The Common-Source Amplifier

Basic Circuit

Fig. 1 shows the circuit diagram of a single stage common-emitter amplifier. The object is to solve for the small-signal voltage gain, input resistance, and output resistance.

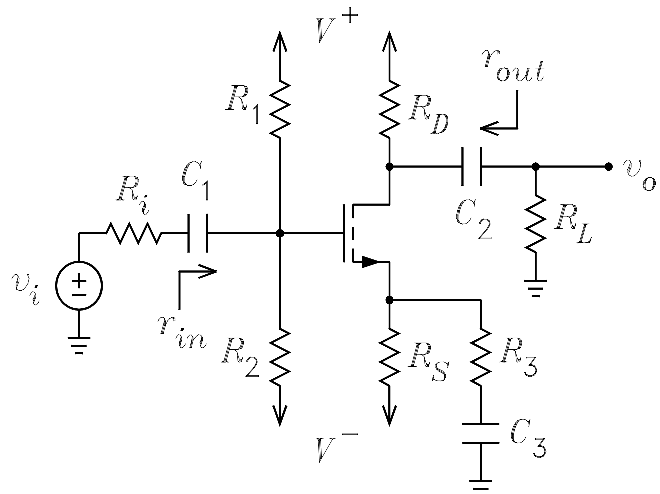

Figure 1: Common-source amplifier.

DC Solution

(a) Replace the capacitors with open circuits. Look out of the 3 MOSFET terminals and make Thévenin equivalent circuits as shown in Fig. 2.

$$V_{GG} = \frac{V^+R_2 + V^-R_1}{R_1 + R_2} \qquad R_{GG} = R_1 \| R_2$$

$$V_{SS} = V^- \qquad R_{SS} = R_S \qquad V_{DD} = V^+ \qquad R_{DD} = R_D$$

(b) Write the loop equation between the V_{GG} and the V_{SS} nodes.

$$V_{GG} - V_{SS} = V_{GS} + I_S R_{SS} = V_{GS} + I_D R_{SS}$$

(c) Use the equation for the drain current to solve for V_{GS}.

$$V_{GS} = \sqrt{\frac{I_D}{K}} + V_{TO}$$

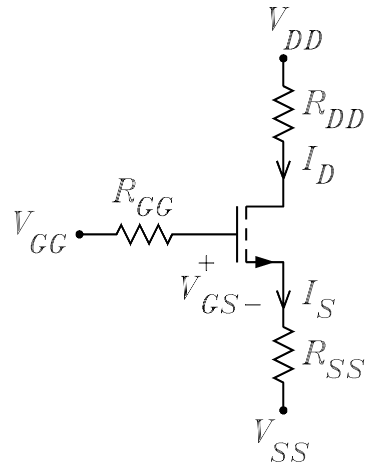

Figure 2: Bias circuit.

(d) Solve the equations simultaneously.

$$I_D R_{SS} + \sqrt{\frac{I_D}{K}} + [(V_{GG} - V_{SS}) - V_{TO}] = 0$$

(e) Let $V_1 = (V_{GG} - V_{SS}) - V_{TO}$. Solve the quadratic for I_D.

$$I_D = \left(\frac{\sqrt{1 + 4KV_1 R_{SS}} - 1}{2\sqrt{K} R_{SS}}\right)^2$$

(d) Verify that $V_{DS} > V_{GS} - V_{TO} = \sqrt{I_D/K}$ for the active mode.

$$V_{DS} = V_D - V_S = (V_{DD} - I_D R_{DD}) - \left(V^- + I_D R_{SS}\right) = V_{DD} - V_{SS} - I_D R_{DD}$$

Small-Signal or AC Solutions

(a) Redraw the circuit with $V^+ = V^- = 0$ and all capacitors replaced with short circuits as shown in Fig. 3.

