Feedback Amplifiers

Collection of Solved Problems

A collection of solved feedback amplifier problems can be found at the below link. The solutions are based on the use of the Mason Flow Graph described below.

http://users.ece.gate.edu/~mleach/ece3050/notes/feedback/FBExamples.pdf

Basic Description of Feedback

A feedback amplifier is one in which the output signal is sampled and fed back to the input to form an error signal that drives the amplifier. The basic block diagrams of non-inverting and inverting feedback amplifiers are shown in Fig. 1. Depending on the type of feedback, the variables \( x, y, \) and \( z \) are voltages or currents. The diagram in Fig. 1(a) represents a non-inverting amplifier. The summing junction at its input subtracts the feedback signal from the input signal to form the error signal \( z = x - by \) which drives the amplifier. If the amplifier has an inverting gain, the feedback signal must be added to the input signal in order for the feedback to be negative. This is illustrated in Fig. 1(b). The summing junction at the input adds the feedback signal to the input signal to form the error signal \( z = x + by \). In both diagrams, the gain around the loop is negative and equal to \( -bA \), where both \( A \) and \( b \) are positive real constants. Because the loop-gain is negative, the feedback is said to be negative. If the gain around the loop is positive, the amplifier is said to have positive feedback which causes it to be unstable.

![Feedback Amplifier Block Diagrams](image-url)

Figure 1: Feedback amplifier block diagrams. (a) Non-inverting. (b) Inverting.

In the non-inverting amplifier of Fig. 1(a), the error signal is given by \( z = x - by \). The output signal can be written.

\[
y = Az = A(x - by)
\]  

This can be solved for the gain to obtain

\[
\frac{y}{x} = \frac{A}{1 + bA}
\]  

We see that the effect of the feedback is to reduce the gain by the factor \( (1 + bA) \). This factor is called the “amount of feedback”. It is often specified in dB by the relation \( 20 \log |1 + bA| \).

In the inverting amplifier of Fig. 1(b), the error signal is given by \( z = x + by \). When \( x \) goes positive, \( y \) goes negative, so that the error signal represents a difference signal. The output signal can be written

\[
y = -Az = -A(x + by)
\]
This can be solved for the gain to obtain
\[ \frac{y}{x} = \frac{-A}{1 + bA} \] (4)

We see that the amount of feedback for the inverting amplifier is the same as for the non-inverting amplifier.

If \( A \) is large enough so that \( bA >> 1 \), the gain of the non-inverting amplifier given by Eq. (2) can be approximated by
\[ \frac{y}{x} \approx \frac{A}{bA} = \frac{1}{b} \] (5)
The gain of the inverting amplifier given by Eq. (4) can be approximated by
\[ \frac{y}{x} \approx \frac{-A}{bA} = -\frac{1}{b} \] (6)

These are important results, for they show that the gain is set by the feedback network and not by the amplifier. In practice, this means that an amplifier without feedback can be designed without too much consideration of what its gain will be as long as the gain is high enough. When feedback is added, the gain can be reduced to any desired value by the feedback network.

The product \( bA \) in Eqs. (2) and (4) must be dimensionless. Thus if \( A \) is a voltage gain (voltage in-voltage out) or a current gain (current in-current out), then \( b \) must be dimensionless. If \( A \) is a transconductance gain (voltage in-current out), \( b \) must have the units of ohms (\( \Omega \)). If \( A \) is a transresistance gain (current in-voltage out), \( b \) must have the units siemens (\( f \)). How these are determined is illustrated below.

We have assumed so far that the gains \( A \) and \( b \) are positive real constants. In general, the gains are phasor functions of frequency. This leads to a stability problem in feedback amplifiers.

As frequency is increased, \( |A| \) must eventually decrease because no amplifier can have an infinite bandwidth. The decrease in \( |A| \) is accompanied with a phase shift so that \( bA \) can be equal to a negative real number at some frequency. Suppose that \( bA = -1 \) at some frequency. Eqs. (3) and (4) show that the gain becomes infinite at that frequency. An amplifier with an infinite gain at any frequency can put out a signal at that frequency with no input signal. In this case, the amplifier is said to oscillate. It can be shown that an amplifier will oscillate if \( |bA| \geq 1 \) at any frequency for which \( bA \) is a negative real number, i.e. the phase of \( bA \) is \( \pm 180^\circ \).

In the block diagrams of Fig. 1, the input and output variables can be modeled as either a voltage or a current. It follows that there are four combinations of inputs and outputs that represent the possible types of feedback. These are summarized in Table 1 where the names for each are given. These names come from the way that the feedback network connects between the input and output stages. This is explained in the following for each type of feedback.

<table>
<thead>
<tr>
<th>Name</th>
<th>Input Variable x</th>
<th>Output Variable y</th>
<th>Error Variable z</th>
<th>Forward Gain A</th>
<th>Feedback Factor b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series-Shunt</td>
<td>Voltage v</td>
<td>Voltage v</td>
<td>Voltage v</td>
<td>Voltage Gain</td>
<td>Dimensionless</td>
</tr>
<tr>
<td>Shunt-Shunt</td>
<td>Current i</td>
<td>Voltage v</td>
<td>Current i</td>
<td>Transresistance</td>
<td>siemens (( f ))</td>
</tr>
<tr>
<td>Series-Series</td>
<td>Voltage v</td>
<td>Current i</td>
<td>Voltage v</td>
<td>Transconductance</td>
<td>ohms (( \Omega ))</td>
</tr>
<tr>
<td>Shunt-Series</td>
<td>Current i</td>
<td>Current i</td>
<td>Current i</td>
<td>Current Gain</td>
<td>Dimensionless</td>
</tr>
</tbody>
</table>

