Frequency Response of the CE Amplifier

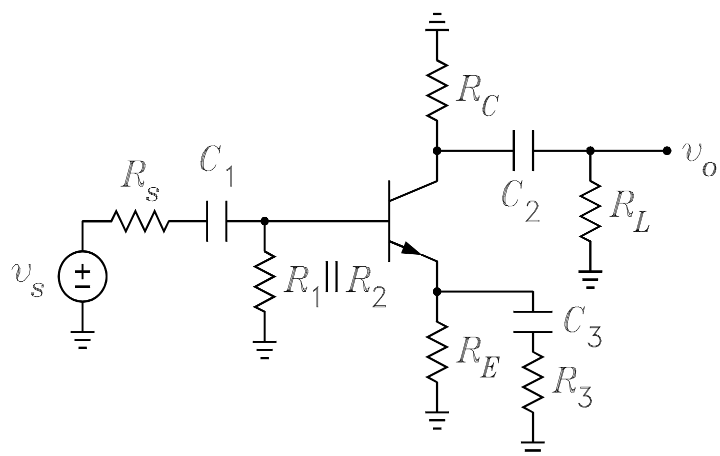
Midband Voltage Gain

The figure shows the signal circuit of the common-emitter amplifier. There are three capacitors in the circuit. At the mid frequency band, these are considered to be short circuits. When r_0 is neglected except in calculating the collector output impedance r_{ic}, the midband voltage gain from v_s to v_o can be written

$$A_v = \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \times \frac{1}{r_{ie} + R_{te}} \times \alpha \times (-r_{ic} \| R_C \| R_L)$$

$$R_{tb} = R_S \| R_1 \| R_2 \qquad R_{te} = R_E \| R_3 \qquad r_{ie} = \frac{R_{tb} + r_x + r_\pi}{1+\beta} \qquad r_{ic} = r_0 \left(1 + \frac{\beta R_{te}}{R_{tb} + r_x + r_\pi + R_{te}}\right) + R_{te}$$

This solution corresponds to the third solution found in the class notes on the common-emitter amplifier.



Effect of C_1

At low frequencies, C_1 is an open circuit and the gain is zero. Thus C_1 has a high pass effect on the gain, i.e. it affects the lower cutoff frequency of the amplifier. To account for C_1, A_v is multiplied by the high-pass transfer function

$$T_1(s) = \frac{\tau_1 s}{1 + \tau_1 s}$$

where τ_1 is the time constant for C_1. The worst case time constant for the calculation of the lower cutoff frequency is the smallest value, i.e. the value which predicts the highest pole frequency. For this to be the case, the base input resistance r_{ib} must be calculated with C_3 a short circuit. This makes r_{ib} its smallest possible value. Imagine C_1 being replaced with an ohmmeter with the source zeroed. The time constant is given by the resistance measured by the ohmmeter multiplied by C_1.

$$\tau_1 = (R_S + R_1 \| R_2 \| r_{ib})\, C_1 \qquad r_{ib} = r_x + r_\pi + (1+\beta)\, R_{te}$$

The pole frequency is given by

$$f_1 = \frac{1}{2\pi\tau_1}$$

Effect of C_2

Capacitor C_2 also has a high pass effect on the gain. To account for C_2, A_v is multiplied by the high-pass transfer function

$$T_2(s) = \frac{\tau_2 s}{1 + \tau_2 s}$$

where τ_2 is the time constant for C_2. The worst case time constant for the calculation of the lower cutoff frequency is the smallest value, i.e. the value which predicts the highest pole frequency. For this to be the case, the collector input resistance r_{ic} must be calculated with C_1 and C_3 short circuits. This makes r_{ic} its smallest possible value. Imagine C_2 being replaced with an ohmmeter with the source zeroed. The time constant is given by the resistance measured by the ohmmeter multiplied by C_2.

$$\tau_2 = (R_C \| r_{ic} + R_L) C_2 \qquad r_{ic} = r_0 \left(1 + \frac{\beta R_{te}}{R_{tb} + r_x + r_\pi + R_{te}}\right) + R_{te}$$

The pole frequency is given by

$$f_2 = \frac{1}{2\pi\tau_2}$$

Effect of C_3

When capacitor C_3 is an open circuit or a short circuit, the gain is not zero. Thus C_3 must have the effect of a shelving transfer function. The gain is the highest when R_{te} is has the smallest value. This occurs when C_3 is a short circuit. Thus C_3 must have a high pass shelving effect on the gain. The expression for A_v above shows that the gain is inversely proportional to $r_{ie} + R_{te}$. Let this impedance be denoted by Z_e. When C_3 is included, the two-terminal impedance theorem can be used to write

$$Z_e = (r_{ie} + R_E) \frac{1 + (r_{ie} \| R_E + R_3) C_3 s}{1 + (R_E + R_3) C_3 s}$$

