On the Specification of Moving-Coil Drivers for Low-Frequency Horn-Loaded Loudspeakers *

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A procedure is presented for the design from specifications of moving-coil drivers for low-frequency horn-loaded loudspeakers. The method permits specification of the upper and lower system cutoff frequencies, the volume of the cavity behind the driver, the driver area, the horn throat area, and the desired system electrical impedance. From these specifications, the required Thiele – Small small-signal parameters and the electromechanical parameters of the driver are determined under the condition of a maximum-sensitivity constraint on the system. The procedure can be easily modified for a maximum-efficiency constraint.

0 INTRODUCTION

Horn-loaded loudspeakers have been in widespread use for many years, especially in applications where large acoustic powers must be radiated and where control of the directivity pattern of the radiated sound is desired. Compared to the direct radiator, the high efficiency of a horn-loaded loudspeaker is probably its best known advantage. For example, a moving-coil driver that has an electroacoustic efficiency of less than 1% when used as a piston radiator can easily achieve an efficiency in the 10 – 50% range when horn loaded.

When a given moving-coil driver is used in a horn system, the input impedance to the driver is increased by the horn loading over the useful frequency band. This is because the system compliance must be chosen so that the resonant frequency lies at the geometric mean between the lower and upper cutoff frequencies. Although the system total quality factor must be low for acceptable bandwidths, the in-band resonance of the horn-loaded driver can increase the input impedance sufficiently so that the system sensitivity (that is, its acoustic output for a constant input voltage) becomes unacceptable. Thus an important consideration in a horn synthesis is the system impedance. This must be kept acceptably low so that adequate electrical input power can be obtained from modern amplifiers which are essentially constant-voltage sources with negligible output impedance.

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This paper investigates the system design aspects of low-frequency horn-loaded loudspeakers. First, as a background, a complete electroacoustic analysis of horn-loaded systems is given. Based on this analysis, two synthesis procedures are then presented, one for a given driver and one from specifications. The latter procedure allows the designer to specify the Thiele – Small small-signal parameters for the driver from the system specifications while at the same time constraining the system impedance to lie in an acceptable range for modern power amplifiers. To illustrate the synthesis procedures, two numerical examples are given.

1 GLOSSARY OF SYMBOLS

B	Magnetic flux density in driver air gap
c	Velocity of sound in air (345 m/s)
C_{AB}	Acoustical compliance of air in box
C_{AF}	Acoustical compliance of air in front chamber
C_{AT}	Total acoustical compliance of driver and box
C_{MD}	Mechanical compliance of driver suspension
e_g	Open-circuit output voltage of electrical source
f_c	Resonant frequency of driver on box
f_D	Mechanical force on driver diaphragm
f_L	Lower midband cutoff frequency of system
f_H	Upper midband cutoff frequency of system
f_s	Driver free-air resonant frequency
f_0	Horn cutoff frequency
f_3	High-frequency band upper minus 3-dB frequency

$F(s)$	Midband system transfer function
$F_H(s)$	High-frequency band system transfer function
i_g	Voice-coil current supplied by electrical source
l	Effective length of voice-coil conductor in magnetic gap
L_E	Inductance of driver voice coil
M_{AD}	Acoustical mass of driver diaphragm assembly
M_{AL}	Acoustical mass of horn throat impedance
M_{MD}	Mechanical mass of driver diaphragm assembly
P_{AR}	Acoustical power radiated
P_E	Electrical input power to driver
Q_{EC}	Quality factor of system at f_c considering electrical losses only
Q_{ES}	Quality factor of driver at f_s considering electrical losses only
Q_{LC}	Quality factor of system at f_c considering acoustical radiation losses only
Q_{MC}	Quality factor of system at f_c considering mechanical losses in driver and acoustical losses in box
Q_{TC}	Total quality factor of system at f_c
R_{AB}	Acoustical resistance of box losses caused by internal energy absorption
R_{AE}	Acoustical resistance of driver electrical losses
R_{AL}	Acoustical resistance of horn throat impedance
R_{AM}	Acoustical resistance of driver mechanical losses and box acoustical losses
R_{AT}	Total acoustical resistance ($R_{AE} + R_{AM}$)
R_E	Dc resistance of driver voice coil plus output resistance of source
R_{ES}	Electrical resistance of driver suspension losses, box acoustical losses, and acoustical radiation losses
R_{MD}	Mechanical resistance of driver suspension
s	Complex frequency ($\sigma + j\omega$)
S_D	Effective piston area of driver diaphragm
S_T	Area of horn throat
u_D	Mechanical velocity of driver diaphragm
U_D	Volume velocity emitted by driver diaphragm
V_{AS}	Volume of air having same acoustical compliance as driver suspension
V_B	Volume of box that loads rear of driver diaphragm
V_F	Volume of front chamber between driver diaphragm and throat of horn
α	Volume compliance ratio (V_{AS}/V_B)
η	Power efficiency ratio
ρ_0	Density of air (1.18 kg/m^3)
$\rho_0 c$	Characteristic impedance of air (407 mks rayls)
ω_c	Angular resonant frequency of driver on box
ω_L	Lower midband angular cutoff frequency of system
ω_H	Upper midband angular cutoff frequency of system
ω_s	Driver free-air angular resonant frequency
ω_0	Horn angular cutoff frequency
ω_3	High-frequency band upper minus 3-dB angular frequency

2 HORN ELECTROACOUSTIC CIRCUIT

The point of departure is a basic introduction (or review, as the case may be) of the horn-loaded loudspeaker. With two exceptions, the approach is based on the electroacoustic circuit model of a moving-coil loudspeaker loaded by an acoustic horn as described in [1] and [2]. The first exception is that a gyrator model is used for the voice coil of the driver. This eliminates the confusing parallel-element mobility and admittance circuits from the analysis. The second is that the output impedance of the driving source is assumed to be negligible, for this is the case with modern power amplifiers. In contrast, the early analyses of horn loudspeakers driven from amplifiers with moderately high output impedances calculated the efficiency as the ratio of acoustic output power to maximum power available from the source [1]. The maximum available source power is inversely proportional to the source output impedance. Since modern amplifiers exhibit a very low output impedance which is typically less than 0.1 Ω, the maximum available output power is a meaningless specification, for amplifiers are not designed to drive load impedances this low. Thus an alternate definition of efficiency is used. To correspond with the modern notation used in loudspeaker analyses, the small-signal parameters defined by Thiele [3] and by Small [4] for direct-radiator loudspeakers and later applied to horn loudspeakers by Small [5] and Keele [6] will be used.

Fig. 1 shows the basic configuration of a horn-loaded moving-coil loudspeaker driver. The driver is modeled as having a piston area S_D. The rear of the driver is loaded into a box of volume V_B. Its front is loaded into a chamber of volume V_F which is coupled into the horn with a throat area S_T. The electro-mechano-acoustical equivalent circuit of the system is given in Fig. 2.

Fig. 2 is divided into three parts: electrical, mechanical, and acoustical. The electrical part shows the moving-coil driver connected to a generator of voltage e_g which supplies a current i_g. R_E and L_E are the voice-coil resistance and inductance, respectively. A gyrator with impedance Bl is used to couple the voice-coil circuit to the mechanical circuit as shown in the Appendix, where B is the magnetic flux density in the air gap and l is the length of the voice-coil conductor in the magnetic field.