The Mason Signal-Flow Graph

The analysis of feedback amplifiers is facilitated by the use of the Mason signal-flow graph. A signal-flow graph is a graphical representation of a set of linear equations which can be used to
write by inspection the solution to the set of equations. For example, consider the set of equations

\[x_2 = Ax_1 + Bx_2 + Cx_5\]  
(7)

\[x_3 = Dx_1 + Ex_2\]  
(8)

\[x_4 = Fx_3 + Gx_5\]  
(9)

\[x_5 = Hx_4\]  
(10)

\[x_6 = Ix_3\]  
(11)

where \(x_1\) through \(x_6\) are variables and \(A\) through \(I\) are constants. These equations can be represented graphically as shown in Fig. 2. The graph has a node for each variable with branches connecting the nodes labeled with the constants \(A\) through \(I\). The node labeled \(x_1\) is called a source node because it has only outgoing branches. The node labeled \(x_6\) is called a sink node because it has only incoming branches. The path from \(x_1\) to \(x_2\) to \(x_3\) to \(x_6\) is called a forward path because it originates at a source node and terminates at a non-source node and along which no node is encountered twice. The path gain for this forward path is \(AEI\). The path from \(x_2\) to \(x_3\) to \(x_4\) to \(x_5\) is called a feedback path because it originates and terminates on the same node and along which no node is encountered more than once. The loop gain for this feedback path is \(EFHC\).

![Flow graph for the equations.](image)

Mason’s formula can be used to calculate the transmission gain from a source node to any non-source node in a flow graph. The formula is

\[T = \frac{1}{\Delta} \sum_k P_k \Delta_k\]  
(12)

where \(P_k\) is the gain of the \(k\)th forward path, \(\Delta\) is the graph determinant, and \(\Delta_k\) is the determinant with the \(k\)th forward path erased. The determinant is given by

\[\Delta = 1 - \text{(sum of all loop gains)} + \left(\text{sum of the gain products of all possible combinations of two non-touching loops}\right) - \left(\text{sum of the gain products of all possible combinations of three non-touching loops}\right) + \left(\text{sum of the gain products of all possible combinations of four non-touching loops}\right) - \cdots\]  
(13)
For the flow graph in Fig. 2, the objective is to solve for the gain from node $x_1$ to node $x_6$. There are two forward paths from $x_1$ to $x_6$ and three loops. Two of the loops do not touch each other. Thus the product of these two loop gains appears in the expression for $\Delta$. The path gains and the determinant are given by

$$P_1 = AEI$$  \hfill (14)  

$$P_2 = DI$$  \hfill (15)  

$$\Delta = 1 - (B + CEFH + GH) + B \times GH$$  \hfill (16)  

Path $P_1$ touches two loops while path $P_2$ touches one loop. The determinants with each path erased are given by

$$\Delta_1 = 1 - GH$$  \hfill (17)  

$$\Delta_2 = 1 - (B + GH) + B \times GH$$  \hfill (18)  

Thus the overall gain from $x_1$ to $x_6$ is given by

$$\frac{x_6}{x_1} = \frac{AEI \times (1 - GH) + DI \times [1 - (B + GH) + B \times GH]}{1 - (B + CEFH + GH) + B \times GH}$$  \hfill (19)  

This equation can also be obtained by algebraic solution of the equations in Eqs. (7) through (11).

**Review of Background Theory**

This section summarizes several BJT small-signal ac equivalent circuits which are used to write the circuit equations in the following sections. Fig. 3(a) shows an npn BJT with Thévenin sources connected to its base and emitter and a load resistor connected to its collector. First we define the emitter intrinsic resistance $r_e$, the collector-emitter resistance $r_0$, the resistance $r_e'$, and the transconductance $G_m$. These are given by

$$r_e = \frac{V_T}{I_E}$$  \hfill (20)  

$$r_0 = \frac{V_{CE} + V_A}{I_C}$$  \hfill (21)  

$$r'_e = \frac{R_{th} + r_x}{1 + \beta} + r_e$$  \hfill (22)  

$$G_m = \frac{a}{r'_e + R_{te}}$$  \hfill (23)  

where $V_T$ is the thermal voltage, $I_E$ is the emitter bias current, $V_{CE}$ is the quiescent collector-emitter voltage, $V_A$ is the Early voltage, $I_C$ is the quiescent collector current, $r_x$ is the base spreading resistance, $\beta$ is the base-collector current gain, and $\alpha$ is the emitter-collector current gain. The latter two are related by $\alpha = \beta / (1 + \beta)$ and $\beta = \alpha / (1 - \alpha)$.

