The BJT Differential Amplifier

Basic Circuit

Figure 1 shows the circuit diagram of a differential amplifier. The tail supply is modeled as a current source I_Q. The object is to solve for the small-signal output voltages and output resistances. It will be assumed that the transistors are identical.

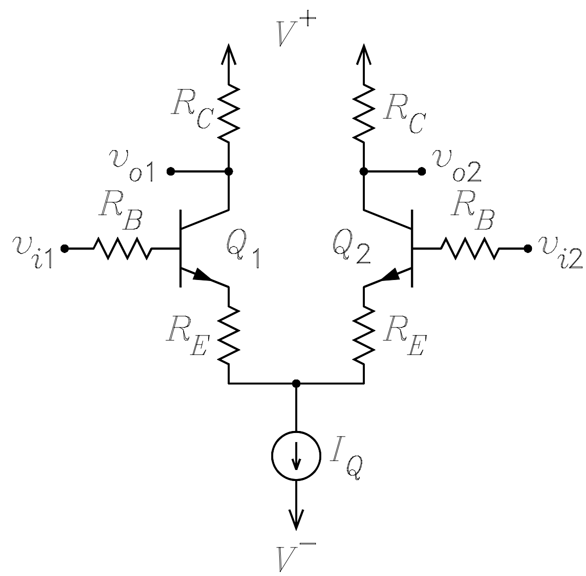

Figure 1: Circuit diagram of the differential amplifier.

DC Solution

Zero both base inputs. For identical transistors, the current I_Q divides equally between the two emitters.

(a) The dc currents are given by

$$I_{E1} = I_{E2} = \frac{I_Q}{2} \qquad I_{B1} = I_{B2} = \frac{I_Q}{2(1+\beta)} \qquad I_{C1} = I_{C2} = \frac{\alpha I_Q}{2}$$

(b) Verify that $V_{CB} > 0$ for the active mode.

$$V_{CB} = V_C - V_B = \left(V^+ - \alpha I_E R_C\right) - \left(-\frac{I_E}{1+\beta} R_B\right) = V^+ - \alpha I_E R_C + \frac{I_E}{1+\beta} R_B$$

(e) Calculate the collector-emitter voltage.

$$V_{CE} = V_C - V_E = V_C - (V_B - V_{BE}) = V_{CB} + V_{BE}$$

Small-Signal AC Solution using the Emitter Equivalent Circuit

This solution uses the r_0 approximations and assumes that the base spreading resistance r_x is not zero.

(a) Calculate g_m, r_π, r_e, and r_{ie}.

$$g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{V_T}{I_B} \qquad r_e = \frac{V_T}{I_E} \qquad r_{ie} = \frac{R_B + r_x + r_\pi}{1+\beta} \qquad r_0 = \frac{V_A + V_{CE}}{I_E}$$

(b) Redraw the circuit with $V^+ = V^- = 0$. Replace the two BJTs with the emitter equivalent circuit. The emitter part of the circuit obtained is shown in Fig. 2(a).

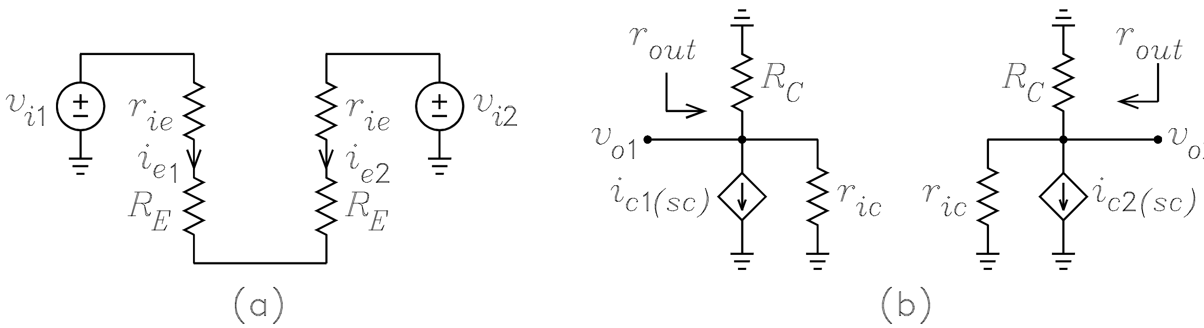

Figure 2: (a) Emitter equivalent circuit for i_{e1} and i_{e2}. (b) Collector equivalent circuits.