In the mechanical part of the circuit, force is voltage and velocity is current. The current u_D is the velocity with which the driver cone moves. The elements M_{MD}, R_{MD}, and C_{MD} are the mechanical mass, resistance, and compliance, respectively, associated with the driver cone and its suspension. A transformer with a turns ratio equal to the piston area S_D of the cone couples the mechanical circuit to the acoustical circuit.

In the acoustical part of the circuit, pressure is voltage and volume velocity is current. The current U_D is the volume velocity emitted by the driver cone. In this circuit R_{AB} and C_{AB} are the acoustical resistance and compliance, respectively, of the box of volume V_B, and C_{AF} is the acoustical compliance of the front chamber. The input impedance to the horn is modeled as an acoustical mass M_{AL} in parallel with an acoustical resistance R_{AL}. For an infinite exponential horn, for example, this model is valid only above the cutoff frequency of the horn. Also, both M_{AL} and R_{AL} are functions of frequency. However, for frequencies greater than the horn cutoff frequency they approach constants. For the exponential horn these asymptotic values are [1]

$$M_{AL} = \frac{\rho_0 c}{\omega_0 S_T} \tag{1}$$

$$R_{AL} = \frac{\rho_0 c}{S_T} \tag{2}$$

where $\rho_0 c$ is the characteristic impedance of air and ω_0 is the horn cutoff frequency. At low frequencies the exact horn impedance expression must be used. For the infinite exponential horn this is [1]

$$Z_{AL} = R_{AL}\left[\sqrt{1-\left(\frac{\omega_0}{\omega}\right)^2} + j\,\frac{\omega_0}{\omega}\right] \tag{3}$$

where R_{AL} is given by Eq. (2). It can be shown that Eq. (2) gives the correct horn acoustical resistance for all horn geometries provided the frequency is sufficiently higher than the cutoff frequency.

It is convenient to divide R_{AT} into two parts—one part associated with the electrical losses and the other part associated with the driver mechanical losses plus box acoustical losses. This will be done by defining

$$R_{AE} = \frac{B^2 l^2}{S_D^2 R_E} \tag{7}$$

$$R_{AM} = \frac{R_{MD}}{S_D^2} + R_{AB}\,. \tag{8}$$

The analysis of the circuit in Fig. 4 can be simplified by defining three frequency ranges [1]. This approach is valid if the upper system cutoff frequency is sufficiently higher than the lower cutoff frequency. This is the case if the system is to exhibit acceptable bandwidth. For low frequencies, C_{AF} can be replaced by an open circuit and M_{AD} by a short circuit. For mid-band frequencies M_{AL} and C_{AF} are replaced by open circuits. For high frequencies C_{AT} is replaced by a short circuit and M_{AL} by an open circuit.

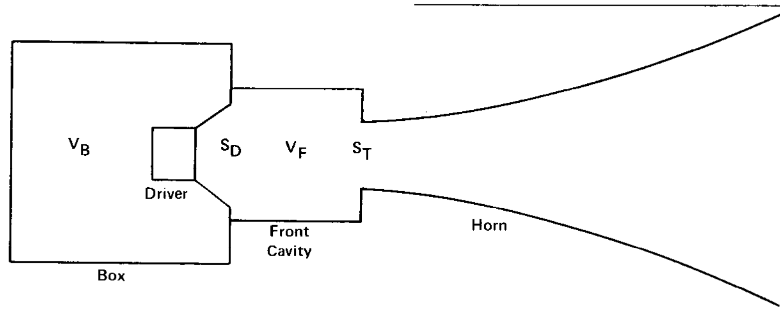

Fig. 1. Basic configuration of a low-frequency horn-loaded loudspeaker.

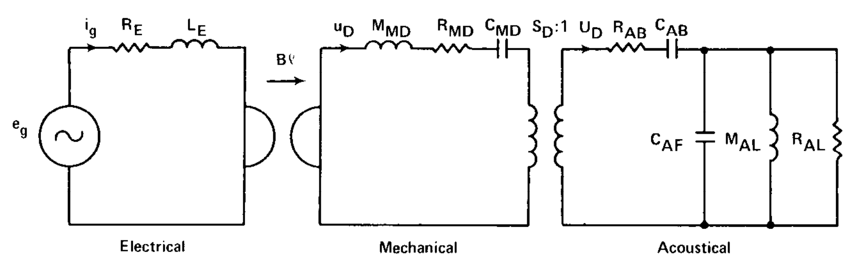

Fig. 2. Complete electro-mechano-acoustical equivalent circuit of horn-loaded system.

The acoustical output of the horn modeled by the circuit in Fig. 2 is the power dissipated in the resistor R_{AL}. To calculate this, it is convenient to transform the electrical and mechanical parts of the circuit into the acoustical part. This is given in Fig. 3, where the familiar impedance transformation for a transformer and that for a gyrator given in the Appendix have been used. The voice-coil inductance L_E is neglected in most low-frequency loudspeaker analyses and will be neglected in the following. When this is done, the circuit of Fig. 3 can be reduced to that given in Fig. 4, where

$$M_{AD} = \frac{M_{MD}}{S_D^2} \tag{4}$$

$$R_{AT} = \frac{B^2 l^2}{S_D^2 R_E} + \frac{R_{MD}}{S_D^2} + R_{AB} \tag{5}$$

$$C_{AT} = \frac{S_D^2 C_{MD} C_{AB}}{S_D^2 C_{MD} + C_{AB}}\,. \tag{6}$$

3 THE MID-FREQUENCY RANGE

It is the mid-frequency range that is of most interest because it is in this band that the system efficiency is defined. The equivalent circuit for this range is given in Fig. 5. A straightforward analysis of this circuit shows that the power delivered to the horn, that is, the power dissipated in R_{AL}, as a function of frequency is

$$P_{AR} = \frac{1}{2}\,\frac{B^2 l^2 e_g^2}{S_D^2 R_E^2}\,\frac{R_{AL}}{(R_{AT}+R_{AL})^2}\,|F(j\omega)|^2$$

$$= \frac{1}{2}\,\frac{e_g^2}{R_E}\,\frac{R_{AE} R_{AL}}{(R_{AT}+R_{AL})^2}\,|F(j\omega)|^2 \tag{9}$$

where R_{AE} is defined by Eq. (7) and where $F(s)$ is the system transfer function, which is given by

$$F(s) = \frac{(\omega_c/Q_{TC})s}{s^2 + (\omega_c/Q_{TC})s + \omega_c^2}\,. \tag{10}$$

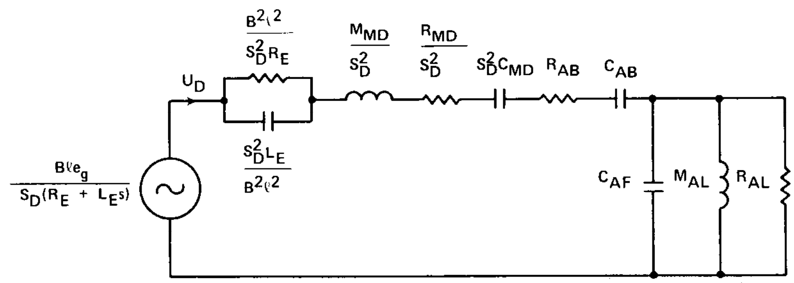

Fig. 3. Complete acoustical equivalent circuit of horn-loaded system.

