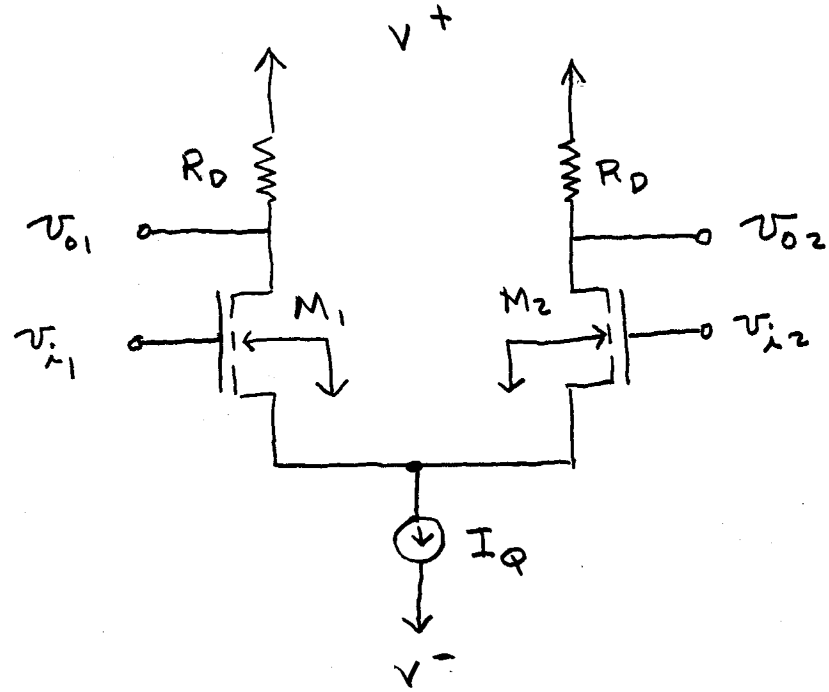
If $R_{ts} = 0$, the gains are the same. As R_{ts} increases, the CG gain decreases. This if a high gain is desired, the CG stage should not be used with a source having a high R_{ts}.

The MOSFET Diff Amp with Body Effect

The differential amplifier or diff amp is a basic building block of analog electronics. It is an amplifier that has an output that is proportional to the difference between two input signals. The diff amp requires a dc current source to set its bias current. This can be realized with a MOSFET, a JFET, or a BJT. In some cases, a resistor is used.



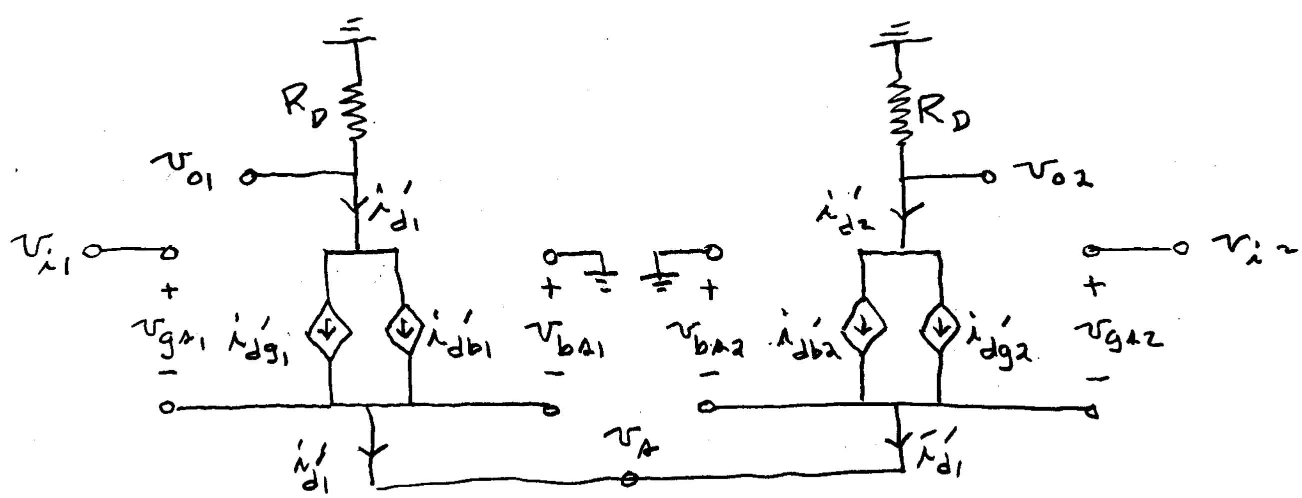
We assume M1 and M2 are identical. For $v_{i1} = v_{i2} = 0$, I_Q divides equally between M1 and M2 so that