(b) Calculate g_m, r_s, and r_0 from the DC solution.

$$g_m = 2\sqrt{KI_D} \qquad r_s = \frac{1}{g_m} \qquad r_0 = \frac{\lambda^{-1} + V_{DS}}{I_D}$$

(c) Replace the circuits looking out of the gate and source with Thévenin equivalent circuits as shown in Fig. 4.

$$v_{tg} = v_i \frac{R_1 \| R_2}{R_i + R_1 \| R_2} \qquad R_{tg} = R_1 \| R_2 \qquad v_{te} = 0 \qquad R_{ts} = R_S \| R_3$$

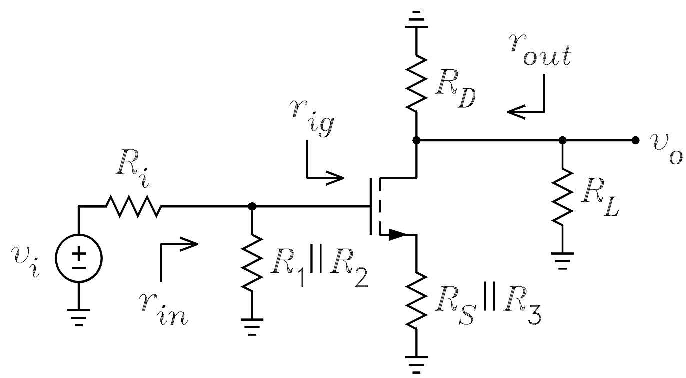

Figure 3: Signal circuit.

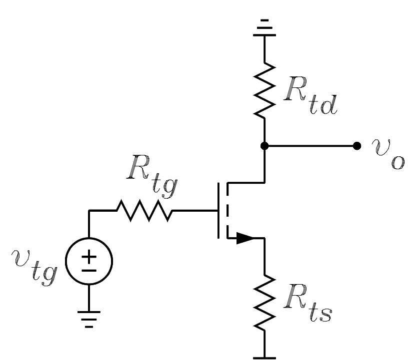

Figure 4: Signal circuit with Thévenin gate circuit.

Exact Solution

(a) Replace the circuit seen looking into the drain with its Norton equivalent circuit as shown in Fig. 5. Solve for $i_{d(sc)}$.

$$i_{d(sc)} = G_{mg} v_{tg} = G_{mg} v_i \frac{R_1 \| R_2}{R_i + R_1 \| R_2}$$

$$G_{mg} = \frac{1}{r_s + R_{ts} \| r_0} \frac{r_0}{r_0 + R_{ts}}$$

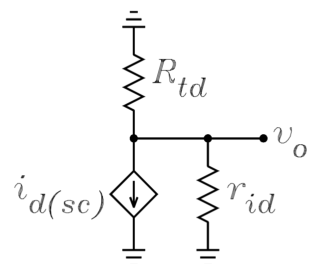

Figure 5: Norton drain circuit.

(b) Solve for v_o.

$$v_o = -i_{d(sc)} r_{id} \| R_D \| R_L = -G_{mg} v_i \frac{R_1 \| R_2}{R_i + R_1 \| R_2} r_{id} \| R_D \| R_L$$

$$r_{id} = \frac{r_0 + r_s \| R_{ts}}{1 - R_{ts} / (r_s + R_{te})} = r_0 \left(1 + \frac{R_{ts}}{r_s'}\right) + R_{ts}$$

(c) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_i} = -G_{ms} \frac{R_1 \| R_2}{R_i + R_1 \| R_2} r_{id} \| R_D \| R_L$$

(d) Solve for r_{in}.

$$r_{in} = R_1 \| R_2$$

(e) Solve for r_{out}.

$$r_{out} = r_{id} \| R_D$$

(d) Special case for $R_{ts} = 0$.