The parameters ω_c and Q_{TC} are defined as the system resonant frequency and the total quality factor Q, respectively. These are given by

$$\omega_c = \frac{1}{\sqrt{M_{AD}C_{AT}}} \tag{11}$$

$$Q_{TC} = \frac{1}{R_{AT}+R_{AL}}\sqrt{\frac{M_{AD}}{C_{AT}}} \,. \tag{12}$$

In a conventional design the poles of $F(s)$ are real (or equivalently $Q_{TC} \le 0.5$) if the system is to have an acceptable bandwidth. In this case $F(s)$ can be written

$$F(s) = \frac{(\omega_L + \omega_H)s}{(s+\omega_L)(s+\omega_H)} \tag{13}$$

where $\omega_L < \omega_H$ and

$$\omega_c = \sqrt{\omega_L\omega_H} \tag{14}$$

$$Q_{TC} = \frac{\omega_c}{\omega_L + \omega_H} \,. \tag{15}$$

A typical plot of the magnitude of $F(j\omega)$ as a function of frequency is shown in Fig. 6. The frequencies ω_L and ω_H will be defined as the lower and upper minus 3-dB cutoff frequencies, respectively, of the system. These are given by

$$\omega_L = \frac{\omega_c}{2Q_{TC}}\,[1 - \sqrt{1-4Q_{TC}^2}] \tag{16}$$

$$\omega_H = \frac{\omega_c}{2Q_{TC}}\,[1 + \sqrt{1-4Q_{TC}^2}]. \tag{17}$$

The mid-band efficiency is defined as the ratio of acoustical power radiated to total electrical input power at $\omega = \omega_c$. Because the system transfer function is unity for $\omega = \omega_c$, the acoustical power radiated is given by Eq. (9) with the substitution $|F(j\omega_c)|^2 = 1$. The electrical input power must be calculated from the electrical equivalent circuit which is obtained by reflecting all mechanical and acoustical circuit elements in Fig. 2 back into the electrical circuit. For simplicity this will be done only for $\omega = \omega_c$, because this is the frequency at which the efficiency is defined. With this assumption and the preceding midband approximations, the electrical equivalent circuit is purely resistive. It is given in Fig. 7. In this figure R_{ES} is given by

$$R_{ES} = \frac{B^2l^2}{R_{MD} + S_D^2(R_{AB}+R_{AL})}$$

$$= \frac{B^2l^2/S_D^2}{R_{AL} + R_{AM}} \tag{18}$$

where R_{AM} is defined by Eq. (8). The electrical input power is thus given by

$$P_E = \frac{1}{2}\frac{e_g^2}{R_E + R_{ES}}$$

$$= \frac{1}{2}\frac{e_g^2}{R_E}\frac{R_{AL}+R_{AM}}{R_{AL}+R_{AT}} \,. \tag{19}$$

It thus follows that the mid-band efficiency is given by

$$\eta = \frac{P_{AR}}{P_E} = \frac{R_{AE}R_{AL}}{(R_{AT}+R_{AL})(R_{AM}+R_{AL})} \,. \tag{20}$$

Because R_{AL} is inversely proportional to the horn throat area S_T, it is of interest to determine what value of R_{AL} maximizes the efficiency. By differentiating Eq. (20) with respect to R_{AL} and equating the derivative to zero, the

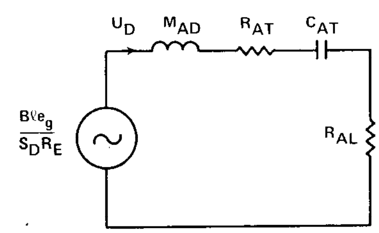

Fig. 5. Simplified mid-frequency acoustical circuit.

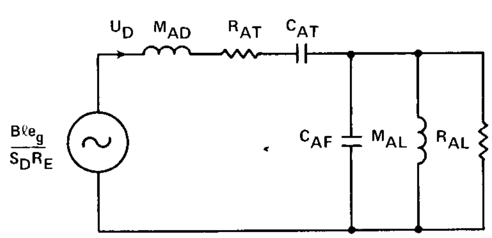

Fig. 4. Reduced circuit of Fig. 3 neglecting the voice-coil inductance.

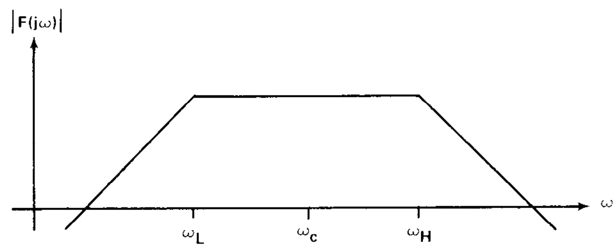

Fig. 6. Asymptotic plot of magnitude of Eq. (11) for mid-band response of horn-loaded system.

maximum efficiency occurs when

$$R_{AL} = \sqrt{R_{AM} R_{AT}} \; . \tag{21}$$

The required throat area can be determined from Eq. (2). Quite often, however, it may not be desired to design a horn system for maximum efficiency. Instead, it may be desired to design the system for maximum sensitivity, that is, for maximum acoustical output for a given electrical input voltage. By differentiating Eq. (9) with respect to R_{AL} and equating the derivative to zero, it follows that maximum sensitivity occurs when

$$R_{AL} = R_{AT} \; . \tag{22}$$

The values of R_{AL} for maximum efficiency and maximum sensitivity are not the same because R_{AL} affects the system electrical input impedance and, consequently, the value of input power P_E for a constant input voltage. Because the value of R_{AL} is smaller for maximum efficiency than for maximum sensitivity, it can be seen from Eq. (18) that the electrical input impedance is higher for the maximum efficiency condition. Thus a higher amplifier output voltage is required to drive this increased impedance if the same acoustical output is to be obtained as for the maximum sensitivity condition. However, the amplifier output power required to drive this impedance is less.

Three loss mechanisms can be identified in the midband acoustical circuit of Fig. 5. These are the power dissipated in the two parts of R_{AT} defined by Eqs. (7) and (8) and the power dissipated in R_{AL}. Thus three quality factors can be defined:

$$Q_{EC} = \frac{1}{\omega_c R_{AE} C_{AT}} \tag{23}$$

$$Q_{MC} = \frac{1}{\omega_c R_{AM} C_{AT}} \tag{24}$$

$$Q_{LC} = \frac{1}{\omega_c R_{AL} C_{AT}} \; . \tag{25}$$

These are the quality factors associated with the losses in R_{AE}, R_{AM}, and R_{AL}, respectively. The total quality factor for the circuit is given by

$$Q_{TC} = \frac{1}{\omega_c (R_{AE} + R_{AM} + R_{AL}) C_{AT}} \; . \tag{26}$$

With these definitions the efficiency expression of Eq. (20) can be rewritten:

$$\eta = \frac{Q_{TC}}{Q_{LC}} \frac{Q_{TC}}{Q_{EC} - Q_{TC}} \; . \tag{27}$$

The value of R_{AL} which maximizes η can be written

$$R_{AL} = \frac{1}{\omega_c C_{AT}} \sqrt{\frac{1}{Q_{MC}} \left(\frac{1}{Q_{EC}} + \frac{1}{Q_{MC}}\right)} \; . \tag{28}$$

The corresponding efficiency is

$$\eta = \frac{Q_{TC}}{Q_{EC} - Q_{TC}} \sqrt{\frac{1}{Q_{MC}} \left(\frac{1}{Q_{EC}} + \frac{1}{Q_{MC}}\right)} \; . \tag{29}$$

The value of R_{AL} which maximizes the sensitivity can be written

$$R_{AL} = \frac{1}{2 Q_{TC} \omega_c C_{AT}} \; . \tag{30}$$

The corresponding efficiency is

$$\eta = \frac{1}{2} \frac{Q_{TC}}{Q_{EC} - Q_{TC}} \; . \tag{31}$$

4 THE LOW-FREQUENCY RANGE

As defined in Section 2, the low-frequency range is that frequency band below which M_{AD} in Fig. 4 can be approximated by a short circuit and C_{AF} by an open circuit. The low-frequency acoustical equivalent circuit is given in Fig. 8. A generalized analysis of the low-frequency range is impossible because of the dependence of M_{AL} and R_{AL} on the horn geometry and on frequency. Therefore only the analysis for an infinite exponential horn will be presented.