$$I_{D1} = I_{D2} = \frac{1}{2} I_Q$$

The drain voltages are thus given by

$$V_{D1} = V_{D2} = V^+ - \frac{1}{2} I_Q R_D$$

For the ac signal analysis, we zero V^+, V^-, and I_Q and replace the MOSFETs with their pi models. Because neither the drain or the source of M_1 or M_2 connect to ac ground, we omit r_o for an approximate analysis.



$v_{o1} = -i'_{d1} R_D$ $\qquad$ $v_{o2} = -i'_{d2} R_D$

$i'_{d1} = i'_{dg1} + i'_{db1}$ $\qquad$ $i'_{d2} = i'_{dg2} + i'_{db2}$

$= g_m v_{gs1} + g_{mb} v_{bs1}$ $\qquad$ $= g_m v_{gs2} + g_{mb} v_{bs2}$

$v_{gs1} = v_{i1} - v_s \qquad v_{gs2} = v_{i2} - v_s$

$v_{bs1} = -v_s \qquad v_{bs2} = -v_s$

$\Rightarrow \; i'_{d1} = g_m v_{i1} - (g_m + g_{mb}) v_s$

$i'_{d2} = g_m v_{i2} - (g_m + g_{mb}) v_s$

Now $i'_{d1} + i'_{d2} = 0$

$\Rightarrow \; g_m (v_{i1} + v_{i2}) - 2(g_m + g_{mb}) v_s = 0$

$\Rightarrow \; v_s = \frac{g_m}{g_m + g_{mb}} \frac{v_{i1} + v_{i2}}{2}$

$\Rightarrow \; i'_{d1} = g_m v_{i1} - (g_m + g_{mb}) \frac{g_m}{g_m + g_{mb}} \frac{v_{i1} + v_{i2}}{2}$

$= g_m \frac{v_{i1} - v_{i2}}{2}$

Because $i'_{d1} + i'_{d2} = 0$

$\Rightarrow \; i'_{d2} = -g_m \frac{v_{i1} - v_{i2}}{2}$

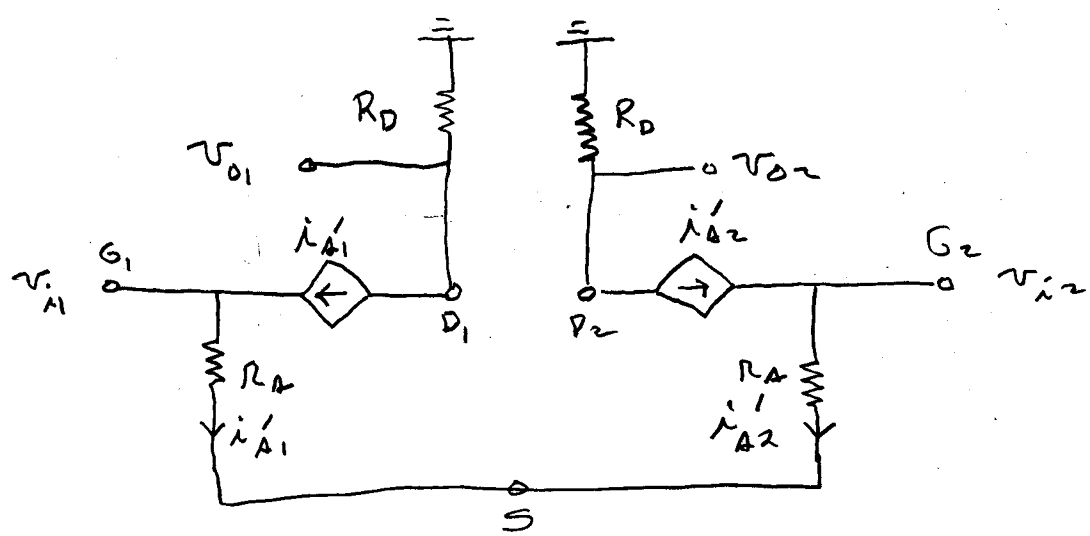
Thus for v_{o1} and v_{o2}, we have

$$v_{o1} = -\frac{g_m R_D}{2}(v_{i1} - v_{i2})$$

$$v_{o2} = +\frac{g_m R_D}{2}(v_{i1} - v_{i2})$$

Note that the body effect has cancelled. Thus we could have omitted it from the pi model from the start.