$$G_{mg} = \frac{1}{r_s} = g_m \qquad r_{id} = r_0$$

Example 1 *For the CS amplifier of Fig.* **??**, *it is given that* $R_i = 5\,\mathrm{k\Omega}$, $R_1 = 5\,\mathrm{M\Omega}$, $R_2 = 1\,\mathrm{M\Omega}$, $R_D = 10\,\mathrm{k\Omega}$, $R_S = 3\,\mathrm{k\Omega}$, $R_3 = 50\,\Omega$, $R_L = 20\,\mathrm{k\Omega}$, $V^+ = 24\,\mathrm{V}$, $V^- = -24\,\mathrm{V}$, $K_0 = 0.001\,\mathrm{A/V^2}$, $V_{TO} = 1.75\,\mathrm{V}$, $\lambda = 0.016\,\mathrm{V^{-1}}$. *Solve for the gain* $A_v = v_o/v_i$, *the input resistance* r_{in}, *and the output resistance* r_{out}. *The capacitors can be assumed to be ac short circuits at the operating frequency.*

Solution. For the dc bias solution, replace all capacitors with open circuits. The Thévenin voltage and resistance seen looking out of the gate are

$$V_{GG} = \frac{V^+ R_2 + V^- R_1}{R_1 + R_2} = -16\,\mathrm{V} \qquad R_{BB} = R_1 \| R_2 = 833.3\,\mathrm{k\Omega}$$

The Thévenin voltage and resistance seen looking out of the source are $V_{SS} = V^-$ and $R_{SS} = R_S$. To calculate I_D, we neglect the Early effect by setting $K = K_0$. The bias equation for I_D is

$$I_D = \left(\frac{\sqrt{1 + 4KV_1 R_{SS}} - 1}{2\sqrt{K} R_{SS}} \right)^2 = 1.655\,\mathrm{mA}$$

To test for the active mode, we calculate the drain-source voltage

$$V_{DS} = V_D - V_S = \left(V^+ - I_D R_D\right) - \left(V^- + I_D R_{SS}\right) = 26.491\,\mathrm{V}$$

This must be greater than $V_{GS} - V_{TO} = \sqrt{I_D/K} = 1.286\,\mathrm{V}$. It follows that the MOSFET is biased in its active mode.

For the small-signal ac analysis, we need g_m, r_s, and r_0. When the Early effect is accounted for, the new value of K is given by

$$K = K_0 \left(1 + \lambda V_{DS}\right) = 1.424 \times 10^{-3}\,\mathrm{A/\,V^2}$$

Note that this is an approximation because the Early effect was neglected in calculating V_{DS}. However, the approximation should be close to the true value. It follows that g_m, r_s, and r_0 are given by

$$g_m = 2\sqrt{K I_D} = 3.07 \times 10^{-3}\,\mathrm{A/\,V} \qquad r_s = \frac{1}{g_m} = 325.758\,\Omega$$

$$r_0 = \frac{\lambda^{-1} + V_{DS}}{I_D} = 53.78\,\mathrm{k\Omega}$$

For the small-signal analysis, V^+ and V^- are zeroed and the three capacitors are replaced with ac short circuits. The Thévenin voltage and resistance seen looking out of the gate are given by

$$v_{tg} = v_i \frac{R_1 \| R_2}{R_i + R_1 \| R_2} = 0.994 v_i \qquad R_{tg} = R_i \| R_1 \| R_2 = 4.97\,\mathrm{k\Omega}$$

The Thévenin resistances seen looking out of the source and the drain are

$$R_{ts} = R_S \| R_3 = 49.18\,\Omega \qquad R_{td} = R_D \| R_L = 6.667\,\mathrm{k\Omega}$$

Next, we calculate G_{mg} and r_{id}

$$G_{mg} = \frac{1}{r_s + R_{ts} \| r_0} \, \frac{r_0}{r_0 + R_{ts}} = \frac{1}{375.237}\,\mathrm{S}$$

$$r_{id} = r_0 \left(1 + \frac{R_{ts}}{r_s}\right) + R_{ts} = 61.95\,\mathrm{k\Omega}$$

The output voltage is given by

$$v_o = -G_{mg} \times (r_{id} \| R_{td})\, v_{tg} = -G_{mg} \times (r_{id} \| R_{td}) \times 0.916 v_i = -15.945 v_i$$

Thus the voltage gain is

$$A_v = \frac{v_o}{v_i} = -15.945$$

The input and output resistances are given by

$$r_{in} = R_1 \| R_2 = 833.3\,\mathrm{k\Omega} \qquad r_{out} = r_{id} \| R_D = 8.61\,\mathrm{k\Omega}$$

Approximate Solutions

These solutions assume that $r_0 = \infty$ except in calculating r_{id}. In this case, $i_{d(sc)} = i'_d = i'_s$.