(c) Using Ohm's Law, solve for i_{e1} and i_{e2}.

$$i_{e1} = \frac{v_{i1} - v_{i2}}{2\,(r_{ie} + R_E)} \qquad i_{e2} = -i_{e1}$$

(d) The circuit for v_{o1}, v_{o2}, and r_{out} is shown in Fig. 2(b).

$$v_{o1} = -i_{c1(sc)} \times r_{ic}\|R_C = -\alpha \times i_{e1} \times r_{ic}\|R_C = \frac{-\alpha \times r_{ic}\|R_C}{2\,(r_{ie} + R_E)}\,(v_{i1} - v_{i2})$$

$$v_{o2} = -i_{c2(sc)} \times r_{ic}\|R_C = -\alpha \times i_{ie2} \times r_{ic}\|R_C = \frac{-\alpha \times r_{ic}\|R_C}{2\,(r_{ie} + R_E)}\,(v_{i2} - v_{i1})$$

$$r_{out1} = r_{out2} = r_{ic}\|R_C$$

$$r_{ic} = r_0\left[1 + \frac{\beta\,(2R_E + r_{ie})}{R_B + r_\pi + r_x + 2R_E + r_{ie}}\right] + (R_B + r_x + r_\pi)\,\|\,(2R_E + r_{ie})$$

(e) The resistance seen looking into either input with the other input zeroed is

$$r_{in} = R_B + r_x + r_\pi + (1+\beta)\,(2R_E + r_{ie})$$

The differential input resistance r_{ind} is the resistance between the two inputs for differential input signals. For an ideal current source tail supply, this is the same as the input resistance r_{in} above.

Diff Amp with Non-Perfect Tail Supply

Fig. 3 shows the circuit diagram of a differential amplifier. The tail supply is modeled as a current source I_Q' having a parallel resistance R_Q. In the case of an ideal current source, R_Q is an open circuit. Often a diff amp is designed with a resistive tail supply. In this case, $I_Q' = 0$. The solutions below are valid for each of these connections. The object is to solve for the small-signal output voltages and output resistances.

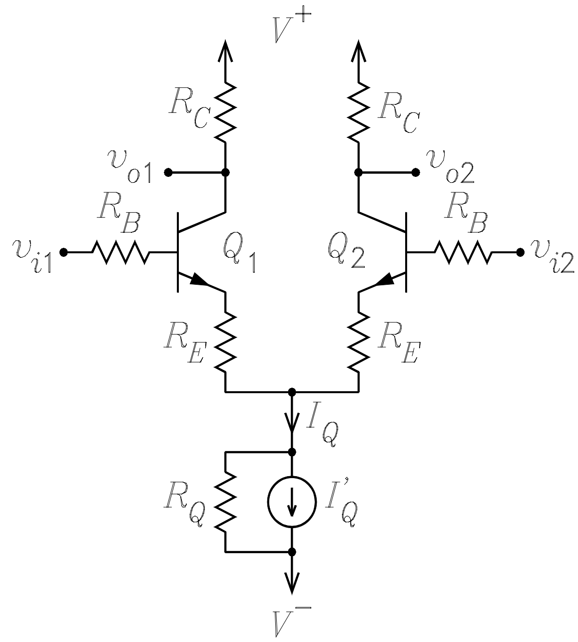

Figure 3: BJT Differential amplifier.

DC Solutions

This solution assumes that I'_Q is known. If I_Q is known, the solutions are the same as above.

(a) Zero both inputs. Divide the tail supply into two equal parallel current sources having a current $I'_Q/2$ in parallel with a resistor $2R_Q$. The circuit obtained for Q_1 is shown on the left in Fig. 4. The circuit for Q_2 is identical. Now make a Thévenin equivalent as shown in on the right in Fig. 4. This is the basic bias circuit.

