ECE 6416 Quiz 1
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Instructions. Print your name in the space above and at the top of all other pages in your quiz. Place a box around each answer. Express each numerical answer as a decimal number. Numerical values are $4kT_0 = 1.6 \times 10^{-20}$ J and $q = 1.6 \times 10^{-19}$ C. Honor Code: I have neither given nor received help on this quiz. Initials

1. A 10 kΩ resistor has a noise index of $-15$ dB. The dc current flowing through the resistor is $I_{DC} = 1$ mA. The frequency band of interest is the band from 20 Hz to 20 kHz.

(a) Solve for the rms thermal noise voltage $v_t$ in the band.

$$ v_t = \sqrt{4kT(f_2 - f_1)} = 1.79 \mu V $$

(b) Solve for the rms excess noise voltage $v_{ex}$ in the band.

$$ v_{ex} = \sqrt{\frac{10^{N/10} (I_{DC}R)^2 \log (f_2/f_1)}{I_{DC}R}} = 3.08 \mu V $$

(c) Solve for the total rms noise voltage $v_n$ in the band.

$$ v_n = \sqrt{v_t^2 + v_{ex}^2} = 3.56 \mu V $$

2. A 0.01 μF capacitor is connected in parallel with a 10 kΩ resistor. Solve for the rms thermal noise voltage generated by the circuit in the band from 10 kHz to 100 kHz. You are given the following integral for reference:

$$ Z = R||1 = \frac{R}{1 + j\omega RC} $$

$$ \text{Re}(Z) = \text{Re}\left(\frac{R}{1 + j\omega RC}\right) = \frac{R}{1 + (\omega RC)^2} $$

$$ v_t^2 = 4kT \int_{10kHz}^{100kHz} \frac{df}{1 + (2\pi fRC)^2} $$

$$ = \frac{2kT}{\pi C} [\arctan (2\pi f_2 RC) - \arctan (2\pi f_1 RC)] = 3.61 \times 10^{-14} V^2 $$

$$ v_t = \sqrt{v_t^2} = 190 nV $$

3. A non-inverting amplifier has the gain with feedback $A_v = 100$ and the input resistance $R_i = \infty$. With a source resistor $R_S = 100 k\Omega$ connected from input to ground, the rms noise voltage at the output over the band $\Delta f = 100 kHz$ is found to be $v_{no1} = 1.5 mV$. When a resistor $R_1 = 20 k\Omega$ is connected in parallel with $R_S$ the noise output voltage drops to $v_{no2} = 0.6 mV$. If the correlation between $v_n$ and $i_n$ is neglected, solve for the rms values of $v_n$ and $i_n$ over the band $\Delta f$ at the amplifier input.

$$ v_n^2 + i_n^2 R_S^2 = \frac{v_{no1}^2}{A_v^2} - 4kT R_S \Delta f \implies v_n^2 + 10^{10} i_n^2 = 6.5 \times 10^{-11} $$

$$ v_n^2 + i_n^2 (R_S||R_1)^2 = \frac{v_{no2}^2}{A_v^2} - 4kT R_S||R_1 \Delta f \implies v_n^2 + 2.778 \times 10^8 i_n^2 = 9.333 \times 10^{-12} $$
\[ \Delta = \begin{vmatrix} 1 & 10^{10} \\ 1 & 2.778 \times 10^8 \end{vmatrix} = 2.778 \times 10^8 - 10^{10} = -9.722 \times 10^9 \]

\[ v_n^2 = \frac{1}{\Delta} \begin{vmatrix} 6.5 \times 10^{-11} & 10^{10} \\ 9.333 \times 10^{-12} & 2.778 \times 10^8 \end{vmatrix} = \frac{6.5 \times 10^{-11} \times 2.778 \times 10^8 - 9.333 \times 10^{-12} \times 2.778 \times 10^8}{-9.722 \times 10^9} = 7.743 \times 10^{-12} \]

\[ v_n = 2.783 \mu V \]

\[ i_n^2 = \frac{1}{\Delta} \begin{vmatrix} 1 & 6.5 \times 10^{-11} \\ 1 & 9.333 \times 10^{-12} \end{vmatrix} = \frac{9.333 \times 10^{-12} - 6.5 \times 10^{-11}}{-9.722 \times 10^9} = 5.726 \times 10^{-21} \]

\[ i_n = 75.67 \text{ pA} \]