When the expression for the input impedance to an infinite exponential horn given by Eq. (2) is used for the impedance of the parallel M_{AL} and R_{AL} combination in Fig. 8, it follows from a straightforward analysis that the real power delivered to this impedance is given by

$$P_{AR} = \frac{1}{2} \frac{B^2 l^2 e_g^2}{S_D^2 R_E^2} \frac{R_{AL} \sqrt{1 - (\frac{\omega}{\omega_0})^2}}{[R_{AT} + R_{AL} \sqrt{1 - (\frac{\omega}{\omega_0})^2}]^2 + [R_{AL} \frac{\omega}{\omega_0} - \frac{1}{\omega C_{AT}}]^2} \tag{32}$$

where ω_0 is the horn cutoff frequency. It can be seen that P_{AR} is zero for $\omega = \omega_0$. Because the horn input impedance is imaginary for $\omega < \omega_0$ [1], no radiation occurs at or below the cutoff frequency.

Klipsch [7] described a novel way to increase the acoustic output of the horn for frequencies just above the horn cutoff frequency that consisted of choosing the total compliance C_{AT} so that $\omega_0 R_{AL} C_{AT} = 1$ in Eq. (32). If this is true, the second term in parentheses in the denominator of

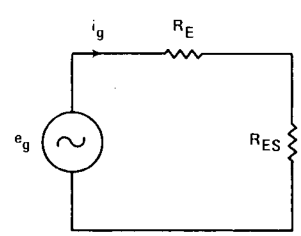

Fig. 7. Mid-band electrical equivalent circuit at system resonant frequency.

this equation is zero when $\omega = \omega_0$. By making the denominator smaller for frequencies at and just above ω_0, the value of P_{AR} is effectively increased. The condition that $\omega_0 R_{AL} C_{AT} = 1$ is equivalent to equating the mass reactance of the horn throat at the horn cutoff frequency to the reactance of the total system compliance at this frequency. This is a powerful technique for improving the low-frequency system efficiency. The technique was later refined by Plach and Williams [8], [9] who called it "reactance annulling."

5 THE HIGH-FREQUENCY RANGE

As defined in Section 2, the high-frequency range is that frequency band above which C_{AT} in Fig. 4 can be approximated by a short circuit and M_{AL} by an open circuit. The high-frequency equivalent circuit is given in Fig. 9. It follows from a straightforward analysis that the power delivered to R_{AL} in this circuit, that is, the acoustical power radiated, is given by

$$P_{AR} = \frac{1}{2}\frac{B^2 l^2 e_g^2}{S_D^2 R_E^2}\frac{R_{AL}}{(R_{AT}+R_{AL})^2}\,|F_H(j\omega)|^2 \tag{33}$$

where $F_H(s)$ is given by

$$F_H(s) = \frac{1}{\frac{R_{AL}}{R_{AT}+R_{AL}} M_{AD} C_{AF} s^2 + \left[\frac{R_{AL} R_{AT} C_{AF}}{R_{AL}+R_{AT}} + \frac{M_{AD}}{R_{AL}+R_{AT}}\right] s + 1} \,. \tag{34}$$

The upper minus 3-dB frequency for this expression would correspond to ω_H of the mid-frequency range if $C_{AF} = 0$. (This assumes that the impedance of C_{AT} in Fig. 5 can be neglected at ω_H.)

Proper choice of C_{AF} in Eq. (34) can extend the high-frequency response of the horn system. Although the analysis may not be accurate if the voice coil inductance cannot be neglected, the values of C_{AF} can be determined from a specification of the quality factor or Q for the second-order low-pass response of $F_H(s)$. Let this be written in the form $Q = 1/\sqrt{k}$. It then follows that the required value for C_{AF} is

$$\begin{aligned} C_{AF} &= \frac{M_{AD}}{R_{AL}}\,\frac{4}{\left(\sqrt{k(R_{AT}+R_{AL})} + \sqrt{k(R_{AT}+R_{AL}) - 4R_{AT}}\right)^2} \\ &= \frac{Q_{LC} C_{AT}}{\left(\frac{1}{Q_{TC}} - \frac{1}{Q_{LC}}\right)}\,\frac{4}{\left(\sqrt{k\left(1+\frac{R_{AL}}{R_{AT}}\right)} + \sqrt{k\left(1+\frac{R_{AL}}{R_{AT}}\right) - 4}\right)^2} \end{aligned} \tag{35}$$

where Q_{TC} and Q_{LC} are defined by Eqs. (12) and (25), respectively. The value of V_{AF} is given by $V_{AF} = \rho_0 c^2 C_{AF}$, where ρ_0 is the density of air and c is the velocity of sound in air. The corresponding upper minus 3-dB frequency is found from

$$\omega_3^2 = \frac{1}{8}\frac{\omega_c^2}{Q_{LC}}\left(\frac{1}{Q_{TC}} - \frac{1}{Q_{LC}}\right)\left(1+\frac{R_{AT}}{R_{AL}}\right)\left(\sqrt{k\left(1+\frac{R_{AL}}{R_{AT}}\right)} + \sqrt{k\left(1+\left(\frac{R_{AL}}{R_{AT}}\right) - 4\right)}\right)^2 \left(2 - k + \sqrt{(2-k)^2 + 4}\right). \tag{36}$$

The value of k which maximizes ω_3 is a function of the ratio R_{AL}/R_{AT}. For example, for the maximum sensitivity condition, $R_{AL} = R_{AT}$ and $Q_{LC} = 2Q_{TC}$. It follows then that the value of k which maximizes ω_3 for this condition is $k = 4$. Thus Q is 0.5, and this corresponds to a critically damped alignment for $F_H(s)$. The corresponding values for C_{AF} and ω_3 are

$$C_{AF} = 0.686\, Q_{TC}^2 C_{AT} \tag{37}$$

$$\omega_3 = \frac{1.099}{Q_{TC}}\,\omega_c \,. \tag{38}$$

6 SYSTEM DESIGN WITH A GIVEN DRIVER

In a system design for a given moving-coil driver a specification of ω_L and ω_H in addition to the driver parameters is sufficient to determine all system parameters. Let f_s be the driver free-air resonant frequency, V_{AS} its volume compliance, and Q_{ES}, Q_{MS}, and Q_{TS} its electrical, mechanical, and total quality factors, respectively. A design procedure based on these parameters is straightforward and is summarized in the following.

1) *Calculate the System Resonant Frequency* ω_c. From a specification of ω_L and ω_H, ω_c can be obtained from Eq. (14).