(b) Make an "educated guess" for V_{BE}. Write the loop equation between the ground node to the left of R_B and V^-. To solve for I_E, this equation is

$$0 - \left(V^- - I'_Q R_Q\right) = \frac{I_E}{1+\beta} R_B + V_{BE} + I_E \left(R_E + 2R_Q\right)$$

(c) Solve the loop equation for the currents.

$$I_E = \frac{I_C}{\alpha} = (1+\beta)\, I_B = \frac{-V^- + I'_Q R_Q - V_{BE}}{R_B/(1+\beta) + R_E + 2R_Q}$$

(d) Verify that $V_{CB} > 0$ for the active mode.

$$V_{CB} = V_C - V_B = \left(V^+ - \alpha I_E R_C\right) - \left(-\frac{I_E}{1+\beta} R_B\right) = V^+ - \alpha I_E R_C + \frac{I_E}{1+\beta} R_B$$

(e) Calculate the collector-emitter voltage.

$$V_{CE} = V_C - V_E = V_C - (V_B - V_{BE}) = V_{CB} + V_{BE}$$

(f) If $R_Q = \infty$, it follows that $I_{E1} = I_{E2} = I'_Q/2$. If the current source is replaced with a resistor R_Q only, the currents are given by

$$I_E = \frac{I_C}{\alpha} = (1+\beta)\, I_B = \frac{-V^- - V_{BE}}{R_B/(1+\beta) + R_E + 2R_Q}$$

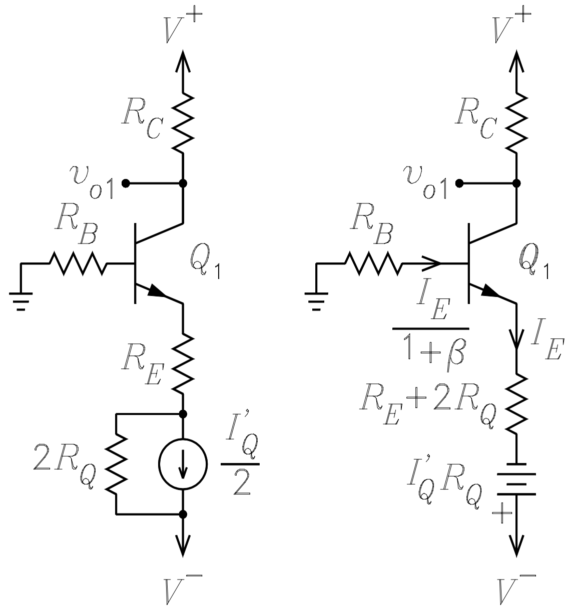

Figure 4: DC bias circuits for Q_1.

Small-Signal or AC Solutions

This solutions use the r_0 approximations.

(a) Calculate g_m, r_π, and r_{ie}.

$$g_m = \frac{\alpha I_E}{V_T} \qquad r_\pi = \frac{(1+\beta)\, V_T}{I_E} \qquad r_{ie} = \frac{R_B + r_x + r_\pi}{1+\beta} \qquad r_0 = \frac{V_A + V_{CE}}{\alpha I_E}$$

(b) Redraw the circuit with $V^+ = V^- = 0$ and $I'_Q = 0$. Replace the two BJTs with the emitter equivalent circuit. The emitter part of the circuit obtained is shown in 5(a).