Source Equivalent Circuit Solution

(a) After making the Thévenin equivalent circuits looking out of the gate and source, replace the MOSFET with the source equivalent circuit as shown in Fig. 6.

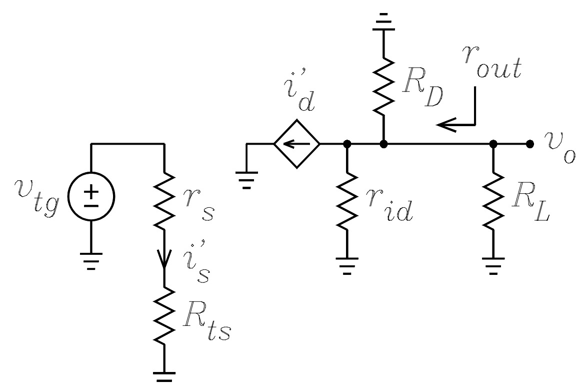

Figure 6: Source equivalent circuit.

(b) Solve for $i'_d = i'_s$ and r_{id}.

$$v_{tg} = i'_s\,(r'_e + R_{te}) = i'_d\,(r_s + R_{ts}) \Longrightarrow i'_d = v_{tg}\frac{1}{r_s + R_{ts}}$$

$$r_{id} = \frac{r_0 + r_s \| R_{ts}}{1 - R_{ts}/\,(r_s + R_{te})} \overset{\text{or}}{=} r_0 \left(1 + \frac{R_{ts}}{r'_s}\right) + R_{ts}$$

(c) Solve for v_o and $A_v = v_o/v_i$.

$$v_o = -i'_d r_{id} \| R_D \| R_L = v_{tg}\frac{-1}{r_s + R_{ts}} r_{id} \| R_D \| R_L = -v_i \frac{R_1 \| R_2}{R_i + R_1 \| R_2} \frac{1}{r_s + R_{ts}} r_{id} \| R_D \| R_L$$

$$R_{ts} = R_S \| R_3$$

$$A_v = \frac{v_o}{v_i} = -\frac{R_1 \| R_2}{R_i + R_1 \| R_2} \frac{1}{r_s + R_{ts}} r_{id} \| R_D \| R_L$$

Note that this is of the form

$$A_v = \frac{v_{tg}}{v_i} \times \frac{i'_s}{v_{tg}} \times \frac{i'_d}{i'_s} \times \frac{v_o}{i'_d}$$

(d) Solve for r_{out}.

$$r_{out} = r_{id} \| R_D$$

Example 2 *Use the simplified T-model solutions to calculate the values of A_v, r_{in}, and r_{out} for Example 1.*

$$A_v = 0.994 \times \left(2.667 \times 10^{-3}\right) \times \left(-6.019 \times 10^3\right) = -15.957$$

$$r_{in} = 833.3\,\mathrm{k\Omega} \qquad r_{id} = 61.95\,\mathrm{k\Omega} \qquad r_{out} = 8.61\,\mathrm{k\Omega}$$

π Model Solution

(a) After making the Thévenin equivalent circuits looking out of the gate and source, replace the MOSFET with the π model as shown in Fig. 7.