2) *Calculate the System Total Quality Factor* Q_{TC}. This is given by Eq. (15).

3) *Calculate the System Compliance Ratio* α. If it is assumed that the change in mass loading on the driver is negligible when it is mounted into the horn system, the required system compliance ratio is given by

$$\alpha = \left(\frac{\omega_c}{\omega_s}\right)^2 - 1 \tag{39}$$

where $\omega_s = 2\pi f_s$.

4) *Calculate the Box Volume* V_B. The effective volume of air in the back cavity behind the driver is given by

$$V_B = \frac{V_{AS}}{\alpha} \,. \tag{40}$$

5) *Calculate the System Electrical Quality Factor* Q_{EC}. Under the assumption stated in step 3, Q_{EC} is given by

$$Q_{EC} = \sqrt{\alpha + 1}\, Q_{ES}. \tag{41}$$

6) *Calculate the System Acoustical Load Quality Factor* Q_{LC}. It follows from the definitions of Q_{TC}, Q_{EC}, Q_{MC}, and Q_{LC} that Q_{LC} is calculated from

$$\frac{1}{Q_{LC}} = \frac{1}{Q_{TC}} - \frac{1}{Q_{EC}} - \frac{1}{Q_{MC}}. \tag{42}$$

To evaluate this, a value for Q_{MC} must be known. For zero losses in the box, this is given by $Q_{MC} = \sqrt{\alpha + 1}\, Q_{MS}$. However, the box will limit this to a lower value because of losses in R_{AB} of Fig. 2. Small [8] gives the typical values for Q_{MC} of 2 to 5 for systems using filling material and 5 to 10 for unfilled systems. His discussion was with reference to closed-box direct-radiator systems which would normally employ a larger V_B than the horn systems. Thus a larger value of Q_{MC} would be expected for the smaller box horn-loaded system. A more accurate method for determining Q_{MC} would be to measure it with the driver mounted on the box but not coupled to the horn. The measurement procedure is given in [10].

7) *Calculate the Horn Throat Area* S_T. The required horn throat area can be calculated from Eqs. (2) and (25). It is given by

$$S_T = \frac{\omega_c V_{AS} Q_{LC}}{(\alpha + 1)c} \tag{43}$$

where c is the velocity of sound in air.

8) *Calculate the System Efficiency* η. The system efficiency can be calculated from Eq. (27). The value should be greater than the efficiency for a closed-box direct-radiator system given in [10]. Otherwise the design has no merit over a closed-box system with the same driver.

9) *Calculate the Optimum Front Cavity Volume* V_{AF}. The front cavity volume which optimizes ω_3 of Eq. (36) should be determined. This is a function of the ratio of R_{AL} to R_{AT} given by

$$\frac{R_{AL}}{R_{AT}} = \frac{Q_{TC}}{Q_{LC} - Q_{TC}}. \tag{44}$$

10) *Design the Horn for Proper Reactance Annulling.*

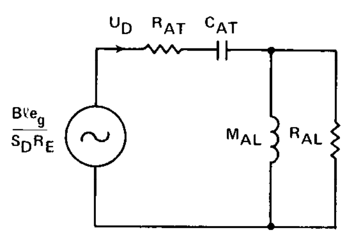

Fig. 8. Simplified low-frequency acoustical circuit.

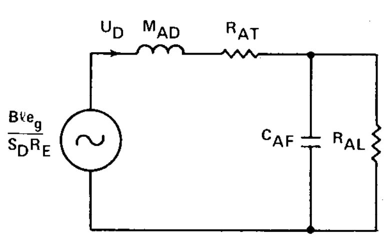

Fig. 9. Simplified high-frequency acoustical circuit.

The details of acoustic horn design are given in [1] and [2]. Reactance annulling can be achieved with an exponential horn by proper choice of the horn flare constant. In a proper design, reactance annulling must occur at a frequency less than ω_L, that is, the horn cutoff frequency must be less than ω_L if ω_L is to be the lower minus 3-dB frequency of the system.

In the following a system design example is given:

6.1 System Specifications

$$f_L = \frac{\omega_L}{2\pi} = 40 \text{ Hz}$$

$$f_H = \frac{\omega_H}{2\pi} = 400 \text{ Hz}.$$

6.2 Driver Specifications

f_s = 45 Hz
Q_{TS} = 0.20
Q_{ES} = 0.21
Q_{MS} = 6.00
V_{AS} = 0.17 m^3.

6.3 Computed System Parameters

1) $f_c = \dfrac{\omega_c}{2\pi} = 126.5$ Hz.
2) $Q_{TC} = 0.29$.
3) $\alpha = 6.90$.
4) $V_B = 2.46 \times 10^{-2}\ m^3$.
5) $Q_{EC} = 0.59$.
6) $Q_{LC} = 0.60$ (assume $Q_{MC} = 10$).
7) $S_T = 0.0297\ m^2$.
8) $\eta = 46.7\%$.
9) The maximum value for ω_3 occurs for $k = 4.1$. The values for ω_3 and V_{AF} are

$$f_3 = \frac{\omega_3}{2\pi} = 476 \text{ Hz}$$

$$V_{AF} = 1.26 \times 10^{-3}\ m^3.$$

10) Reactance annulling can be achieved by designing the horn for a cutoff frequency of

$$f_0 = \frac{(\alpha + 1)S_T c}{2\pi V_{AS}} = 75.8 \text{ Hz}.$$

Because this frequency is greater than ω_L, reactance annulling cannot be used with this design.

7 SYSTEM DESIGN FROM SPECIFICATIONS

In a system design from specifications, the parameters of the driver are obtained which will cause the system to meet the desired specifications. When designing a system from specifications, the most important considerations are its size, bandwidth, and efficiency. In addition the input impedance is also considered to be a necessary specification because it should fall into the range of impedances that audio power amplifiers are designed to drive. If it is too high, for example, the amplifier may be forced into clipping before acceptable acoustic outputs are obtained. Therefore the design procedure to be presented here allows the designer to constrain this parameter. It will be assumed that a maximum sensitivity design is desired rather than a maximum efficiency one. The design procedure for a maximum efficiency design is similar but involves slightly more complicated expressions.

1) *Establish the System Specifications*. The size of the system is affected by the volume V_B of the box behind the driver, the piston area S_D of the driver, and the horn throat area S_T. The performance of the system is determined by its lower and upper cutoff frequencies ω_L and ω_H, respectively. The proper interface to the amplifier is determined by the input resistance R_{in} at the system resonant frequency and the dc voice-coil resistance R_E. The design procedure requires a specification of these seven parameters.

2) *Calculate the System Resonant Frequency* ω_c. From a specification of ω_L and ω_H, ω_c can be obtained from Eq. (14).

3) *Calculate the System Total Quality Factor* Q_{TC}. This is given by Eq. (15).

4) *Calculate the System Electrical Quality Factor* Q_{EC}. For the maximum sensitivity design, $Q_{LC} = 2Q_{TC}$. Q_{EC} is thus calculated from

$$\frac{1}{Q_{EC}} = \frac{1}{2Q_{TC}} - \frac{1}{Q_{MC}} . \tag{45}$$

The value that is used for Q_{MC} is dependent on the box losses. This was discussed in step 6 of the preceding design procedure.

