt{\overline{i_{na}^2}} = \left[\frac{4kT\Delta f}{R_2} + \frac{\overline{v_n^2}}{R_2^2} + 2\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}\,\text{Re}\left(\frac{\gamma}{R_2}\right) + \overline{i_n^2}\right]^{1/2} = 24 \text{ pA}$$

$$\gamma_a = \frac{\gamma\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} + \dfrac{\overline{v_n^2}}{R_2}}{\sqrt{\overline{v_{na}^2}}\sqrt{\overline{i_{na}^2}}} = 0.847 + j6.26 \times 10^{-3}$$

The capacitor impedance is $Z_C = 1/j2\pi fC = -j159\ \Omega$. To the left of C

$$\sqrt{\overline{v_{nb}^2}} = \left[\overline{v_{na}^2} + 2\sqrt{\overline{v_{na}^2}}\sqrt{\overline{i_{na}^2}}\,\text{Re}\left(\gamma_a Z_C^*\right) + \overline{i_n^2}\,|Z_C|^2\right]^{1/2} = 4.3 \text{ nV}$$

$$\sqrt{\overline{i_{nb}^2}} = \sqrt{\overline{i_{na}^2}} = 24 \text{ pA}$$

$$\gamma_b = \frac{\gamma_a\sqrt{\overline{v_{na}^2}}\sqrt{\overline{i_{na}^2}} + \overline{i_{na}^2} Z_C}{\sqrt{\overline{v_{nb}^2}}\sqrt{\overline{i_{nb}^2}}} = 0.394 - j0.885$$

To the left of R_1

$$\sqrt{\overline{v_{nc}^2}} = \sqrt{\overline{v_{nb}^2}} = 4.3 \text{ nV}$$

$$\sqrt{\overline{i_{nc}^2}} = \left(\frac{\overline{v_{nb}^2}}{R_1^2} + \frac{4kT\Delta f}{R_1} + \overline{i_{nb}^2}\right)^{1/2} = 26.3 \text{ pA}$$

$$\gamma_c = \frac{\gamma_b\sqrt{\overline{v_{nb}^2}}\sqrt{\overline{i_{nb}^2}} + \dfrac{\overline{v_n^2}}{R_1}}{\sqrt{\overline{v_{nc}^2}}\sqrt{\overline{i_{nc}^2}}} = 0.523 - j0.807$$

The equivalent noise voltage in series with the source is

$$\begin{aligned}\sqrt{\overline{v_{ni}^2}} &= \left(4kTR_s\Delta f + \overline{v_{nc}^2} + 2\sqrt{\overline{v_{nc}^2}}\sqrt{\overline{i_{nc}^2}}\,\mathrm{Re}\left(\gamma_c R_s\right) + \overline{i_{nc}^2}R_s^2\right)^{1/2} \\ &= 5.69\ \mathrm{nV}\end{aligned}$$

The following example illustrates the calculation of $\sqrt{\overline{v_{ni}^2}}$ for the circuit of Fig. 4 by first calculating the open-circuit input voltage due to all sources in the circuit, factoring out the coefficient of V_s, and assigning all remaining noise terms to V_{ni}.

Example 2 *For the circuit of Example 1, calculate* $\sqrt{\overline{v_{ni}^2}}$ *by calculating the open-circuit input voltage, factoring out the coefficient of* V_s*, and assigning all remaining noise terms to* V_{ni}*.*

Solution. The equivalent source impedance seen by the amplifier is given by

$$Z_{\mathrm{eq}} = R_2 \| \left(\frac{1}{j\omega C} + R_1 \| R_s\right) = 68.6 - j29.4$$

The thermal noise voltage generated by R_s, R_1, and R_2 has a mean-square value of $4kT\,\mathrm{Re}\left(Z_{\mathrm{eq}}\right)$. Denote the phasor value of this voltage by $V_{t(\mathrm{eq})}$. The open-circuit input voltage is given by

$$\begin{aligned}V_{i(oc)} &= V_s\frac{R_1}{R_1+R_s}\,\frac{R_2}{R_2+R_1\|R_s+1/j\omega C} + V_{t(\mathrm{eq})} + V_n + I_n Z_{\mathrm{eq}} \\ &= (0.292 + j0.273)\,V_s + V_{t(\mathrm{eq})} + V_n + I_n Z_{\mathrm{eq}} \\ &= (0.292 + j0.273)\left(V_s + \frac{V_{t(\mathrm{eq})} + V_n + I_n Z_{\mathrm{eq}}}{0.292 + j0.273}\right)\end{aligned}$$

It follows that the equivalent noise input voltage is given by

$$\begin{aligned}\sqrt{\overline{v_{ni}^2}} &= \left[\frac{4kT\,\mathrm{Re}\left(Z_{\mathrm{eq}}\right) + \overline{v_n^2} + 2\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}\,\mathrm{Re}\left(\gamma Z_{\mathrm{eq}}^*\right) + \overline{i_n^2}\left|Z_{\mathrm{eq}}\right|}{\left|0.292 + j0.273\right|^2}\right]^{1/2} \\ &= 5.69\ \mathrm{nV}\end{aligned}$$

This is the same as that found in Example 1. Note that the method used in this example is more straightforward because it is necessary to transform only one source through the network. That source is the signal source V_s.