Now that we see that the body effect cancels, let us use the T model without the body effect to calculate v_{o1} and v_{o2}.



$v_{o1} = -i'_{A1} R_D \qquad v_{o2} = -i'_{A2} R_D$

$$i_{A1} = -i_{A2} = \frac{v_{i1} - v_{i2}}{2r_A}$$

$$\Rightarrow v_{o1} = -\frac{R_D}{2r_A}(v_{i1} - v_{i2})$$

$$v_{o2} = +\frac{R_D}{2r_A}(v_{i1} - v_{i2})$$

Because $r_A = 1/g_m$, these are the same answers.

Often the output of the diff amp is taken differentially.

$$v_o = v_{o1} - v_{o2}$$

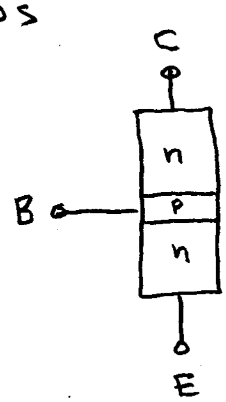
$$= -g_m R_D (v_{i1} - v_{i2})$$

$$\overset{\text{or}}{=} -\frac{R_D}{r_A}(v_{i1} - v_{i2})$$

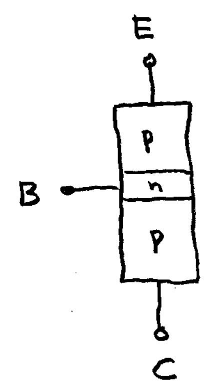
The Bipolar Junction Transistor

The BJT is fabricated as two back-to-back p-n junctions. There are two types, the npn and the pnp. A diagram of each is as follows

npn:

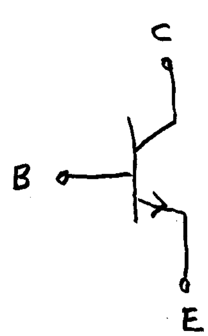


pnp:

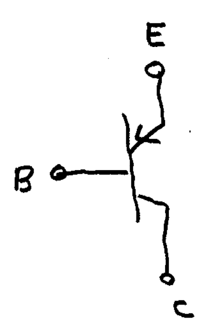


The circuit symbols are

npn:

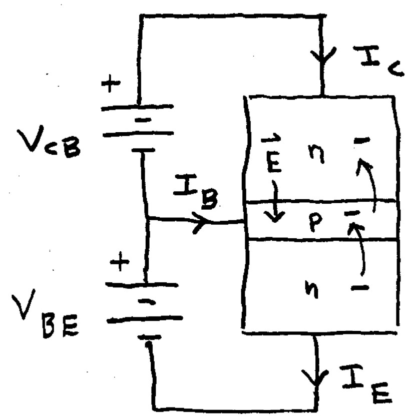


pnp:



The leads are called the collector (C), the base (B), and the emitter E.

To understand its operation, consider the npn circuit



The battery V_{CB} reverse biases the collector-to-base junction. The battery V_{BE} forward biases the base-to-emitter junction. The two n regions are doped very heavily with n-type impurities. The p region is doped very lightly with a p-type impurity. Because of the way it is doped, the majority current carriers are free electrons.

The battery V_{BE} causes free electrons to be injected or emitted from the emitter region into the base region. The battery V_{CB} sets up an electric field $\vec{E}$ from the collecto region into the base region. The base region is very narrow. This electric field attracts the free electrons injected into the base and pulls them into the collector. The fraction of electrons which are collected is denoted by α. Thus we have

$$I_C = \alpha I_E$$

When $I_E = 0$, a small reverse saturation current flows across the reverse biased collecto-to-base region. This current is denoted by I_{CBO}. The current flows from C to B with the E open circuited, i.e. with $I_E = 0$. Thus we add I_{CBO} to the equation for I_C to obtain

$$I_c = \alpha I_E + I_{CBO}$$

The parameter α is called the emitter-to-collector current gain. A typical value for α is $\alpha = 0.99$.