5) *Calculate the Total System Volume Compliance* V_{AT}. The required system volume compliance is obtained from Eq. (25) with the substitution $R_{AL} = \rho_0 c/S_T$, $Q_{LC} = 2Q_{TC}$, and $C_{AT} = V_{AT}/\rho_0 c^2$. The result is

$$V_{AT} = \frac{cS_T}{2Q_{TC}\omega_c} . \tag{46}$$

6) *Calculate the System Compliance Ratio* α. The compliance ratio is given by

$$\alpha = \frac{V_{AT}}{V_B - V_{AT}} . \tag{47}$$

7) *Calculate the Driver Free-Air Electrical Quality Factor* Q_{ES}. Eq. (41) can be used to calculate Q_{ES}. It is assumed that the assumption stated in step 3 of the preceding synthesis procedure holds.

8) *Calculate the Driver Free-Air Resonant Frequency* ω_s. If the driver electrical losses predominate over the mechanical losses, ω_s is approximately given by

$$\omega_s = \frac{\omega_c}{\sqrt{\alpha + 1}} . \tag{48}$$

The conditions under which this relation holds are discussed in [10].

9) *Calculate the Driver Free-Air Volume Compliance* V_{AS}. Eq. (40) may be solved for V_{AS} from the specified V_B and calculated α.

10) *Calculate the Driver Free-Air Mechanical Compliance* C_{MD}. The required driver mechanical compliance is obtained from its volume compliance by the relation [10]

$$C_{MD} = \frac{V_{AS}}{\rho_0 c^2 S_D^2} . \tag{49}$$

11) *Calculate the Driver Moving Mechanical Mass* M_{MD}. The total required moving mass is given by [10]

$$M_{MD} = \frac{1}{\omega_s^2 C_{MD}} . \tag{50}$$

12) *Calculate the Driver Electrical Resistance* R_{ES} *Due to Suspension Losses*. If R_{in} is the desired input resistance at ω_c and R_E is the dc voice-coil resistance, R_{ES} is given by

$$R_{ES} = R_{in} - R_E . \tag{51}$$

13) *Calculate the Driver Bl Product*. The Bl or motor product of the driver is obtained from Eqs. (18), (23), (24), and (26). The relation is

$$Bl = [S_D^2 R_{ES} (\frac{1}{Q_{TC}} - \frac{1}{Q_{EC}}) \frac{\rho_0 c^2}{\omega_c V_{AT}}]^{\frac{1}{2}} . \tag{52}$$

14) *Calculate the System Efficiency* η. This is given by Eq. (31).

15) *Calculate the Volume* V_{AF} *of the Front Cavity*. From Eq. (37) the value of V_{AF} for the maximum sensitivity design is

$$V_{AF} = 0.686 Q_{TC}^2 V_{AT} \tag{53}$$

The new upper minus 3-dB frequency is given by Eq. (38).

16) *Design the Horn for Proper Annulling*. This is discussed in step 10 of the preceding synthesis procedure.

In the following a system design example is given:

7.1 System Specifications

1) $V_B = 0.015\ m^3$
$S_D = 0.073\ m^2$
$S_T = 0.018\ m^2$
$f_L = 40$ Hz
$f_H = 400$ Hz
$R_{in} = 8\ \Omega$
$R_E = 4\ \Omega$.

7.2 Computed Driver and System Parameters

2) $f_c = \frac{\omega_c}{2\pi} = 126.5$ Hz.
3) $Q_{TC} = 0.29$.
4) $Q_{EC} = 0.62$ (assume $Q_{MC} = 10$).
5) $V_{AT} = 0.0135\ m^3$.
6) $\alpha = 8.81$.
7) $Q_{ES} = 0.20$.
8) $f_s = \frac{\omega_s}{2\pi} = 40.4$ Hz.
9) $V_{AS} = 0.132\ m^3$.
10) $C_{MD} = 1.76 \times 10^{-4}$ m/N.
11) $M_{MD} = 0.0882$ kg.
12) $R_{ES} = 4\ \Omega$.
13) $Bl = 22.6$ T·m.
14) $\eta = 43.9\%$.
15) $V_{AF} = 7.79 \times 10^{-4}\ m^3$
$f_3 = \frac{\omega_3}{2\pi} = 479$ Hz.

16) For proper reactance annulling, the horn cutoff frequency must be

$$f_0 = \frac{(\alpha + 1)S_T c}{2\pi V_{AS}} = 73.5 \text{ Hz}.$$

Because this is higher than f_L, proper reactance annulling cannot be achieved without a change in the system specifications.

8 CONCLUSIONS

The basic electroacoustic theory of the horn loudspeaker system has been reviewed. Expressions for the acoustic output from the system have been developed for the low-, mid-, and high-frequency bands. Two system synthesis procedures have been presented, one based on a design with a given driver and the other based on system specifications. Numerical examples have been presented to illustrate application of the synthesis procedures.

9 APPENDIX

THE GYRATOR MODEL OF A MOVING-COIL LOUDSPEAKER DRIVER

The gyrator is a circuit element which makes a convenient model for the moving-coil loudspeaker voice coil. The advantage of this model over the standard electroacoustic transformer model is that the confusing parallel-element admittance and mobility circuits can be circumvented. Because the author had seen no applications in the literature of gyrators to loudspeaker analyses, this appendix is included.

The gyrator is a passive and lossless circuit element that is somewhat like the ideal transformer, except that it interchanges the roles of current and voltage at its output terminal pair. The basic circuit symbol for a gyrator is given in Fig. 10. The equations which define its terminal behavior are

$$v_1 = Ri_2 \tag{54}$$

$$v_2 = Ri_1 \tag{55}$$

where R is called the gyrator impedance. If the gyrator is terminated in an impedance Z_L, then $v_2 = i_2 Z_L$. Simultaneous solution of this equation with Eqs. (54) and (55) yields the input impedance to the loaded gyrator:

$$Z_{in} = \frac{v_1}{i_1} = \frac{R^2}{Z_L}. \tag{56}$$

Thus the input impedance is the constant R^2 times the reciprocal of the load impedance.

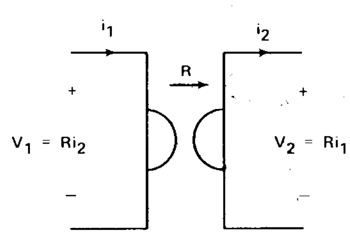

Fig. 10. Circuit symbol of a gyrator used as a two-terminal-pair network.

If the gyrator is driven at its input terminals from a generator with open-circuit voltage v_s and output impedance Z_s, then $v_1 = v_s - i_1 Z_s$. Simultaneous solution of this equation with Eqs. (54) and (55) yields the relationship between v_2 and i_2:

$$v_2 = \frac{R}{Z_s} v_s - \frac{R^2}{Z_s} i_2. \tag{57}$$

This is equivalent to a voltage source of value $(R/Z_s)v_s$ in series with the impedance R^2/Z_s. The equivalent input and output circuits for these two cases are given in Fig. 11.

The basic electroacoustic equations which govern the moving-coil loudspeaker involve the force f_D generated on the voice coil when a current i flows in it and the back electromotive force e generated when the voice coil moves. The equations are [1]:

$$f_D = Bli \tag{58}$$

$$e = Blu_D \tag{59}$$

where B is the magnetic field flux density in the air gap, l is the length of voice-coil wire in the magnetic field, and u_D is the velocity with which the voice coil moves. The gyrator can be used to represent these equations as shown in Fig. 12. In the more familiar transformer representation of these equations the force is the current and the velocity is the voltage at the transformer secondary. This requires the use of parallel-element mobility-type mechanical equivalent circuits. In contrast the gyrator permits forming the more familiar series-element impedance type circuits directly without the need to form circuit duals.