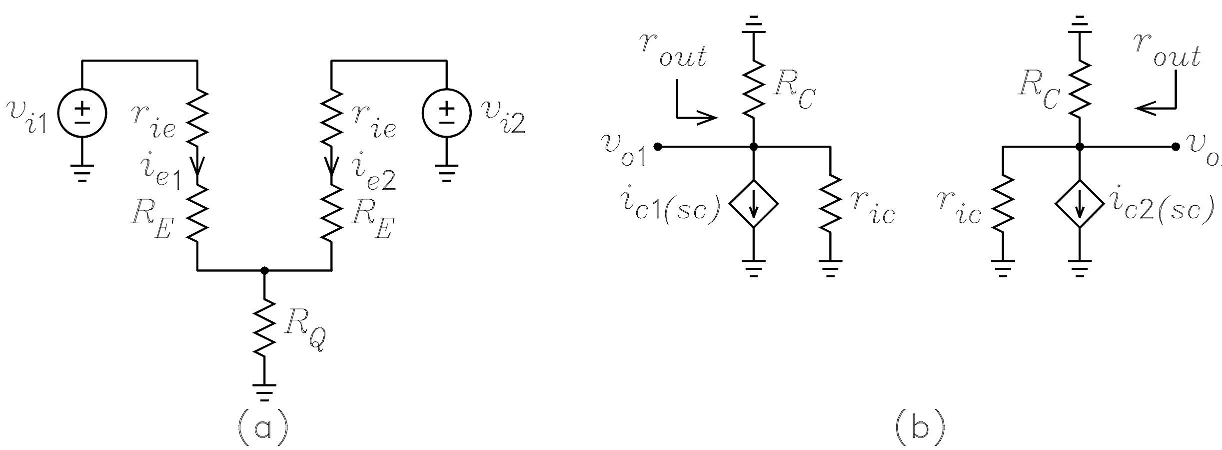

Figure 5: (a) Emitter equivalent circuit. (b) Collector equivalent circuits.

(c) Using superposition, Ohm's Law, and current division, solve for i_{e1} and i_{e2}.

$$i_{e1} = \frac{v_{i1}}{r_{ie} + R_E + R_Q \| \left(r_{ie} + R_E\right)} - \frac{v_{i2}}{r_{ie} + R_E + R_Q \| \left(r_{ie} + R_E\right)} \times \frac{R_Q}{R_Q + r_{ie} + R_E}$$

$$i_{e2} = \frac{v_{i2}}{r_{ie} + R_E + R_Q \| (r_{ie} + R_E)} - \frac{v_{i1}}{r_{ie} + R_E + R_Q \| (r_{ie} + R_E)} \times \frac{R_Q}{R_Q + r_{ie} + R_E}$$

For $R_Q = \infty$, these become

$$i_{e1} = \frac{v_{i1} - v_{i2}}{2 (r_{ie} + R_E)} \qquad i_{e2} = \frac{v_{i2} - v_{i1}}{2 (r_{ie} + R_E)}$$

(d) The circuit for v_{o1}, v_{o2}, r_{out1}, and r_{out2} is shown in Fig. 6.

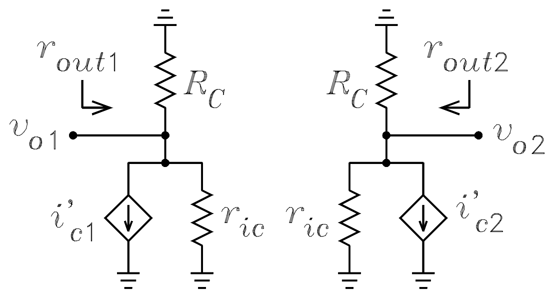

Figure 6: Circuits for calculating v_{o1}, v_{o2}, r_{out1}, and r_{out2}.

$$v_{o1} = -i'_{c1} \times r_{ic} \| R_C = -\alpha \times i_{e1} \times r_{ic} \| R_C = \frac{-\alpha \times r_{ic} \| R_C}{r_{ie} + R_E + R_Q \| (r_{ie} + R_E)} \left(v_{i1} - v_{i2} \frac{R_Q}{R_Q + r_{ie} + R_E} \right)$$

$$v_{o2} = -i'_{c2} \times r_{ic} \| R_C = -\alpha \times i_{e1} \times r_{ic} \| R_C = \frac{-\alpha \times r_{ic} \| R_C}{r_{ie} + R_E + R_Q \| (r_{ie} + R_E)} \left(v_{i2} - v_{i1} \frac{R_Q}{R_Q + r_{ie} + R_E} \right)$$

$$r_{out1} = r_{out2} = r_{ic} \| R_C$$

$$r_{ic} = r_0 \left[1 + \frac{\beta R_{te}}{R_B + r_\pi + R_{te}} \right] + (R_B + r_x + r_\pi) \| R_{te} \qquad R_{te} = R_E + R_Q \| (r_{ie} + R_E)$$