The small-signal ac Thévenin equivalent circuit seen looking into the base is shown in Fig. 3(b), where $v_{b(oc)}$ and $r_{ib}$ are given by

$$v_{b(oc)} = v_{te} \frac{r_0 + R_{tc}}{R_{te} + r_0 + R_{tc}}$$  \hfill (24)  

$$r_{ib} = r_x + (1 + \beta) r_e + R_{te} \frac{(1 + \beta) r_0 + R_{tc}}{r_0 + R_{te} + R_{tc}}$$  \hfill (25)  

4
The small-signal ac Thévenin equivalent circuit seen looking into the emitter is shown in Fig. 3(c), where \( v_{e(oc)} \) and \( r_{ie} \) are given by

\[
v_{e(oc)} = v_{tb} \frac{r_0 + R_{tc}/(1 + \beta)}{r_e' + r_0 + R_{tc}/(1 + \beta)}
\]

\[
r_{ie} = \frac{r_0 + R_{tc}}{r_e'r_{ie} + r_0 + R_{tc}/(1 + \beta)}
\]

The small-signal ac Norton equivalent circuit seen looking into the collector is shown in Fig. 3(d), where \( i_{c(se)} \) and \( r_{ic} \) are given by

\[
i_{c(se)} = G_{mb}v_{tb} - G_{me}v_{te}
\]

\[
r_{ic} = \frac{r_0 + r_e' R_{te}}{1 - G_m R_{te}}
\]

The transconductances \( G_{mb} \) and \( G_{me} \) are given by

\[
G_{mb} = G_m\frac{r_0 - R_{te}/\beta}{r_0 + r_e' R_{te}}
\]

\[
G_{me} = G_m\frac{r_0 + r_e'/\alpha}{r_0 + r_e' R_{te}}
\]

**The \( r_0 \) Approximations**

In some of the examples that follow, the analysis is simplified by making use of the so-called \( r_0 \) approximations. That is, we assume that \( r_0 \rightarrow \infty \) in all equations except in the one for \( r_{ic} \). This assumption makes \( G_{mb}, G_{me}, r_{ib}, \) and \( r_{ie} \) independent of \( r_0 \). In addition, it makes \( G_{mb} = G_{me} \) so that we can denote \( G_{mb} = G_{me} = G_m \). In this case,

\[
i_{c(se)} = G_m(v_{tb} - v_{te}) = \frac{\alpha}{r_e' + R_{te}}(v_{tb} - v_{te})
\]
\[ v_{b(oc)} = v_{te} \]  
\[ r_{ib} = r_x + (1 + \beta)(r_e + R_{te}) \]  
\[ v_{e(oc)} = v_{lb} \]  
\[ r_{ie} = r'_e \]  
\[ i_b = \frac{i_{c(sc)}}{\beta} \]  
\[ i_e = \frac{i_{c(sc)}}{\alpha} \]

If \( v_{te} = 0 \) and \( R_{re} = 0 \), \( r_0 \) appears as a resistor from the collector to ground. If \( R_{tc} = 0 \), \( r_0 \) appears as a resistor from emitter to ground. In either case, \( r_0 \) can be easily included in the analysis by treating it as an external resistor from either the emitter or the collector to ground.

**Series-Shunt Feedback**

A series-shunt feedback amplifier is a non-inverting amplifier in which the input signal \( x \) is a voltage and the output signal \( y \) is a voltage. If the input source is a current source, it must be converted into a Thévenin source for the gain to be in the form of Eq. (2). Because the input is a voltage and the output is a voltage, the gain \( A \) represents a dimensionless voltage gain. Because \( bA \) must be dimensionless, the feedback factor is also dimensionless. Two examples are given below. The first is an op-amp example. The second is a BJT example.

**Op Amp Example**

Fig. 4(a) shows an op amp with a feedback network consisting of a voltage divider connected between its output and inverting input. The input signal is connected to the non-inverting input. Because the feedback does not connect to the same terminal as the input signal, the summing is series. The feedback network connects in shunt with the output node, thus the sampling is shunt.

To analyze the circuit, we replace the circuit seen looking out of the op-amp inverting input with a Thévenin equivalent circuit with respect to \( v_o \) and the circuit seen looking into the feedback network from the \( v_o \) node with a Thévenin equivalent circuit with respect to \( i_1 \). We replace the op amp with a simple controlled source model which models the differential input resistance, the open-loop voltage gain, and the output resistance. A test source \( i_t \) is added at the output in order to calculate the output resistance. The circuit is shown in Fig. 4(b), where \( R_{id} \) is the differential input resistance, \( A_0 \) is the open-loop gain, \( R_0 \) is the output resistance of the op amp. The feedback factor \( b \) is given by

\[ b = \frac{R_1}{R_1 + R_F} \]  

The error signal \( z \) in Fig. 4(b) is a voltage which we denote by \( v_e \). It is the difference between the two voltage sources in the input circuit and is given by

\[ v_e = v_h - bv_o \]

By voltage division, the voltage \( v_i \) which controls the op amp output voltage is

\[ v_i = \frac{v_e R_{id}}{R_S + R_{id} + R_1||R_F} \]
Signal tracing shows that the negative feedback has the effect of making the current $i_1$ smaller. For this reason, we will neglect the $i_1R_1$ source in the output circuit in calculating $v_o$. By superposition, it follows that $v_o$ can be written

$$v_o = A_0v_i \left( \frac{(R_1 + R_F)\|R_L}{R_0 + (R_1 + R_F)\|R_L} \right) + i_tR_0\| (R_1 + R_F)\|R_L$$

To calculate the input resistance, we need the current $i_1$. It is given by

$$i_1 = \frac{v_i}{R_{id}}$$

To simplify the equations, let us define

$$k_1 = \frac{R_{id}}{R_S + R_{id} + R_1\|R_F}$$

$$k_2 = \frac{(R_1 + R_F)\|R_L}{R_0 + (R_1 + R_F)\|R_L}$$

$$R_{eq} = R_0\| (R_1 + R_F)\|R_L$$

The circuit equations can be rewritten

$$v_i = k_1v_e$$

$$v_o = k_2A_0v_i + i_tR_{eq}$$

The flow graph for these equations is shown in Fig. 5. The determinant is given by

$$\Delta = 1 - [k_1k_2A_0 (-b)] = 1 + bk_1k_2A_0 = 1 + bA$$

From the flow graph, the voltage gain is given by

$$\frac{v_o}{v_s} = \frac{1}{\Delta}k_1k_2A_0 = \frac{k_1k_2A_0}{1 + bk_1k_2A_0} = \frac{A}{1 + bA}$$

