Object

The object of this project is to implement and evaluate analog simulations of two of the motor transfer functions that students encounter in Dr. Dorsey’s senior controls laboratory. Three papers concerning analog computer simulations are posted on the class web page. These can be used as references.

Description of the Circuit

Figure 1 shows the diagram of a feedback control system. The input voltage $V_i$ is the input to the system. The output voltage $V_o$ can be thought of as being the analog of either the angular velocity of the shaft of a dc motor or its angular position.

\[ V_o = \frac{G(s)}{V_i} \]

If \(|G(j\omega)| \gg 1\) for all frequencies of interest, it follows that the transfer function becomes

\[ \frac{V_o}{V_i} \approx 1 \]

That is, the output corresponds exactly to the input. This is a characteristic of an ideal control system. In practice, when $V_i$ is changed, $V_o$ cannot change instantaneously, so that there is a time during which the control system regulates itself. An important part of control system design is the prevention of instabilities which appear as unwanted oscillations at the output. We will not attempt to investigate this problem.

There are two transfer functions to be simulated. The first is

\[ G_1(s) = \frac{k_1}{s(1 + s/\omega_1)(1 + s/\omega_2)} \]

where $k_1 = 2.64$, $\omega_1 = 3.2$, and $\omega_2 = 15$. A possible realization of this transfer function is shown in Fig. 2. Note that this circuit realizes $-G_1(s)$. The circuit can be realized with an
inverting integrator and two inverting low-pass filters. Because the circuit realizes \(-G_1(s)\), the inverting input to Fig. 1 must be changed to a non-inverting input in order to make the feedback negative.

The second transfer function is

\[
G_2(s) = \frac{k_2}{(s/\omega_0)^2 + (1/Q)(s/\omega_0) + 1}
\]

where \(k_2 = 0.282\), \(\omega_0 = 10.01\), and \(Q = 11.21\). Because the poles are complex, feedback must be used to realize the transfer function. A possible realization is shown in Fig. 3. This circuit can be realized with two inverting integrators with summing inputs.

You are going to find that the given transfer functions have a very low bandwidth, making the transient response difficult to observe on the oscilloscope because it is so slow. The response time of the circuits can be increased by the factor \(K\) by replacing each occurrence of \(s\) in the transfer functions by \(s/K\). For the initial design, a suggested value of \(K\) is \(K = 100\). This value can be easily changed later by changing only resistors in the circuit. You should not use electrolytic capacitors to set any of the time constants in the circuits for their values are not that accurate. If they could be used, they would have to be non-polar. A suggested value for each \(C\) is \(C = 0.1 \mu F\), but you are free to use any value that is available.

**Procedure Step One**

Determine suitable \(R\) and \(C\) values for the circuits in Figs. 1 and 2. Assemble the circuits. You cannot test the circuit of Fig. 1 by itself because the integrator gives an infinite gain at dc. To test this circuit, remove the feedback capacitor in the integrator stage and replace it with a resistor equal to the input resistor so that the stage has a unity inverting gain. Measure the Bode magnitude plot for both circuits to verify that they are correct. Note that the gain of the circuit in Fig. 2 peaks up quite a bit at \(\omega = \omega_0\). For this reason, the input voltage must be small enough to prevent clipping at this frequency. In the event that this circuit oscillates, the value of \(Q\) can be decreased. However, this changes the transfer function and should be avoided unless it is a last option.
**Procedure Step Two**

Figure 4 shows the system of Figure 1 with an added gain block $k$. The object of the gain block is to speed up the response time of the system. That is, when a step input is applied, the time required for the output to respond can be reduced by adding gain inside the loop. There is one problem with this, however. As $k$ is increased, the system can become unstable. That is, the poles move into the right half plane and the system oscillates. In a controls course, the method of root locus is usually used to plot the location of the poles as a function of $k$. Given one of these plots, the largest value of $k$ can be determined for the system to be stable. When the system is unstable, its poles are on the $j\omega$ axis or in the right-half plane and the output is not related to the input.

![Figure 4: System with feedback and added gain.](image)

In the second part of the experiment, you are to assemble the circuit of Fig. 4 for each of the transfer functions implemented in Procedure Step one. With a square wave in, observe the output and determine the effect of increasing $k$ on the transient response of the system. The variable gain can be realized with a potentiometer. Note that the system of Fig. 2 is inverting. In this case, either $k$ must be negative or the polarity of the feedback summing in Fig. 4 must be changed to $+$ to keep the overall feedback negative.

In documenting the responses of the system, you should obtain a Bode plot of the overall gain versus frequency. Then you should apply a fixed frequency square wave at the input and record the time domain output signal. The frequency of the square wave must be low enough so that you can see the ringing on the output waveform completely damp out in each one-half period of the square wave. You should show these plots for the original systems and for several values of $k$ for the systems with feedback. You should experimentally determine the largest value of $k$ for which the systems stay stable. You should include a SPICE simulation of the systems in your report.