(e) The resistance seen looking into the v_{i1} (v_{i2}) input with $v_{i2} = 0$ ($v_{i1} = 0$) is

$$r_{ib} = R_B + r_x + r_\pi + (1 + \beta) R_{te}$$

(f) Special case for $R_Q = \infty$.

$$v_{o1} = \frac{-\alpha \times r_{ic} \| R_C}{2 (r_{ie} + R_E)} (v_{i1} - v_{i2}) \qquad v_{o2} = \frac{-\alpha \times r_{ic} \| R_C}{2 (r_{ie} + R_E)} (v_{i2} - v_{i1})$$

(g) The equivalent circuit seen looking into the two inputs is shown in Fig. 7. The resistors labeled r'_π are given by

$$r'_\pi = r_x + r_\pi + (1 + \beta) R_E$$

The differential input resistance r_{id} is defined the same way that it is defined for Fig. **??**. That is, it is the resistance seen between the two inputs when $v_{i1} = v_{id}/2$ and $v_{i2} = -v_{id}/2$, where v_{id} is the differential input voltage. In this case, the small-signal voltage at the upper node of the resistor $(1 + \beta) R_Q$ is zero so that no current flows it. It follows that r_{id} is given by

$$r_{id} = 2 \left(R_B + r'_\pi \right)$$

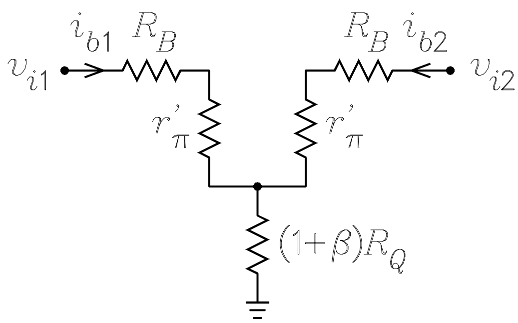

Figure 7: Equivalent circuits for calculating i_{b1} and i_{b2}.

Differential and Common-Mode Gains

(a) Define the common-mode and differential input voltages as follows:

$$v_{id} = v_{i1} - v_{i2} \qquad v_{icm} = \frac{v_{i1} + v_{i2}}{2}$$

With these definitions, v_{i1} and v_{i2} can be written

$$v_{i1} = v_{icm} + \frac{v_{id}}{2} \qquad v_{i2} = v_{icm} - \frac{v_{id}}{2}$$

By linearity, it follows that superposition of v_{icm} and v_{id} can be used to solve for the currents and voltages.

(b) Redraw the emitter equivalent circuit as shown in Fig. 8.

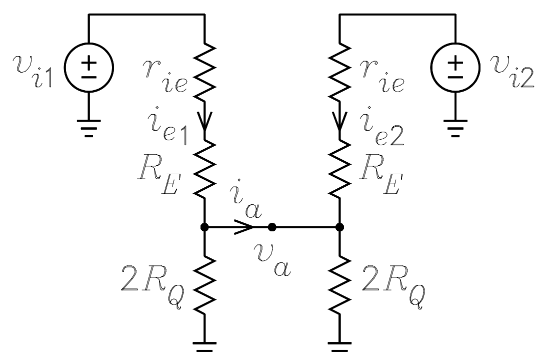

Figure 8: Emitter equivalent circuit.