It follows that $A$ is given by

$$A = k_1k_2A_0$$

This is the gain from $v_s$ to $v_o$ with $b = 0$. If $bA \gg 1$, the gain approaches

$$\frac{v_o}{v_s} \rightarrow \frac{A}{bA} = \frac{1}{b} = 1 + \frac{R_F}{R_1}$$
This is the familiar formula for the gain of the non-inverting op amp.

From the flow graph, the output resistance is given by

\[ r_{out} = \frac{v_o}{i_t} = 1 + \frac{1}{\Delta} R_{eq} = \frac{R_0 (R_1 + R_F) R_L}{1 + bA} \]  

(53)

Similarly, the input resistance is given by

\[ r_{in} = \left( \frac{i_1}{v_s} \right)^{-1} = \left( \frac{1}{\Delta} \frac{k_1}{R_{id}} \right)^{-1} = (1 + bA) \left( R_S + R_{id} + R_1 || R_F \right) \]  

(54)

Note that the voltage gain and the output resistance are decreased by the feedback. The input resistance is increased by the feedback. These are properties of the series-shunt topology.

**Transistor Example**

The ac signal circuit of an example BJT series-shunt feedback amplifier is shown in Fig. 6. We assume that the dc solutions to the circuit are known. The feedback network is in the form of a voltage divider and consists of resistors \( R_{F1} \) and \( R_{F2} \). Because the input to the feedback network connects to the \( v_o \) node, the amplifier is said to employ shunt sampling at the output. The output of the feedback network connects to the emitter of \( Q_1 \). Because this is not the node to which \( v_s \) connects, the circuit is said to have series summing at the input. The following analysis assumes the \( r_0 \) approximations for \( Q_1 \). That is, we neglect \( r_0 \) in all equations except when calculating \( r_{ic1} \).
common-base stage which has a non-inverting voltage gain. Q2 is a common-emitter stage which has an inverting gain. Thus the amplifier has an inverting voltage gain from the emitter of Q1 to the collector of Q2 so that the feedback is negative.

To remove the feedback, we replace the circuit seen looking out of the emitter of Q1 with a Thévenin equivalent circuit with respect to v_o. The circuit seen looking into R_F1 from the v_o node is replaced with a Thévenin equivalent circuit with respect to i_e1. Fig. 7 shows the circuit. A test current source is connected to the v_o node to calculate r_out. The Thévenin voltage looking out of the emitter of Q1 is given by

\[
v_{te1} = v_o \frac{R_1}{R_1 + R_F} = bv_o
\]

where b is the feedback factor given by

\[
b = \frac{R_1}{R_1 + R_F}
\]

For the circuit of Fig. 7, the error voltage v_e is given by

\[
v_e = v_s - bv_o
\]

Signal tracing shows that the negative feedback has the effect of making the current i_e1 smaller. For this reason, we will neglect it in calculating v_o. To circuit equations are

\[
i_{c1(sc)} = G_{m1} (v_s - bv_o) = G_{m1} v_e
\]
\[
v_{tb2} = -i_{c1(sc)} r_{ic1} || R_{C1}
\]
\[
R_{tb2} = r_{ic1} || R_{C1}
\]
\[
i_{c2(sc)} = -G_{mb2} v_{tb2}
\]
\[
v_o = [i_{c2(sc)} + i_t] \times r_{ic2} || R_{C2} || (R_{F1} + R_{F2})
\]

To solve for r_in, we need i_b1. If we use the r_0 approximations, it is given by

\[
i_b1 = \frac{i_{c1(sc)}}{\beta_1}
\]
To simplify the flow graph, let us define

\[ R_{eq1} = r_{ic1} || R_{C1} \]  \hspace{2cm} (64) \\
\[ R_{eq2} = r_{ic2} || R_{C2} \| (R_{F1} + R_{F2}) \]  \hspace{2cm} (65)

The flow graph for the equations is shown in Fig. 8. The determinant is given by

\[ \Delta = 1 - [G_{m1} \cdot (-R_{eq1}) \cdot (-G_{m2}) \cdot R_{eq2} \cdot (-b)] = 1 + bA \]  \hspace{2cm} (66)

where \( A \) is given by

\[ A = G_{m1}R_{eq1}G_{m2}R_{eq2} \]  \hspace{2cm} (67)

This is the gain from \( v_s \) to \( v_o \) with \( b = 0 \). From the flow graph, the voltage gain is given by

\[ \frac{v_o}{v_s} = \frac{1}{\Delta} \frac{G_{m1}R_{eq1}G_{m2}R_{eq2}}{1 + bA} \]  \hspace{2cm} (68)

The output resistance is given by

\[ r_{out} = \frac{v_o}{i_t} = \frac{1}{\Delta} R_{eq2} = \frac{R_{eq2}}{1 + bA} \]  \hspace{2cm} (69)

The input resistance is given by

\[ r_{in} = \frac{i_{b1}}{v_s} = \frac{1}{\Delta} \frac{G_{m1}}{\beta_1} = \Delta \frac{\beta_1}{G_{m1}} \]  \hspace{2cm} (70)