Next, we relate I_c to I_B. By KCL, we have

$$I_E = I_c + I_B$$

$$\Rightarrow \quad I_c = \alpha(I_c + I_B) + I_{CBO}$$

$$\Rightarrow \quad I_c = \frac{\alpha}{1-\alpha} I_B + \frac{I_{CBO}}{1-\alpha}$$

Let β and I_{CEO} be defined as follows:

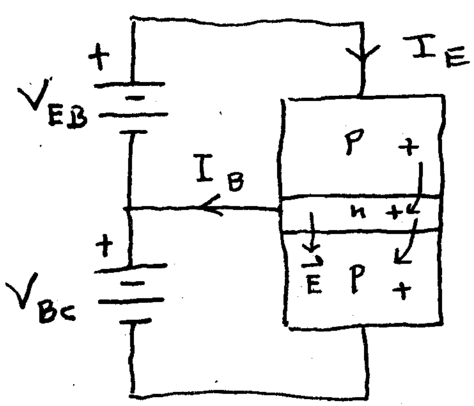
$$\beta = \frac{\alpha}{1-\alpha} \qquad I_{CEO} = \frac{I_{CBO}}{1-\alpha}$$

Thus we can write

$$I_C = \beta I_B + I_{CEO}$$

The parameter β is called the base-to-collector current gain. A typical value is $\beta = 100$. For $\alpha = 0.99$, I_{CEO} is 100 times greater than I_{CBO}.

Next, we look at the pnp device. Consider the circuit



V_{EB} forward biases the emitter-to-base junction. V_{BC} reverse biases the base-to-collector junction. The p-type regions are doped much heavier than the n-type region. Thus holes are the majority current carriers.

Holes that are emitted from the E to the B are collected by the $\vec{E}$ field across the base-to-collector junction and we have

$$I_C = \alpha I_E + I_{CBO}$$

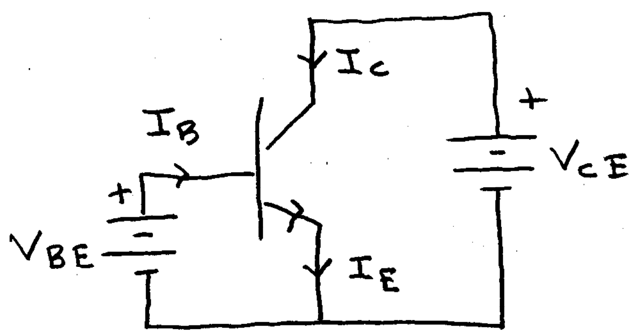
$$= \beta I_B + I_{CEO}$$

where $\beta = \frac{\alpha}{1-\alpha}$ $\quad I_{CEO} = \frac{I_{CBO}}{1-\alpha}$

The equations are identical to those for the npn except the voltage polarities and current directions are reversed.

Next, we wish to relate I_C to the B-E voltage.

Consider the circuit



Let I_{ES} be the saturation current of the B to E diode. We can write

$$I_E = I_{ES}\left(e^{V_{BE}/V_T} - 1\right)$$

Note that $\eta = 1$. This is because there are almost no recombinations due to the very lightly doped p-type base region.