(c) For $v_{i1} = v_{id}/2$ and $v_{i2} = -v_{id}/2$, it follows by superposition that $v_a = 0$ and

$$i_{e1} = \frac{v_{id}/2}{r_{ie} + R_E} \qquad i_{e2} = \frac{-v_{id}/2}{r_{ie} + R_E}$$

$$v_{o1} = -\alpha \times i_{e1} \times r_{ic} \| R_C = \frac{-\alpha \times r_{ic} \| R_C}{r_{ie} + R_E} \frac{v_{id}}{2} = \frac{-\alpha \times r_{ic} \| R_C}{r_{ie} + R_E} \left(\frac{v_{i1} - v_{i2}}{2} \right)$$

$$v_{o2} = -\alpha \times i_{e2} \times r_{ic} \| R_C = \frac{+\alpha \times r_{ic} \| R_C}{r_{ie} + R_E} \frac{v_{id}}{2} = \frac{+\alpha \times r_{ic} \| R_C}{r_{ie} + R_E} \left(\frac{v_{i1} - v_{i2}}{2} \right)$$

The differential voltage gain is given by

$$A_d = \frac{v_{o1}}{v_{id}} = -\frac{v_{o2}}{v_{id}} = -\frac{1}{2}\frac{\alpha r_{ic}\|R_C}{r_{ie}+R_E}$$

(d) For $v_{i1} = v_{i2} = v_{icm}$, it follows by superposition that $i_a = 0$ and

$$i_{e1} = \frac{v_{icm}}{r_{ie}+R_E+2R_Q} \qquad i_{e2} = \frac{v_{icm}}{r_{ie}+R_E+2R_Q}$$

$$v_{o1} = -\alpha \times i_{e1} \times r_{ic}\|R_C = \frac{-\alpha \times r_{ic}\|R_C}{r_{ie}+R_E+2R_Q} v_{icm} = \frac{-\alpha \times r_{ic}\|R_C}{r_{ie}+R_E+2R_Q}\left(\frac{v_{i1}+v_{i2}}{2}\right)$$

$$v_{o2} = -\alpha \times i_{e2} \times r_{ic}\|R_C = \frac{-\alpha \times r_{ic}\|R_C}{r_{ie}+R_E+2R_Q} v_{icm} = \frac{-\alpha \times r_{ic}\|R_C}{r_{ie}+R_E+2R_Q}\left(\frac{v_{i1}+v_{i2}}{2}\right)$$

The common-mode voltage gain is given by

$$A_{cm} = \frac{v_{o1}}{v_{icm}} = \frac{v_{o2}}{v_{icm}} = -\frac{\alpha \times r_{ic}\|R_C}{r_{ie}+R_E+2R_Q}$$

(e) If the output is taken from the collector of Q_1 or Q_2, the common-mode rejection ratio is given by

$$CMRR = \left|\frac{v_{o1}/v_{id}}{v_{o1}/v_{icm}}\right| = \left|\frac{v_{o2}/v_{id}}{v_{o2}/v_{icm}}\right| = \frac{1}{2}\frac{r_{ie}+R_E+2R_Q}{r_{ie}+R_E} = \frac{1}{2} + \frac{R_Q}{r_{ie}+R_E}$$

This can be expressed in dB.

$$CMRR_{dB} = 20\log\left(\frac{1}{2} + \frac{R_Q}{r_{ie}+R_E}\right)$$

Example 1 *For $I'_Q = 2\,\text{mA}$, $R_Q = 50\,\text{k}\Omega$, $R_B = 1\,\text{k}\Omega$, $R_E = 100\,\Omega$, $R_C = 10\,\text{k}\Omega$, $V^+ = 20\,\text{V}$, $V^- = -20\,\text{V}$, $V_T = 0.025\,\text{V}$, $r_x = 20\,\Omega$, $\beta = 99$, $V_{BE} = 0.65\,\text{V}$, and $V_A = 50\,\text{V}$, calculate v_{o1}, v_{o2}, v_{od}, r_{out}, and $CMRR$.*

Solution.

$$I_E = \frac{0 - \left(V^- - I'_Q R_Q\right) - V_{BE}}{R_B/(1+\beta) + R_E + 2R_Q} = 1.192\,\text{mA}$$

$$V_{CB} = V_C - V_B = \left(V^+ - \alpha I_E R_C\right) - \left(-\frac{I_E}{1+\beta}R_B\right) = 8.209\,\text{V}$$

$$g_m = \frac{\alpha I_E}{V_T} = 0.0472\,\text{S} \qquad r_\pi = \frac{(1+\beta)V_T}{I_E} = 2.097\,\text{k}\Omega$$