But \( \beta_1/G_{m1} \) is given by

\[ \frac{\beta_1}{G_{m1}} = \frac{r_{eq1} + R_{te1}}{\alpha_1} = R_S + r_{x1} + (1 + \beta_1)(r_{eq1} + R_{te1}) \]  \hspace{2cm} (71)

Thus \( r_{in} \) is given by

\[ r_{in} = (1 + bA) [R_S + r_{x1} + (1 + \beta_1)(r_{eq1} + R_{te1})] \]  \hspace{2cm} (72)

**Summary of the Effects of Series-Shunt Feedback**

From the examples above, it can be seen that the voltage gain is divided by the factor \((1 + bA)\). The input resistance is multiplied by the factor \((1 + bA)\). And the output resistance is divided by the factor \((1 + bA)\).
Shunt-Shunt Feedback

A shunt-shunt feedback amplifier is an inverting amplifier in which the input signal \( x \) is a current and the output signal \( y \) is a voltage. If the input source is a voltage source, it must be converted into a Norton source for the gain to be in the form of Eq. (4). Because the input is a current and the output is a voltage, the gain \( A \) represents a transresistance with the units \( \Omega \). Because \( bA \) must be dimensionless, the feedback factor has the units of \( \Omega \). Two examples are given below. The first is an op-amp example. The second is a BJT example.

Op Amp Example

Fig. 9(a) shows an op amp with a feedback network consisting of a resistor connected between its output and its inverting input. The input signal is connected through a resistor to the inverting input. Because the feedback connects to the same terminal as the input signal, the summing is shunt. The feedback network connects in shunt with the output node, thus the sampling is shunt.

To analyze the circuit, we replace the circuit seen looking out of the \( v_i \) node through \( R_S \) with a Norton equivalent circuit with respect to \( v_s \) and the circuit seen looking out of the \( v_i \) node through \( R_F \) with a Norton equivalent circuit with respect to \( v_o \). This must always be done with the shunt-shunt amplifier in order for the gain with feedback to be of the form \(-A/(1 + bA)\), where \( A \) and \( b \) are positive and the \(-\) sign is necessary because the circuit has an inverting gain. In addition, we replace the circuit seen looking out of the \( v_o \) node through \( R_F \) with a Thévenin equivalent circuit with respect to \( v_i \).

The circuit with feedback removed is shown in Fig. 9(b), where \( r_{in} \) is the input resistance seen by the source current \( i_s \). The Norton current seen looking out of the \( v_i \) node is represented by the \( bv_o \) source, where \( b \) is the feedback factor. The current \( i_s \) and the feedback factor are given by

\[
i_s = \frac{v_s}{R_S} \quad (73)
\]

\[
b = \frac{1}{R_F} \quad (74)
\]

The error current \( i_e \) is the total current delivered to the \( v_i \) node. It is given by

\[
i_e = i_s + bv_o \quad (75)
\]

Because \( v_o \) is negative when \( i_s \) is positive, the current \( bv_o \) subtracts from \( i_s \) to cause \( i_e \) to be decreased.

![Figure 9: (a) Shunt-shunt op amp circuit. (b) Circuit with feedback removed.](image)

Fig. 10 shows the circuit with the op amp replaced with a controlled source model which models the differential input resistance \( R_{id} \), the open-loop voltage gain \( A_0 \), and the output resistance \( R_0 \).
A test source $i_t$ is added at the output in order to calculate the output resistance. The voltage $v_i$ controls the op-amp output voltage. It is given by

$$v_i = (i_s + b v_o) R_S \| R_F \| R_{id} = i_e R_S R_F R_{id} \quad \text{(76)}$$

Signal tracing shows that the negative feedback has the effect of making the voltage $v_i$ smaller. For this reason, we will neglect the $v_i$ source in calculating $v_o$. It follows that $v_o$ is given by

$$v_o = -A_0 v_i \frac{R_F \| R_L}{R_0 + R_F \| R_L} + i_t R_0 \| R_F \| R_L \quad \text{(77)}$$

Figure 10: Shunt-shunt circuit with the op amp replaced with a controlled source model.

To simplify the equations, let us define

$$R_{eq1} = R_S \| R_F \| R_{id} \quad \text{(78)}$$
$$R_{eq2} = R_0 \| R_F \| R_L \quad \text{(79)}$$
$$k = \frac{R_F \| R_L}{R_0 + R_F \| R_L} \quad \text{(80)}$$

The circuit equations can be rewritten

$$i_e = i_s + b v_o \quad \text{(81)}$$
$$v_i = i_e R_{eq1} \quad \text{(82)}$$
$$v_o = -k A_0 v_i + i_t R_{eq2} \quad \text{(83)}$$

The flow graph for these equations is shown in Fig. 11. The determinant is given by

$$\Delta = 1 - \left[ R_{eq1} (-A_0 k) b \right] = 1 + b R_{eq1} A_0 k \quad \text{(84)}$$

From the flow graph, the transresistance gain is given by

$$\frac{v_o}{i_s} = \frac{1}{\Delta} (-R_{eq1} A_0 k) = \frac{-R_{eq1} A_0 k}{1 + b R_{eq1} A_0 k} = \frac{-A}{1 + b A} \quad \text{(85)}$$

It follows that $A$ is given by

$$A = R_{eq1} A_0 k \quad \text{(86)}$$

If $b A \gg 1$, the transresistance gain approaches

$$\frac{v_o}{i_s} \rightarrow -\frac{A}{b A} = -\frac{1}{b} = -R_F \quad \text{(87)}$$
Figure 11: Flow graph for the shunt-shunt amplifier.