For I_c, we can write

$$I_C = \alpha I_{ES}\left(e^{V_{BE}/V_T} - 1\right) + I_{CBO}$$

Let us define the BJT saturation current I_S as

$$I_S = \alpha I_{ES}$$

Thus we have

$$I_C = I_S e^{V_{BE}/V_T} + (I_{CBO} - I_S)$$

For I_B, we have

$$I_B = \frac{I_C - I_{CEO}}{\beta}$$

$$= \frac{1}{\beta}\left[I_S e^{V_{BE}/V_T} + (I_{CBO} - I_S) - I_{CEO}\right]$$

But $I_{CBO} - I_{CEO} = I_{CBO} - \dfrac{I_{CBO}}{1-\alpha}$

$$= I_{CBO}\left(1 - \frac{1}{1-\alpha}\right)$$

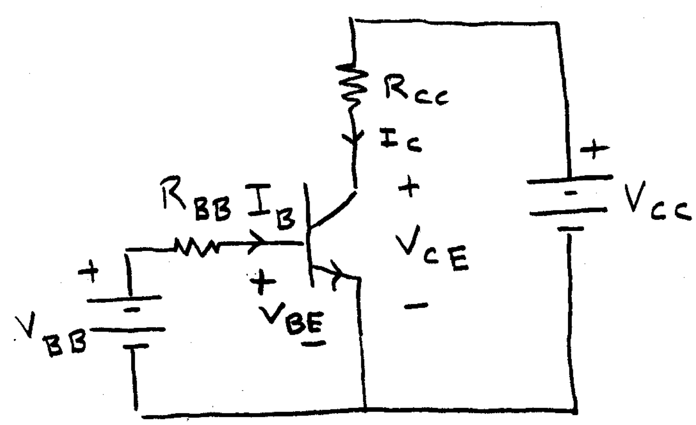
$$= -\beta I_{CBO}$$

Thus we can write

$$I_B = \frac{I_S}{\beta} e^{V_{BE}/V_T} - \left(I_{CBo} + \frac{I_S}{\beta}\right)$$

The Modes of Operation

Consider the circuit



We can write

$$I_B = \frac{V_{BB} - V_{BE}}{R_{BB}}$$

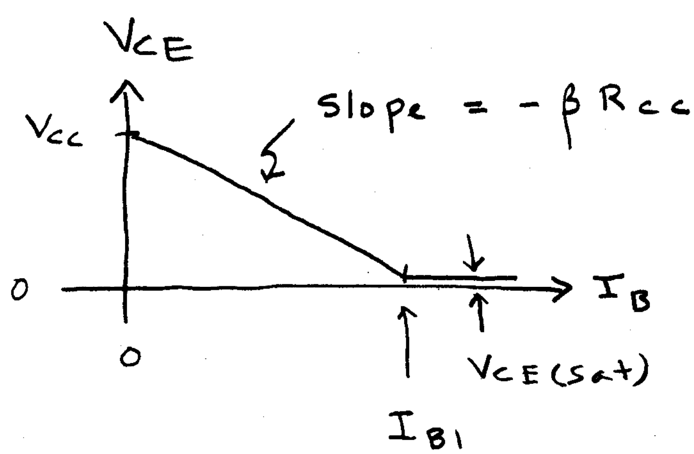
$$I_C = \beta I_B + I_{CEo}$$

$$V_{CE} = V_{cc} - I_c R_{cc}$$

To see how V_{CE} varies with I_B, let us assume I_{CEo} is small so that we can write

$$I_c = \beta I_B$$

$$\Rightarrow \quad V_{CE} = V_{cc} - \beta I_B R_{cc}$$



As I_B is increased, V_{CE} decreases, but it cannot go negative because the B-C junction becomes forward biased. When this happens, V_{CE} levels off at a value labeled

$V_{CE(SAT)}$. This is given by

$$V_{CE(SAT)} = V_{BE(SAT)} - V_{BC(SAT)}$$

The transistor is said to be saturated. Typically, for most BJTs

$$0 < V_{CE(SAT)} < 0.2 \text{ V}$$

The value of I_B at which saturation occurs is obtained from

$$V_{CE(SAT)} = V_{CC} - \beta I_{B1} R_{CC}$$

$$\Rightarrow \quad I_{B1} = \frac{V_{CC} - V_{CE(SAT)}}{\beta R_{CC}}$$

For $I_B > I_{B1}$, the BJT is saturated. For $0 < I_B < I_{B1}$, the BJT is said to be in its linear or active mode.

The base current is given by

$$I_B = \frac{V_{BB} - V_{BE}}{R_{BB}}$$

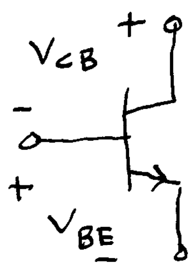
For $V_{BB} = 0$, I_B is zero. As V_{BB} is increased, I_B will begin to flow when the B-E junction turns on. We denote the value of V_{BE} at which the BJT turns on by V_γ. This is also called the cutin voltage. Typically $V_\gamma \approx 0.5$ V to 0.6 V.