It follows that C_3 can be accounted for by multiplying A_v by the shelving transfer function

$$T_3(s) = K \frac{1 + \tau_{3z} s}{1 + \tau_{3p} s} \qquad K = \frac{\tau_{3p}}{\tau_{3z}}$$

The value of K is chosen to make the high-frequency asymptotic value of $T_3(j\omega)$ unity. From the expression for Z_e, the time constants τ_{3p} and τ_{3z} are given by

$$\tau_{3p} = (r_{ie} \| R_E + R_3) C_3 \qquad r_{ie} = \frac{R_{tb} + r_x}{1 + \beta} + r_e \qquad \tau_{3z} = (R_E + R_3) C_3$$

The pole and zero frequencies are given by

$$f_{3p} = \frac{1}{2\pi\tau_{3p}} \qquad f_{3z} = \frac{1}{2\pi\tau_{3z}}$$

Worst-Case Lower Cutoff Frequency f_L

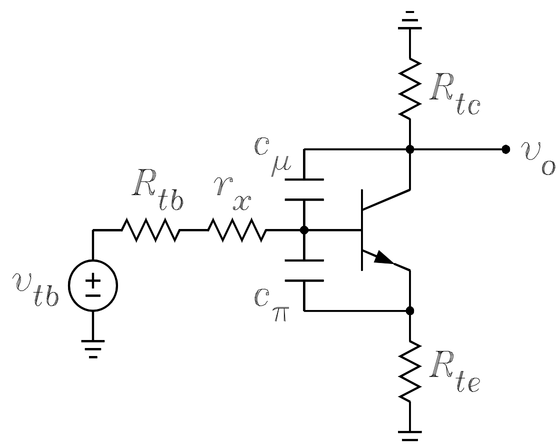
The lower cutoff frequency of the amplifier is approximately given by

$$f_L \simeq \sqrt{\sum_i f_{pi}^2 - 2\sum_i f_{zi}^2}$$

where f_{pi} are the pole frequencies and f_{zi} are the zero frequencies. This equation gives the worst case value for f_L. That is, the actual lower cutoff frequency cannot be larger than the value predicted by this equation. The frequency that dominates is the highest pole frequency.

High-Frequency Circuit

The figure shows that high-frequency equivalent circuit. The internal capacitors c_π and c_μ cause the high-frequency gain to roll off. Each has a low-pass effect on the voltage gain. Note that both connect to the internal base node (the B' node). At high frequencies, C_1 through C_3 are all short circuits. The time constant for c_π is calculated with c_μ an open circuit and the time constant for c_μ is calculated with c_π an open circuit. Because c_μ connects the collector output back into the base circuit, it must be replaced by separate capacitors from base to ground and from collector to ground using the Miller theorem.



Effect of c_μ

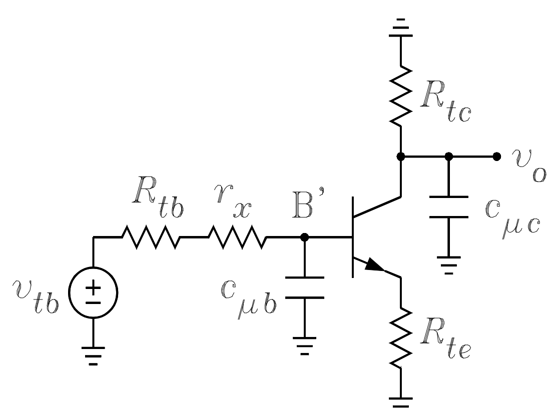
In applying the Miller theorem, a capacitor $c_{\mu b}$ is placed from the B' node to ground and a capacitor $c_{\mu c}$ is placed from collector node to ground. These are given by

$$c_{\mu b} = (1 - K)\, c_\mu \qquad c_{\mu c} = c_\mu$$

where K is the voltage gain from the B' node to the collector node. This is given by the equation for A_v with $R_{tb} = 0$ and $r_x = 0$.

$$K = \frac{-r_{ic} \| R_C \| R_L}{\dfrac{1}{g_m} + \dfrac{R_{te}}{\alpha}} = \frac{-r_{ic} \| R_C \| R_L}{\dfrac{r_e + R_{te}}{\alpha}}$$

Because K is negative, $1 - K$ is a positive number. The equivalent circuit is shown in the figure.