We note that $A$ is the negative of the gain from $i_s$ to $v_o$ calculated with $b = 0$. Also, $-bA$ is the loop gain in the flow graph.

From the flow graph, the output resistance is given by

$$r_{out} = \frac{v_o}{i_t} = \frac{1}{\Delta R_{eq2}} = \frac{R_O \| R_F \| R_L}{1 + bA}$$

(88)

Similarly, the input resistance is given by

$$r'_{in} = \frac{v_i}{i_s} = \frac{1}{\Delta R_{eq1}} = \frac{R_S \| R_F \| R_{id}}{1 + bA}$$

(89)

We note that $R_{eq2}$ is the output resistance with the $v_i$ source zeroed at the output. Also, $R_{eq1}$ is the input resistance with the $bv_o$ source zeroed at the input.

The voltage gain of the original circuit in Fig. 9(a) is given by

$$\frac{v_o}{v_s} = \frac{i_s}{v_s} = \frac{1}{R_S} \frac{-A}{1 + bA} \approx \frac{1}{R_S} \frac{-A}{bA} = \frac{-R_F}{R_S}$$

(90)

where the approximation holds for $bA \gg 1$. This is the familiar gain expression for the inverting op amp amplifier. The input resistance is obtained from $r'_{in}$ with the relation

$$r_{in} = R_S + \left( \frac{1}{r'_{in}} - \frac{1}{R_S} \right)^{-1}$$

(91)

Transistor Example

Fig. 12 shows the ac signal circuit of an example BJT shunt-shunt feedback amplifier. The feedback network consists of the resistor $R_F$ which connects between the output and input nodes. Because $R_F$ connects to the output node, the amplifier is said to have shunt sampling. Because the current fed back through $R_F$ to the input node combines in parallel with the source current, the circuit is said to have shunt summing. Thus the amplifier is said to have shunt-shunt feedback. In order for the feedback to be negative, the voltage gain from $v_i$ to $v_o$ must be inverting. $Q_1$ is a common-emitter stage which has an inverting gain. $Q_2$ is a common-collector stage which has a non-inverting gain. Thus the overall voltage gain is inverting so that the feedback is negative.

Fig. 13 shows the equivalent circuit with feedback removed. The circuits seen looking out of the $v_i$ node through $R_S$ and through $R_F$, respectively, are converted into Norton equivalent circuits with respect to $v_s$ and $v_o$. The source current $i_s$ and the feedback factor $b$ are given by

$$i_s = \frac{v_s}{R_S}$$

(92)

$$b = \frac{1}{R_F}$$

(93)
Figure 12: Example BJT shunt-shunt amplifier.

Figure 13: Shunt-shunt amplifier with feedback removed.
The feedback network at the output is modeled by a Thévenin equivalent circuit with respect to $v_i$. The external current source $i_t$ is added to the circuit so that the output resistance can be calculated. Signal tracing shows that the input voltage $v_i$ is reduced by the feedback. Therefore, we neglect the effect of the $v_i$ controlled source in the output circuit when calculating $v_o$.

The circuit equations are

$$v_{tb1} = i_e R_S\| R_F = i_e R_{eq1}$$
$$i_e = i_s + b v_o$$
$$v_i = i_e R_S\| R_F \| r_{ib1} = i_e R_{eq2}$$
$$i_{cl(oc)} = G_{mb1} v_{tb1}$$
$$v_{tb2} = -i_{cl(oc)} r_{ic1} \| R_C1 = -i_{cl(oc)} R_{eq3}$$
$$v_{v2(oc)} = \frac{r_{i2}}{r_{i2} + r_{i2}} v_{tb2} = k_{1} v_{tb2}$$
$$v_o = \frac{R_E2\| R_f}{r_{i2} + R_E2\| R_f} v_{v2(oc)} + i_t \times r_{i2}\| R_E2\| R_F = k_{2} v_{v2(oc)} + i_t R_{eq4}$$

The flow graph for the equations is shown in Fig. 14. The determinant is given by

$$\Delta = 1 - [R_{eq1} G_{mb1} (-R_{eq3}) k_{1} k_{2}] = 1 + b R_{eq1} G_{mb1} R_{eq4} k_{1} k_{2}$$

The transresistance gain is given by

$$\frac{v_o}{i_s} = \frac{1}{\Delta} R_{eq1} G_{mb1} (-R_{eq3}) k_{1} k_{2} = \frac{-R_{eq1} G_{mb1} R_{eq4} k_{1} k_{2}}{1 + b R_{eq1} G_{mb1} R_{eq4} k_{1} k_{2}} = -A$$

It follows that $A$ is given by

$$A = R_{eq1} G_{mb1} R_{eq4} k_{1} k_{2}$$

The input and output resistances are given by

$$r_{in}' = \frac{v_i}{i_s} = \frac{R_{eq2}}{\Delta} = \frac{R_1\| R_F \| r_{ib1}}{\Delta}$$
$$r_{out} = \frac{v_o}{i_t} = \frac{R_{eq4}}{\Delta} = \frac{r_{i2}\| R_E2\| R_F}{\Delta}$$

It can be seen from these expressions that the transresistance gain, the input resistance, and the output resistance are all decreased by a factor equal to the amount of feedback.