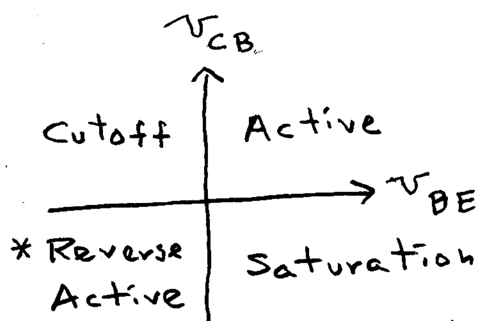
As V_{BB} is increased, I_B begins to flow for $V_{BB} > V_\gamma$. The BJT enters the active region. In this region $V_{BE\,(active)} = 0.6$ V to 0.7 V. For $I_B > I_{B1}$, the BJT is saturated. In this case, both junctions are forward biased.

Regions of operation

1. Active – $V_{BE} > 0$ and $V_{CB} > 0$
2. Saturation – $V_{BE} > 0$ and $V_{CB} < 0$
3. Cutoff – $V_{BE} < 0$ and $V_{CB} > 0$
4. Reverse Active – $V_{BC} > 0$ and $V_{BE} < 0$

Summary Graph





* Not allowed

The Early Effect

When V_{CE} changes, the width of the base region changes. This is called base width modulation or the Early effect. It causes both I_S and β to vary with V_{CE}.

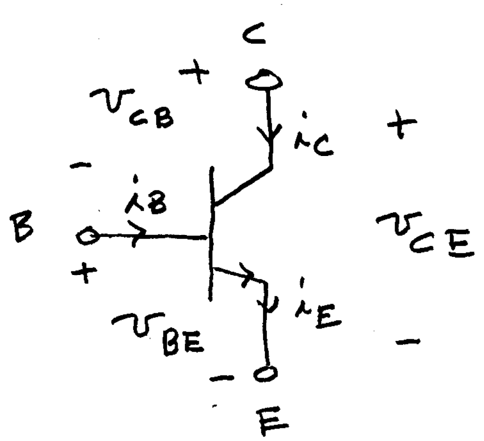
They are given by

$$I_S = I_{SO} \left(1 + \frac{V_{CE}}{V_A}\right)$$

$$\beta = \beta_o \left(1 + \frac{V_{CE}}{V_A}\right)$$

where V_A is the Early voltage, I_{SO} is the value of I_S with $V_{CE} = 0$, and β_o is the value of β with $V_{CE} = 0$.

General Current Equations for the BJT



We will assume that the device is in its active mode and that the leakage currents can be neglected.