The pole time constant for c_μ is given by the sum of the time constants for $c_{\mu b}$ and $c_{\mu c}$. Imagine $c_{\mu b}$ being replaced with an ohmmeter. The time constant for $c_{\mu b}$ is given by the resistance measured by the ohmmeter multiplied by $c_{\mu b}$.

$$\tau_{\mu b} = \left[(R_{tb} + r_x) \parallel (r'_{ib}) \right] c_{\mu b} \qquad r'_{ib} = r_\pi + (1+\beta) R_{te}$$

Imagine $c_{\mu c}$ being replaced with an ohmmeter. The time constant for $c_{\mu c}$ is given by the resistance measured by the ohmmeter multiplied by $c_{\mu c}$.

$$\tau_{\mu c} = (r_{ic} \| R_C \| R_L) c_{\mu c}$$

The time constant for c_μ is the sum of these two time constants.

$$\tau_\mu = \left[(R_{tb} + r_x) \parallel (r_{ib} - r_x) \right] c_{\mu b} + (r_{ic} \| R_C \| R_L) c_{\mu c}$$

The pole frequency caused by c_μ is given by

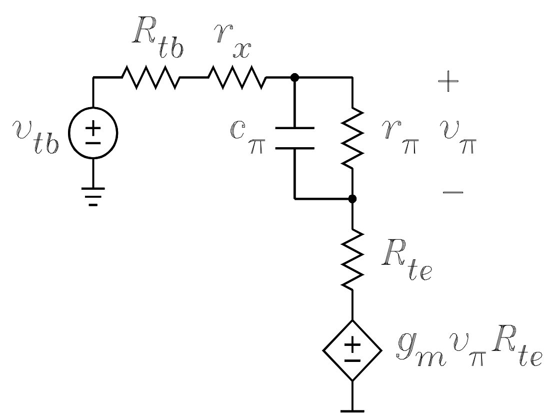
$$f_\mu = \frac{1}{2\pi \tau_\mu}$$

Effect of c_π

In the π model, c_π is in parallel with r_π. The collector current is proportional to the voltage v_π across this parallel combination. When c_π becomes a short circuit at high frequencies, the voltage v_π is zero. Thus c_π must have a low-pass filter effect. To calculate the time constant, it will be assumed that r_0 is an open circuit in the small-signal model. Looking out of the emitter in the π model, the Thévenin voltage and resistance are given by

$$v_{th} = g_m v_\pi R_{te} \qquad R_{th} = R_{te}$$

The figure shows the π model of the base-emitter circuit with the Thévenin equivalent emitter circuit.



Voltage division can be used to write the equation by inspection for v_π as a function of the difference voltage $(v_{tb} - g_m v_\pi R_{te})$. The pole time constant is the time constant for c_π. The equation for v_π is

$$v_\pi = (v_{tb} - g_m v_\pi R_{te}) \frac{\dfrac{r_\pi}{1 + r_\pi c_\pi s}}{R_{tb} + r_x + \dfrac{r_\pi}{1 + r_\pi c_\pi s} + R_{te}}$$

Note that v_π occurs on both sides of the equal sign. The equation can be solved for v_π to obtain

$$v_\pi = v_{tb} \frac{\dfrac{r_\pi}{R_{tb} + r_x + r_\pi + R_{te}}}{1 + g_m R_{te} \dfrac{r_\pi}{R_{tb} + r_x + r_\pi + R_{te}}} \frac{1}{1 + \tau_\pi s}$$

where

$$\tau_\pi = \frac{(R_{tb} + r_x + R_{te}) r_\pi}{R_{tb} + r_x + r_\pi + (1+\beta) R_{te}} c_\pi$$

The pole frequency caused by c_π is

$$f_\pi = \frac{1}{2\pi\tau_\pi}$$

For the case $R_{te} = 0$, c_π connects to ground from the B' node. In this case c_π can be added to the capacitor $c_{\mu b}$ obtained above with the Miller Theorem. The time constant for the sum of the two capacitors is

$$\tau_{sum} = [(R_{tb} + r_x) \| r_\pi] (c_\pi + c_{\mu b})$$

Worst-Case Upper Cutoff Frequency f_U

The upper cutoff frequency of the amplifier is approximately given by

$$f_U \simeq \left(\sqrt{\sum_i f_{pi}^{-2} - 2 \sum_i f_{zi}^{-2}} \right)^{-1}$$

where f_{pi} are the pole frequencies and f_{zi} are the zero frequencies. Note that there are no zero frequencies predicted by the analysis. This equation gives the worst case value for f_U. That is, the actual upper cutoff frequency cannot be smaller than the value predicted by the equation. The frequency that dominates is the lowest pole frequency.