The voltage gain of the original circuit in Fig. 12 is given by

$$\frac{v_o}{v_s} = \frac{i_s}{i_s} = \frac{1}{R_S} \frac{1}{1 + b A} \simeq \frac{1}{R_S} \frac{-A}{b A} = -\frac{R_F}{R_S}$$
where the approximation holds for \( bA \gg 1 \). This is the familiar gain expression for the inverting op amp amplifier. The input resistance is obtained from \( r'_{in} \) with the relation
\[
 r_{in} = R_S + \left( \frac{1}{r'_{in}} - \frac{1}{R_S} \right)^{-1}
\]  

(107)

Summary of the Effects of Shunt-Shunt Feedback

Notice from the examples above that the transresistance gain and the voltage gain are divided by the factor \((1 + bA)\). The input resistance \( r'_{in} \) is divided by the factor \((1 + bA)\). And the output resistance is divided by the factor \((1 + bA)\).

Series-Series Feedback

A series-series feedback amplifier is a non-inverting amplifier in which the input signal \( x \) is a voltage and the output signal \( y \) is a current. If the input source is a current source, it must be converted into a Thévenin source for the gain to be in the form of Eq. (2). Because the input is a voltage and the output is a current, the gain \( A \) represents a transconductance with the units \( \Omega \). Because \( bA \) must be dimensionless, the feedback factor has the units of \( \Omega \). An op-amp example is given below.

Fig. 15(a) shows an op-amp circuit in which a resistor \( R_1 \) is in series with the load resistor \( R_L \). The voltage across \( R_1 \) is fed back into the inverting op-amp input. The voltage across \( R_1 \) is proportional to the output current \( i_o \). This is said to be series sampling at the output. Because the feedback does not connect to the same op-amp input as the source, the circuit is said to have series summing. Thus the circuit is called a series-series feedback amplifier.

![Series-Series Feedback](image)

Figure 15: (a) Example series-series feedback amplifier. (b) Circuit with feedback removed.

To remove the feedback, the circuit seen looking out of the op-amp inverting input is replaced with a Thévenin equivalent circuit with respect to \( i_o \). The circuit seen looking below \( R_L \) is replaced with a Thévenin equivalent circuit with respect to \( i_1 \). The circuit with feedback removed is shown in Fig. 15(b), where the op amp is replaced with a controlled source model that models its differential input resistance \( R_{id} \), its gain \( A_0 \), and its output resistance \( R_0 \). The feedback factor \( b \) is given by
\[
 b = R_1
\]

A test voltage source \( v_t \) is added in series with \( R_L \) to calculate the output resistance. We define the resistance \( r_{out} \) as the effective series resistance in the output circuit, including \( R_L \). It is given by \( r_{out} = v_t/i_o \). The output resistance seen by \( R_L \) is given by \( r'_{out} = r_{out} - R_L \). Because \( R_L \) is floating, it is not possible to label \( r_{out} \) on the diagram.
Signal tracing shows that the negative feedback has the effect of reducing $i_1$. For this reason, the $i_1R_1$ source in the output circuit will be neglected in solving for $i_o$. We can write the following equations

$$v_e = v_s - bi_o \quad (108)$$

$$v_i = \frac{R_{id}}{R_S + R_{id} + R_1} v_e \quad (109)$$

$$i_o = \frac{A_0 v_i}{R_0 + R_L + R_1} + \frac{v_i}{R_0 + R_L + R_1} \quad (110)$$

$$i_1 = \frac{v_i}{R_{id}} \quad (111)$$

To simplify the equations, let us define

$$k = \frac{R_{id}}{R_S + R_{id} + R_1} \quad (112)$$

$$R_{eq} = R_0 + R_L + R_1 \quad (113)$$

The circuit equations can be rewritten

$$v_i = kv_e \quad (114)$$

$$i_o = \frac{A_0 v_i}{R_{eq}} + \frac{v_i}{R_{eq}} \quad (115)$$

The flow graph for these equations is shown in Fig. 16. The determinant is given by

$$\Delta = 1 - \left[ k \frac{A_0}{R_{eq}} (-b) \right] = 1 + bk \frac{A_0}{R_{eq}} = 1 + bA \quad (116)$$

From the flow graph, the transconductance gain is given by

$$\frac{i_o}{v_s} = \frac{1}{\Delta} \frac{A_0}{R_{eq}} = \frac{kA_0}{1 + bkA_0/R_{eq}} = \frac{A}{1 + bA} \quad (117)$$

It follows that $A$ is given by

$$A = k \frac{A_0}{R_{eq}} \quad (118)$$

This is the gain from $v_s$ to $i_o$ with $b = 0$. If $bA \gg 1$, the gain approaches

$$\frac{i_o}{v_s} \rightarrow \frac{A}{bA} = \frac{1}{b} = \frac{1}{R_1} \quad (119)$$

This is the gain obtained by assuming the op amp is ideal. Because it has negative feedback, there is a virtual short between its two inputs so that the voltage at the upper node of $R_1$ is $v_s$ and the current through $R_1$ is $v_s/R_1$. Thus the output current is $i_o = v_s/R_1$.