$$r_e = \frac{V_T}{I_E} = 20.97\,\Omega \qquad r_{ie} = \frac{R_B + r_x}{1+\beta} + r_e = 31.17\,\Omega$$

$$r_0 = \frac{V_A + V_{CE}}{I_C} = 49.869\,\text{k}\Omega \qquad R_{te} = R_E + R_Q\|(r_{ie}+R_E) = 230.83\,\Omega$$

$$r_{ic} = r_0\left[1 + \frac{\beta(2R_E + r_{ie})}{R_B + r_\pi + 2R_E + r_{ie}}\right] + (R_B + r_\pi)\|(2R_E + r_{ie}) = 390.5\,\text{k}\Omega$$

$$v_{o1} = \frac{-\alpha \times r_{ic}\|R_C}{r_{ie}+R_E+R_Q\|(r_{ie}+R_E)}\left(v_{i1} - v_{i2}\frac{R_Q}{R_Q+r_{ie}+R_E}\right) = -36.84v_{i1} + 36.75v_{i2}$$

$$v_{o2} = -36.84v_{i2} + 36.75v_{i1}$$

$$r_{out} = r_{ic} \| R_C = 9.75\,\mathrm{k\Omega}$$

$$A_{vd} = -\frac{1}{2} \frac{\alpha \times r_{ic} \| R_C}{r_{ie} + R_E} = -36.80$$

$$A_{vcm} = \frac{-\alpha \times r_{ic} \| R_C}{r_{ie} + R_E + 2R_Q} = -0.0964$$

$$CMRR_{dB} = 20 \log \left| \frac{A_{vd}}{A_{vcm}} \right| = 51.63\,\mathrm{dB}$$

The Diff Amp with an Active Load

Figure 9 shows a BJT diff amp with an active load formed by a current mirror with base current compensation. Similar circuits are commonly seen as the input stages of operational amplifiers and audio amplifiers. The object is to solve for the open-circuit output voltage v_{oc}, the short-circuit output current i_{sc}, and the output resistance r_{out}. By Thévenin's theorem, these are related by the equation $v_{oc} = i_{sc} r_{out}$. It will be assumed that the current mirror consisting of transistors $Q_3 - Q_5$ is perfect so that its output current is equal to its input current, i.e. $i_{c4} = i_{c1}$. In addition, the r_0 approximations will be used in solving for the currents. That is, the Early effect will be neglected except in solving for r_{out}. For the bias solution, it will be assumed that the tail bias current I_Q splits equally between Q_1 and Q_2 so that $I_{E1} = I_{E2} = I_Q/2$.

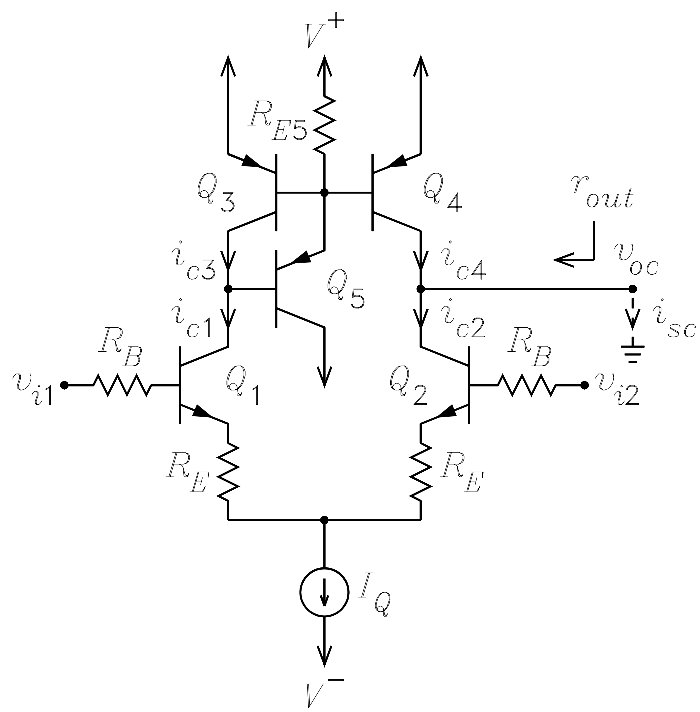

Figure 9: Diff amp with active current-mirror load.