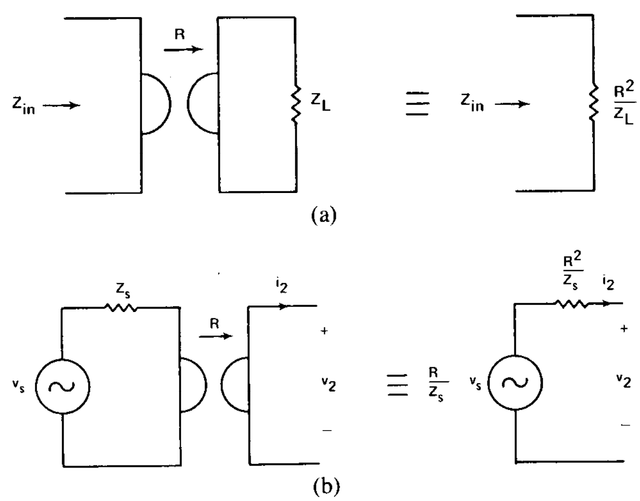

Fig. 11. Equivalent input and output circuits of the gyrator. (a) Equivalent input circuit. (b) Equivalent output circuit.

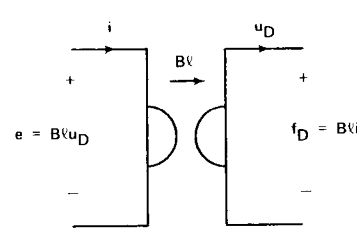

Fig. 12. Gyrator model for the electromagnetic-mechanical transducer of a moving-coil loudspeaker driver.

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[3] A. N. Thiele, "Loudspeakers in Vented Boxes, Parts I and II," *J. Audio Eng. Soc.*, vol. 19, pp. 382–391 (1971 May); pp. 471–483 (1971 June).

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[5] R. H. Small, "Suitability of Low-Frequency Drivers for Horn-Loaded Loudspeaker Systems," presented at the 57th Convention of the Audio Engineering Society, Los Angeles, 1977 May, preprint 1251.

[6] D. B. Keele, "Low-Frequency Horn Design Using Thiele/Small Driver Parameters," presented at the 57th Convention of the Audio Engineering Society, Los Angeles, 1977 May, preprint 1250.

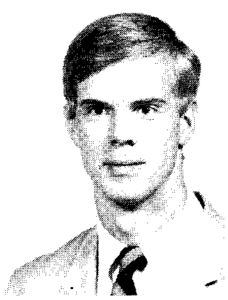
[7] P. W. Klipsch, "A Low-Frequency Horn of Small Dimensions," *J. Acoust. Soc. Am.*, vol. 13, pp. 137–144 (1941 Oct.).

[8] D. J. Plach, "Design Factors in Horn-Type Speakers," *J. Audio Eng. Soc.*, vol. 1, pp. 276–281 (1953 Oct.).

[9] D. J. Plach and P. B. Williams, "Reactance Annulling for Horn-Type Loudspeakers," *Radio-Electron. Eng.* pp. 15–18 (1955 Feb.).

[10] R. H. Small, "Closed-Box Loudspeaker Systems, Part I: Analysis," *J. Audio Eng. Soc.*, vol. 20, pp. 798–808 (1972 Dec.); "Part II: Synthesis," vol. 21, pp. 11–18 (1973 Jan./Feb.).

THE AUTHOR



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In 1964 he worked at NASA in Hampton, Virginia. From 1965 to 1968 he served as an officer in the U.S. Air Force. Since 1972 he has been a faculty member at the Georgia Institute of Technology where he is presently associate professor of electrical engineering. His interests are applied electromagnetics, audio and electroacoustics, and electronic circuit design.

Dr. Leach is a member of the IEEE, the Audio Engineering Society, Sigma Xi, Tau Beta Pi, Eta Kappa Nu, Omicron Delta Kappa, and Phi Beta Kappa.

LETTERS TO THE EDITOR

COMMENTS ON REACTANCE ANNULLING IN HORN-LOADED LOUDSPEAKER SYSTEMS

In a recent paper,[1] Leach mentions that the method of using system compliance reactance to cancel horn throat mass reactance in the region above horn flare cutoff is "a powerful technique." I could not agree more since this is perhaps the only case where the laws of physics aid the designer rather than frustrating him. However, Leach credits Klipsch[2] as the originator of the idea, and while his work was no doubt independent, Wente and Thuras[3] used the same technique in the low-frequency horns for the Philadelphia–Washington "Audio Perspective" demonstration in the early 1930s.

In the interest of historical accuracy I believe that it was actually the late Al Thuras who first became aware of the potential of the method.[4] His patent[5] contains a complete and lucid description in the specification. It is strange, however, that the technique was completely ignored in the claims of his patent. As Bell Labs. was wont to do, perhaps there is a separate patent covering this particular innovation.

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Author's Reply

I greatly appreciate Mr. McClain's comments on my paper and his historical perspective on the use of reactance annulling in horn-loaded loudspeaker systems. When I wrote the paper I had intended to cover only the specification of the driver small-signal parameters in a system design and no details that pertain to acoustic horn design. However, because reactance annulling is such a simple concept, I did include a brief description of it and its motivation. When I attempted to generate two design examples to illustrate the synthesis procedures, I was unable to incorporate the concept as I understood it into my examples. That is, I could not force reactance annulling to occur at or below the desired lower midband cutoff frequency. I was convinced that the laws of physics were working against me.

After the paper was published, I made a second attempt to incorporate reactance annulling at or below the lower midband cutoff frequency into the synthesis procedures. However, I met with no success. Indeed, I found it straightforward to show that it is impossible to do with an exponential horn. I came to the conclusion that the lower midband cutoff frequency, which I denoted by f_L, must be specified artificially low in order to obtain a reactance annulling frequency that occurs at or below the desired lower cutoff frequency for the system.

Although this modified procedure could be used for an acceptable design, the artificially low choice for f_L would result in a reduced efficiency. In addition, it results in an electroacoustic driver system which is capable of a wider bandwidth than the horn to which it is coupled. Fortunately this problem can be eliminated by choice of a hyperbolic horn[6,7] rather than an exponential horn. The following argument follows Plach[7] and Salmon.[8] Salmon's patent states, "Where the cut-off frequency of the horn (the frequency below which the acoustic resistance of the horn is zero) is below the resonant frequency of the driving motor with the impedance on the horn side of the diaphragm reduced to zero, it is possible to substantially match and annul the reactance of the driver, which, in this range, is capacitative, by means of a properly selected member of the horn family herein described." The parameter M in the following is identical to Plach's and Salmon's parameter T.

The general formula for the cross-sectional area of the hyperbolic family of horns is given by

$$S = S_T \left(\cosh\left(\frac{x}{x_0}\right) + M \sinh\left(\frac{x}{x_0}\right)\right)^2 \qquad (1)$$

where S_T is the cross-sectional area of the throat, x is the axial distance from the throat, x_0 is the reference axial

[1] W. M. Leach, Jr., "On the Specification of Moving-Coil Drivers for Low-Frequency Horn-Loaded Loudspeakers," *J. Audio Eng. Soc.*, vol. 27, pp. 950–959 (1979 Dec.).