The input resistance $r_{in}$ and the output resistance $r_{out}$ are given by

$$r_{in} = \left( \frac{i_1}{v_s} \right)^{-1} = \left( \frac{1}{\Delta} \frac{k}{R_{id}} \right)^{-1} = \Delta \frac{R_{id}}{k} = (1 + bA) (R_S + R_{id} + R_1) \quad (120)$$

$$r_{out} = \left( \frac{i_o}{v_i} \right)^{-1} = \left( \frac{1}{\Delta} \frac{1}{R_{eq}} \right)^{-1} = (1 + bA) (R_0 + R_L + R_1) \quad (121)$$

Note that the effect of the feedback is to reduce the gain, increase the input resistance and increase the output resistance. In the case $bA \rightarrow \infty$, the output resistance becomes infinite and the load resistor $R_L$ is driven by an ideal current source.
Shunt-Series Feedback

A shunt-series feedback amplifier is an inverting amplifier in which the input signal $x$ is a voltage and the output signal $y$ is a current. If the input source is a voltage source, it must be converted into a Norton source for the gain to be in the form of Eq. (4). Because the input is a current and the output is a current, the gain $A$ represents a dimensionless current gain. Because $bA$ must be dimensionless, the feedback factor is dimensionless. An op-amp example is given below.

Fig. 17(a) shows an op-amp circuit in which a resistor $R_1$ connects from the lower node of the load resistor $R_L$ to ground. The voltage across $R_1$ causes a current to flow through the feedback resistor $R_F$ into the inverting op-amp input. The current through $R_F$ is proportional to the output current $i_o$. This is said to be series sampling at the output. Because the feedback connects to the same op-amp input as the source, the circuit is said to have shunt summing at the input. Thus the circuit is called a shunt-series feedback amplifier.

$$b = \frac{R_1}{R_1 + R_F}$$  \hspace{1cm} (122)
A test voltage source \( v_t \) is added in series with \( R_L \) to calculate the output resistance. We define the resistance \( r_{out} \) as the effective series resistance in the output circuit, including \( R_L \). It is given by \( r_{out} = v_t/i_o \). The output resistance seen by \( R_0 \) is given by \( r'_{out} = r_{out} - R_L \). Because \( R_L \) is floating, it is not possible to label \( r_{out} \) on the diagram.

Signal tracing shows that the negative feedback has the effect of reducing \( v_i \). For this reason, the \( v_i \) source in the output circuit will be neglected in solving for \( i_o \). The error current \( i_e \) is the sum of the two current sources in the input circuit. We can write the following equations

\[
i_e = i_s + bi_o \tag{123}
\]

\[
v_i = i_e R_S (R_1 + R_F) = i_e R_{eq1} \tag{124}
\]

\[
i_o = \frac{-A_0 v_i}{R_0 + R_L + R_1 / R_F} + \frac{v_t}{R_0 + R_L + R_1 / R_F} = \frac{-A_0 v_i}{R_{eq2}} + \frac{v_t}{R_{eq2}} \tag{125}
\]

The flow graph for these equations is shown in Fig. 18. The determinant is given by

\[
\Delta = 1 - \left[ R_{eq1} \left( \frac{-A_0}{R_{eq2}} \right) \right] b = 1 + b R_{eq1} \frac{A_0}{R_{eq2}} = 1 + bA \tag{126}
\]

From the flow graph, the current gain is given by

\[
\frac{i_o}{i_s} = \frac{1}{\Delta} R_{eq1} \left( \frac{-A_0}{R_{eq2}} \right) = \frac{-R_{eq1} A_0 / R_{eq2}}{1 + b R_{eq1} A_0 / R_{eq2}} = \frac{-A}{1 + bA} \tag{127}
\]

It follows that \( A \) is given by

\[
A = R_{eq1} \frac{A_0}{R_{eq2}} \tag{128}
\]

This is the negative of the gain from \( i_s \) to \( i_o \) with \( b = 0 \). If \( bA \gg 1 \), the gain approaches

\[
\frac{i_o}{i_s} \to \frac{-A}{bA} = \frac{-1}{b} = -\left( 1 + \frac{R_F}{R_1} \right) \tag{129}
\]

This is the gain obtained by assuming the op amp is ideal. In this case, there is a virtual ground at its inverting input which causes \( i_e = i_s + bi_o = 0 \). Solution for \( i_o \) yields \( i_o = -i_s/b \).

![Flow graph for the shunt-series amplifier.](image)

The input resistance \( r'_{in} \) and the output resistance \( r_{out} \) are given by

\[
r'_{in} = \frac{v_i}{i_s} = \frac{1}{\Delta} R_{eq1} = \frac{R_S (R_1 + R_F) R_{id}}{1 + bA} \tag{130}
\]

\[
r_{out} = \left( \frac{i_o}{v_t} \right)^{-1} = \left( \frac{1}{\Delta} \frac{1}{R_{eq2}} \right)^{-1} = \left( 1 + bA \right) R_0 + R_L + R_1 / R_F \tag{131}
\]
Note that the effect of the feedback is to reduce the gain, decrease the input resistance and increase the output resistance. In the case \( bA \to \infty \), the output resistance becomes infinite and the load resistor \( R_L \) is driven by an ideal current source. The gain from \( v_s \) to \( i_o \) in the original circuit, the input resistance \( r_{in} \), and the output resistance \( r_{out}' \) seen by \( R_L \) are given by

\[
\frac{i_o}{v_s} = \frac{i_s}{v_s} = \frac{1}{R_S} \frac{-A}{1 + bA} = \frac{1}{R_S} \frac{-A}{1 + bA} \tag{132}
\]

\[
r_{in} = R_S + \left( \frac{1}{r_{in}'} - \frac{1}{R_S} \right)^{-1} \tag{133}
\]

\[
r_{out}' = r_{out} - R_L \tag{134}
\]