Noise Factor and Noise Figure

The noise factor F of an amplifier is defined as the ratio of its actual SNR and the SNR if the amplifier is noiseless, where the temperature is taken to be the standard temperature T_0. When it is expressed in dB, it is called noise figure and is given by $NF = 10\log\left(F\right)$. Consider the amplifier model in Fig. 1. If the amplifier is noiseless, the signal-to-noise ratio given by $SNR = \overline{v_s^2}/\overline{v_{ts}^2}$, where $\overline{v_s^2}$ is the mean-square source voltage and $\overline{v_{ts}^2}$ is the mean-square thermal noise voltage generated by the source impedance. When the amplifier noise is included, the signal-to-noise ratio is given by $SNR = \overline{v_s^2}/\overline{v_{ni}^2}$. Thus the noise factor is given by

$$F = \frac{\left(\overline{v_s^2}/\overline{v_{ts}^2}\right)}{\left(\overline{v_s^2}/\overline{v_{ni}^2}\right)} = \frac{\overline{v_{ni}^2}}{\overline{v_{ts}^2}} = 1 + \frac{\overline{v_n^2} + 2\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}\,\mathrm{Re}\left(\gamma Z_s^*\right) + \overline{i_n^2}\left|Z_s\right|^2}{4kT_0R_s\Delta f} \tag{21}$$

It follows from this expression that a noiseless amplifier has the noise factor $F = 1$.

A useful relation which follows from the definition of F is

$$\overline{v_{ni}^2} = F \times \overline{v_{ts}^2} = F \times 4kT_0 \operatorname{Re}(Z_s) \tag{22}$$

This relation is used below in the method for measuring F.

Example 3 *Calculate F and NF for the amplifier in Example 1 for which $R_s = 75\ \Omega$ and $\sqrt{\overline{v_{ni}^2}} =$ 5.69 nV. Assume $\Delta f = 1$ Hz and $T = T_0 = 290$ K.*

Solution. The mean-square thermal noise voltage of the source is $\overline{v_{ts}^2} = 4kTR_s = 1.20 \times 10^{-18}$ V^2. Thus the noise factor and noise figure are

$$F = \frac{\left(5.69 \times 10^{-9}\right)^2}{1.2 \times 10^{-18}} = 27.0 \qquad NF = 10 \log 27.0 = 14.3 \text{ dB}$$

When $\Delta f = 1$ Hz, as in this example, F is called the spot-noise factor and NF is called the spot-noise figure.

Measuring the Noise Factor

This method is the most general one because it does not require knowledge of either the amplifier gain or its noise bandwidth. Consider the noise model of an amplifier given in Fig. 5. Consider the source to be a white noise source having the spectral density $S_v(f) = \overline{V_s V_s^*}/\Delta f = \overline{v_s^2}/\Delta f$. The total noise voltage at the output can be written

$$\begin{aligned} V_o &= A\left[(V_s + V_{ts} + V_n)\frac{Z_i}{Z_s + Z_i} + I_n (Z_s \| Z_i)\right] \\ &= \frac{AZ_i}{Z_s + Z_i}(V_s + V_{ts} + V_n + I_n Z_s) \end{aligned} \tag{23}$$

The mean-square value is given by

$$\begin{aligned} \overline{v_o^2} &= \left|\frac{AZ_i}{Z_s + Z_i}\right|^2 \left[S_v(f) B_n + 4kT_0 \operatorname{Re}(Z_s) B_n + \overline{v_n^2} \right. \\ &\quad \left. + 2\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}} \operatorname{Re}(\gamma Z_s^*) + \overline{i_n^2}\,|Z_s|^2\right] \\ &= \left|\frac{AZ_i}{Z_s + Z_i}\right|^2 \left[S_v(f) B_n + \overline{v_{ni}^2}\right] \\ &= \left|\frac{AZ_i}{Z_s + Z_i}\right|^2 \left[S_v(f) B_n + F \times 4kT_0 \operatorname{Re}(Z_s) B_n\right] \end{aligned}$$

where B_n is the amplifier noise bandwidth and Eq. (22) has been used to relate $\overline{v_{ni}^2}$ to the thermal noise voltage of the source.

Let $\overline{v_{o1}^2}$ be the value of $\overline{v_o^2}$ with the noise source at the input set to zero, i.e. $S_v(f) = 0$. Now, let $S_v(f)$ be increased until the rms output voltage increases by a factor r, i.e. $\sqrt{\overline{v_o^2}} = r\sqrt{\overline{v_{o1}^2}}$. It follows by taking the ratio of the two mean-square voltages that

$$r^2 = 1 + \frac{S_v(f) B_n}{F \times 4kT \operatorname{Re}(Z_s) B_n} = 1 + \frac{S_v(f)}{F \times 4kT_0 \operatorname{Re}(Z_s)} \tag{24}$$

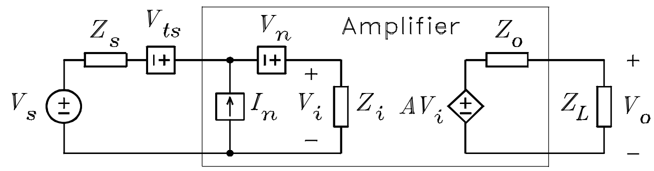

Figure 5: Amplifier driven by a white noise source.

Note that the noise bandwidth B_n cancels.

The above equation can be solved for F to obtain

$$F = \frac{S_v(f)}{(r^2 - 1) \times 4kT_0 \operatorname{Re}(Z_s)} \tag{25}$$

In making measurements, a commonly used value for r is $r = \sqrt{2}$. In this case, the output noise voltage increases by 3 dB when the source is activated. Note that the expression for F is independent of B_n, A, and Z_i. Other methods of measuring F require knowledge of these parameters.