Because the tail supply is assumed to be a current source, the common-mode gain of the circuit is zero when the r_0 approximations are used. In this case, it can be assumed that the two input signals are pure differential signals that can be written $v_{i1} = v_{id}/2$ and $v_{i2} = -v_{id}/2$. For differential input signals, it follows by symmetry that the signal voltage is zero at the node above the tail current supply I_Q. Following the analysis above, the small-signal collector currents in Q_1 and Q_2 are given by

$$i_{c1(sc)} = \frac{\alpha}{r_{ie} + R_E} \frac{v_{id}}{2} \qquad i_{c2(sc)} = -\frac{\alpha}{r_{ie} + R_E} \frac{v_{id}}{2}$$

where

$$r_{ie} = \frac{R_B + r_\pi}{1 + \beta}$$

The short-circuit output current is given by

$$i_{sc} = i_{c4} - i_{c2}$$

With $i_{c4} = i_{c3} = i_{c1}$ and $i_{c2} = -i_{c1}$, this becomes

$$i_{sc} = 2i_{c1(sc)} = \frac{\alpha}{r_{ie} + R_E} v_{id} = \frac{\alpha}{r_{ie} + R_E} (v_{i1} - v_{i2})$$

The output resistance is given by

$$r_{out} = r_{04} \| r_{ic2}$$

where r_{ic2} is given by

$$r_{ic2} = r_0 \left[1 + \frac{\beta (2R_E + r_{ie})}{R_B + r_\pi + 2R_E + r_{ie}}\right] + (R_B + r_\pi) \| (2R_E + r_{ie})$$

By Thévenin's theorem, the small-signal open-circuit output voltage is given by

$$v_{oc} = i_{sc} r_{out} = \frac{\alpha (r_{04} \| r_{ic2})}{r_{ie} + R_E} (v_{i1} - v_{i2})$$

Example 2 *For $I_Q = 2\,\text{mA}$, $R_B = 100\,\Omega$, $R_E = 51\,\Omega$, $V^+ = 15\,\text{V}$, $V^- = -15\,\text{V}$, $V_T = 0.025\,\text{V}$, $r_x = 50\,\Omega$, $\beta = 99$, $\alpha = 0.99$ $V_{BE1} = V_{BE2} = 0.65\,\text{V}$, $V_{EB3} = V_{EB4} = V_{EB5} = 0.65\,\text{V}$, $V_{C2} = V_{C4} = 13.7\,\text{V}$, and $V_A = 50\,\text{V}$, calculate i_{sc}, r_{out}, and v_{oc}. Because $r_x > 0$, we add it to R_B in the equations above.*

Solution.

$$r_{e1} = r_{e2} = \frac{2V_T}{I_Q} = 25\,\Omega \qquad r_{ie1} = r_{ie2} = \frac{R_B + r_x}{1 + \beta} + r_e = 26.5\,\Omega$$

$$R_{te2} = 2R_E + r_{ie1} = 128.5\,\Omega \qquad i_{sc} = \frac{\alpha}{r_{ie} + R_E} (v_{i1} - v_{i2}) = 0.0128\,(v_{i1} - v_{i2})$$

$$r_{02} = \frac{V_A + (V_{C2} + V_{BE})}{\alpha I_Q / 2} = 65\,\text{k}\Omega \qquad r_{ic2} = r_0 \left[1 + \frac{\beta R_{te}}{R_B + r_\pi + R_{te}}\right] + (R_B + r_\pi) \| R_{te} = 362.7\,\text{k}\Omega$$

$$r_{04} = \frac{V_A + (V^+ - V_{C4})}{I_Q} = 51.82\,\text{k}\Omega \qquad r_{out} = r_{04} \| r_{ic2} = 45.34\,\text{k}\Omega$$

$$v_{oc} = i_{sc} r_{out} = 579.2\,(v_{i1} - v_{i2})$$

This is a dB gain of 55.3 dB.