$$i_C = I_S \, e^{v_{BE}/V_T}$$

$$i_B = \frac{i_C}{\beta}$$

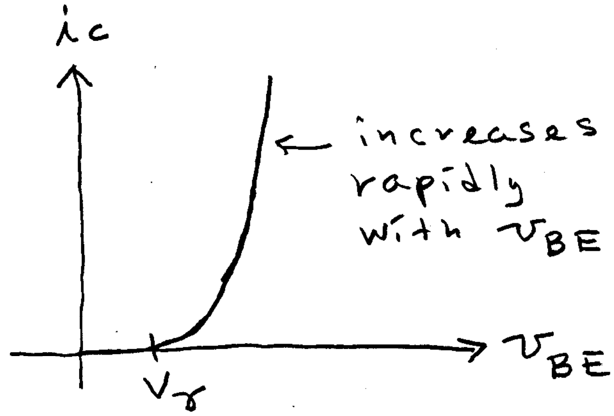
$$i_E = i_C + i_B$$

$$I_S = I_{SO}\left(1 + \frac{v_{CE}}{V_A}\right)$$

$$\beta = \beta_o\left(1 + \frac{v_{CE}}{V_A}\right)$$

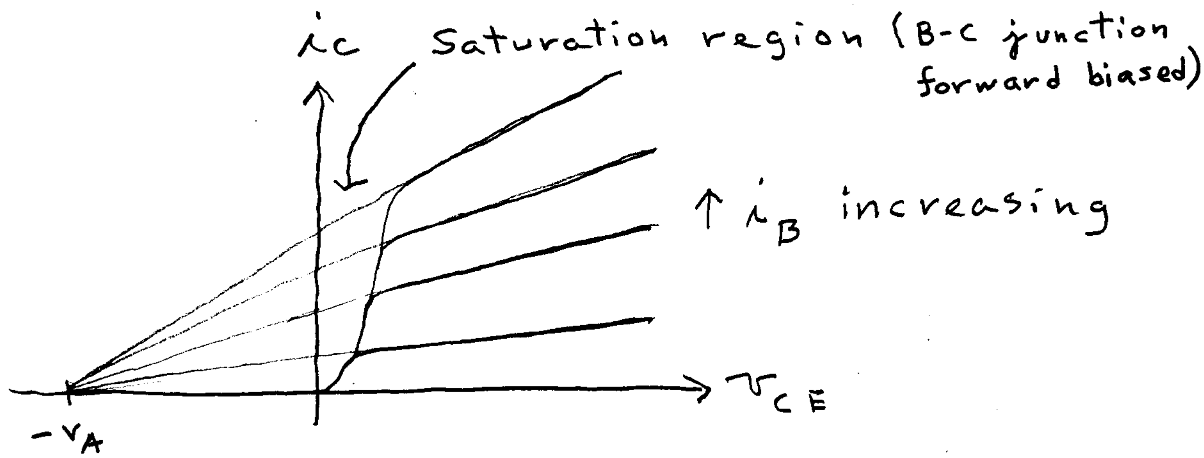
The transfer characteristic is a plot of i_C versus v_{BE} with v_{CE} = constant. If v_{CE} = constant, then I_S = constant

$$i_C = I_S \, e^{v_{BE}/V_T}$$



The output characteristics are plots of i_C versus v_{CE} for various values of i_B.

$$i_C = \beta i_B = \beta_0 \left(1 + \frac{v_{CE}}{V_A}\right) i_B$$

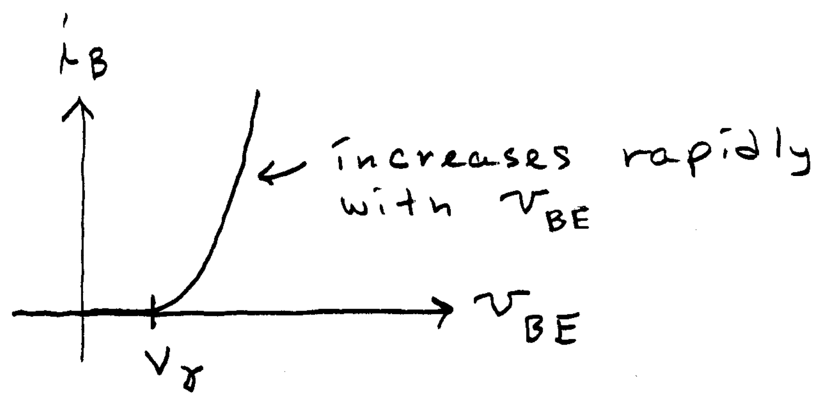


In the saturation region $v_{CB} < 0$ so that $v_{CE} = v_{CB} + v_{BE} < v_{BE}$. That is $v_{CE} < v_{BE}$.

The input characteristics are a plot of i_B versus v_{BE} for v_{CE} = constant.

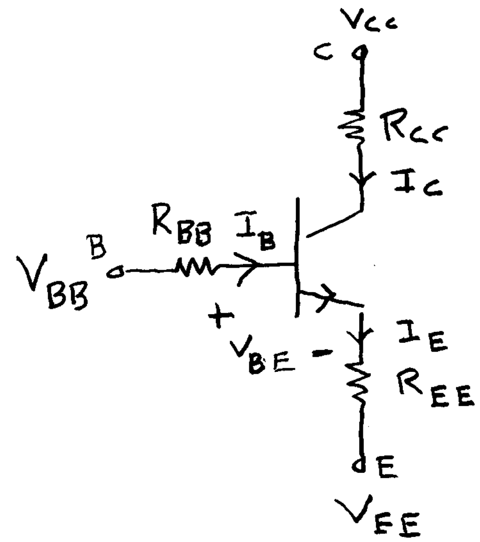
$$i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta} e^{v_{BE}/V_T} = \frac{I_{SO}}{\beta_o} e^{v_{BE}/V_T}$$

Note that the Early effect cancels out so that i_B is not a function of v_{CE}.