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[3] E. C. Wente and A. L. Thuras, "Auditory Perspective—Loudspeakers and Microphones," *Trans. AIEE*, vol. 53, pp. 17–24, (1934 Jan.); reprinted *J. Audio Eng. Soc.*, vol. 26, pp. 518–525 (1978 July/Aug.). (See first paragraph, right-hand column, page 521.)

[4] In accordance with still another feature of this invention, the chamber adjacent one surface of the diaphragm is proportioned so that the stiffness thereof is substantially equal in magnitude to the mean reactance component of the horn load impedance throughout a range of audio frequencies, whereby the reactance component of the horn load impedance is substantially neutralized throughout this range of frequencies (from Thuras[5]).

[5] A. L. Thuras, U. S. Patent 2,037,185, filed March 28, 1933, issued April 14, 1936. Note that this patent had not been issued when the Wente and Thuras paper was published.

[6] L. L. Beranek, *Acoustics* (McGraw-Hill, New York, 1954), ch. 9.

[7] D. J. Plach, "Design Factors in Horn-Type Speakers," *J. Audio Eng. Soc.*, vol. 1, pp. 276–281 (1953 Oct.), and *Loudspeakers* (Audio Engineering Society, New York, 1980).

[8] V. Salmon, U.S. Patent 2,338,262, filed 1942 July 23.

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distance, and M is a parameter such that $0 \leq M \leq 1$. The acoustical throat impedance of the hyperbolic family of horns is given by

$$Z_{AL} = R_{AL} \frac{\sqrt{1 - (\omega_0/\omega)^2} + jM(\omega_0/\omega)}{1 - (1 - M^2)(\omega_0/\omega)^2} \quad (2)$$

where $R_{AL} = \rho_0 c/S_T$ and ω_0 is the angular cutoff frequency of the horn given by $\omega_0 = c/x_0$. Both Eqs. (1) and (2) reduce to the familiar relations for an exponential horn when $M = 1$.

The expression corresponding to Eq. (32) of my paper for the acoustical power delivered to the hyperbolic horn in the low-frequency range is too cumbersome to give here. However, it is straightforward to show that the reactance annulling condition is

$$\omega_0 R_{AL} C_{AT} = M, \quad (3)$$

which reduces to

$$f_0 = \frac{\omega_0}{2\pi} = \frac{M(\alpha + 1) S_T c}{2\pi V_{AS}} \quad (4)$$

where the undefined parameters in these equations are defined in my paper. It can be seen from Eq. (4) that the M parameter can be varied to make the reactance annulled horn cutoff frequency f_0 correspond to the desired lower midband cutoff frequency f_L. This condition can be stated succinctly as follows: With V_B (the box volume) chosen to resonate the system at $f_c = (f_L f_H)^{1/2}$, the system cannot, in general, be also resonant, i.e., reactance annulled, at $f_1 \leq f_L$ unless the horn reactance is adjusted by appropriate choice of the parameter M.

As an example, a hyperbolic horn with $M = 0.51$ would have a reactance annulled cutoff frequency of 40 Hz for the example in Section 6.3 of my paper. A hyperbolic horn with $M = 0.54$ would have a reactance annulled cutoff frequency of 40 Hz for the example in Section 7.2. In each case the reactance annulled cutoff frequency of the horn corresponds to the specified lower midband cutoff frequency of the system. Figures 1 and 2 are plots of the normalized acoustical throat resistance and reactance, respectively, of the hyperbolic horn for several values of the parameter M. It can be seen that the resistive loading is much more uniform for $0.5 \leq M \leq 1/\sqrt{2}$ than for the exponential horn for which $M = 1$. Thus the $M = 0.51$ and $M = 0.54$ values in the preceding examples would result in an almost optimal horn loading for the design examples given in my paper.

While I have the opportunity and because so many people have shown an interest by contacting me, I would like to point out the errors that appeared in my paper. The Q_{TC} in the numerator of Eq. (29) should be Q_{TC}^2. The square-root radical in the denominator of Eq. (32) should not extend over the square bracket. The square-root radical in the denominator of the first line of Eq. (35) should extend over the $-4R_{AT}$. The second square-root radical in the second line of Eq. (35) and the second square-root radical in Eq. (36) should extend over the -4 in each.

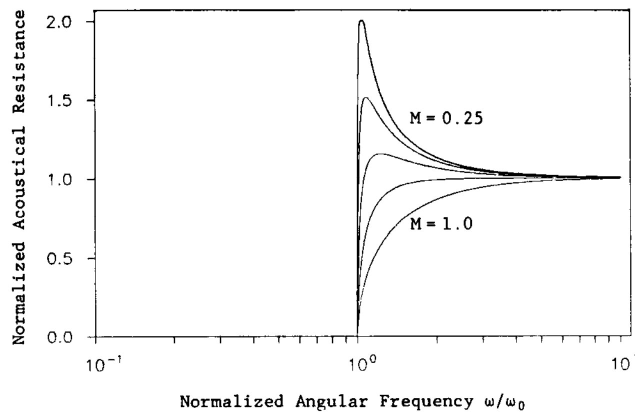

Fig. 1. Normalized acoustical throat resistance of the hyperbolic horn versus normalized angular frequency ω/ω_0, where ω_0 is the angular cutoff frequency of the horn. Curves are given for $M = 0.25, 0.354, 0.5, 0.707$, and 1.0.

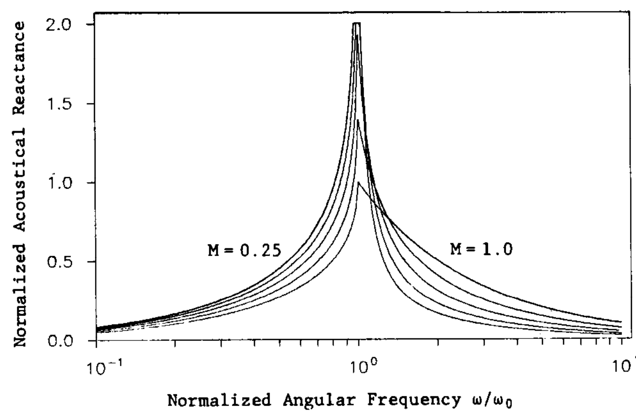

Fig. 2. Normalized acoustical throat reactance of the hyperbolic horn versus normalized angular frequency ω/ω_0, where ω_0 is the angular cutoff frequency of the horn. Curves are given for $M = 0.25, 0.354, 0.5, 0.707$, and 1.0. For $\omega/\omega_0 < 1.0$, the reactance values are capacitive. for $\omega/\omega_0 > 1.0$, the reactive values are inductive.

ACKNOWLEDGMENT

The author would like to gratefully acknowledge the assistance of Mr. Richard F. Long of Richard Long and Associates, New York, in finding the errors in the author's paper and for suggesting the use of the hyperbolic family of horns to achieve reactance annulling. He would also like to acknowledge the aid of the reviewers in providing additional background material on reactance annulling.

COMMENTS ON THE HISTORY OF MAGNETIC RECORDING

In all the accounts of the history of magnetic recording that I have come across until now (e.g., the excellent and personal accounts of John T.Mullin), the development is being described as having been continuous and universal until World War II, when high-frequency bias was developed and kept secret in Germany. It is said that high-frequency bias had not been rediscovered and made commercially feasible until 1945, when analy-

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