The BJT Bias Equation

Assume the external circuits are represented by Thévenin equivalent circuits. For the npn device, the general dc bias circuit is



Neglect leakage currents so that $I_c = \beta I_B = \alpha I_E$

The loop equation for the B to E loop is

$$V_{BB} - V_{EE} = I_B R_{BB} + V_{BE} + I_E R_{EE}$$

To solve for I_c, let $I_B = I_c/\beta$ and $I_E = I_c/\alpha$

$$\Rightarrow \quad V_{BB} - V_{EE} = I_c \frac{R_{BB}}{\beta} + V_{BE} + I_c \frac{R_{EE}}{\alpha}$$

$$\Rightarrow \quad I_c = \frac{V_{BB} - V_{EE} - V_{BE}}{\frac{R_{BB}}{\beta} + \frac{R_{EE}}{\alpha}}$$

Often it is desired to solve for I_E. To do this, we write I_B as follows

$$I_B = I_E - I_C = I_E - \alpha I_E = (1-\alpha) I_E$$

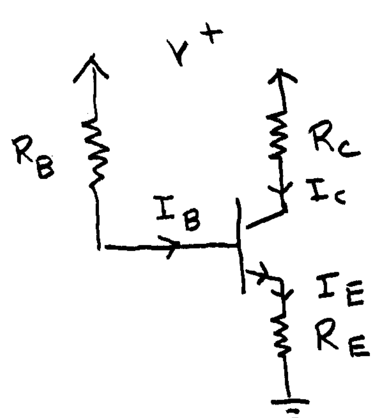
But $1-\alpha = 1 - \dfrac{\beta}{1+\beta} = \dfrac{1}{1+\beta}$

$$\Rightarrow \quad I_B = \frac{I_E}{1+\beta}$$

$$\Rightarrow \quad V_{BB} - V_{EE} = I_E \frac{R_{BB}}{1+\beta} + V_{BE} + I_E R_{EE}$$

$$\Rightarrow \quad I_E = \frac{V_{BB} - V_{EE} - V_{BE}}{\dfrac{R_{BB}}{1+\beta} + R_{EE}}$$

Example 1



$V^+ = 12$ V.

$R_B = 2\ M\Omega$

$R_C = 10\ k\Omega$

$R_E = 1\ k\Omega$

$\beta = 99$

$V_{BE} = 0.65$ V

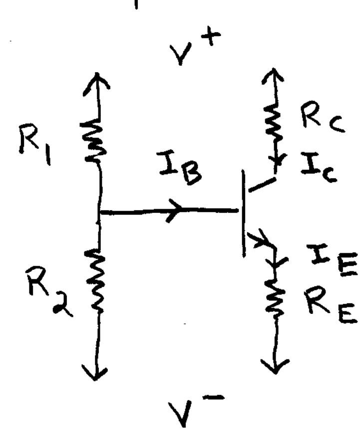
$$I_E = \frac{V^+ - V_{BE}}{\frac{R_B}{1+\beta} + R_E} = 0.5405 \text{ mA}$$

Next, we test for the active mode. This requires $V_{CB} > 0$.

$$\begin{aligned} V_{CB} &= V_C - V_B \\ &= (V^+ - I_C R_C) - (V_{BE} + I_E R_E) \\ &= (V^+ - \alpha I_E R_C) - (V_{BE} + I_E R_E) \\ &= 5.459 \text{ V.} \end{aligned}$$

Thus the BJT is in the active mode.

Example 2



$V^+ = +12$ V.

$V^- = -12$ V.

$R_1 = 300\ K\Omega$

$R_2 = 20\ K\Omega$

$R_C = 5.6\ K\Omega$

$R_E = 2 K\Omega$

$\beta = 99$

$V_{BE} = 0.65$ V.

$\alpha = 0.99$

$$V_{BB} = V^{+}\frac{R_2}{R_1+R_2} + V^{-}\frac{R_1}{R_1+R_2} = -10.5 \text{ V}$$

$$R_{BB} = R_1 \parallel R_2 = 18.75 \text{ K}\Omega$$

$$I_C = \frac{V_{BB} - V^{-} - V_{BE}}{\frac{R_{BB}}{\beta} + \frac{R_E}{\alpha}} = 0.3847 \text{ mA}$$

Next, we test for the active mode.

$$V_{CB} = (V^{+} - I_C R_C) - (V_{BE} + \frac{I_C}{\alpha} R_E + V^{-})$$

$$= 14.88 \text{ V.}$$

Because $V_{CB} > 0$, the BJT is in the active mode.