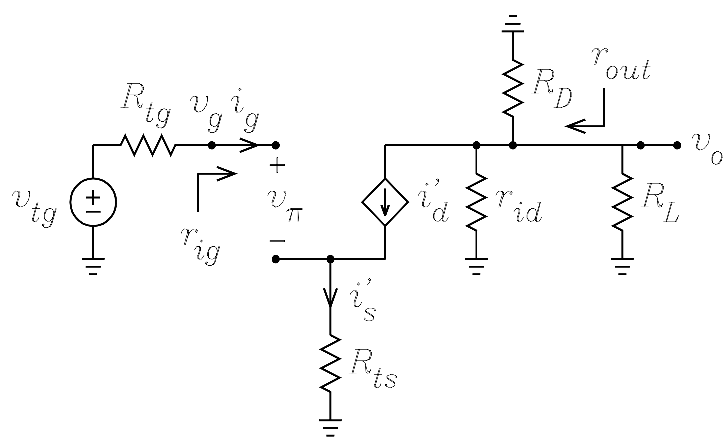

Figure 7: Hybrid π model circuit.

(b) Solve for i'_d and r_{id}.

$$v_{tg} = v_\pi + i'_s R_{ts} = \frac{i'_d}{g_m} + i'_d R_{ts} \Longrightarrow i'_d = \frac{v_{tg}}{\dfrac{1}{g_m} + R_{ts}}$$

$$r_{id} = \frac{r_0 + r_s \| R_{ts}}{1 - R_{ts}/\left(r_s + R_{te}\right)} = r_0 \left(1 + \frac{R_{ts}}{r'_s}\right) + R_{ts}$$

(c) Solve for v_o.

$$v_o = -i'_d r_{id} \| R_D \| R_L = -\frac{v_{tg}}{\dfrac{1}{g_m} + R_{ts}} r_{id} \| R_D \| R_L = v_i \frac{R_1 \| R_2}{R_i + R_1 \| R_2} \frac{-r_{id} \| R_D \| R_L}{\dfrac{1}{g_m} + R_{ts}}$$

(d) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_i} = \frac{R_1 \| R_2}{R_i + R_1 \| R_2} \frac{1}{\frac{1}{g_m} + R_{ts}} \left(-r_{id} \| R_D \| R_L\right)$$

This is of the form

$$A_v = \frac{v_{tg}}{v_i} \times \frac{i'_d}{v_{tg}} \times \frac{v_o}{i'_d}$$

(e) Solve for r_{in}.

$$r_{in} = R_1 \| R_2$$

(f) Solve for r_{out}.

$$r_{out} = r_{id} \| R_D$$

Example 3 *Use the* π*-model solutions to calculate the values of* A_v*,* r_{in}*, and* r_{out} *for Example 1.*

$$A_v = 0.994 \times \left(2.667 \times 10^{-3}\right) \times \left(-6.019 \times 10^3\right) = -15.957$$

$$r_{in} = 833.3\,\mathrm{k\Omega} \qquad r_{id} = 61.95\,\mathrm{k\Omega} \qquad r_{out} = 8.61\,\mathrm{k\Omega}$$

T Model Solution

(a) After making the Thévenin equivalent circuits looking out of the gate and source, replace the MOSFET with the T model as shown in Fig. 8.

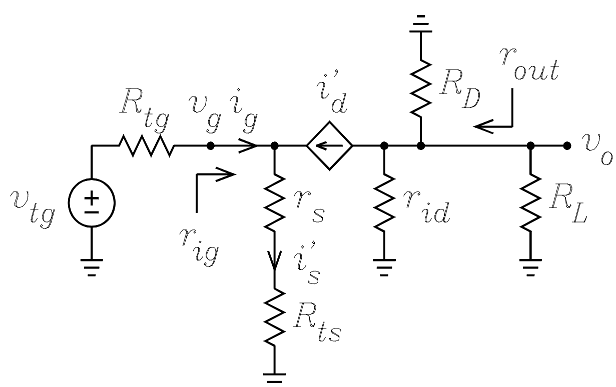

Figure 8: T model circuit.

(b) Solve for i'_d.

$$v_{tg} = i'_s\left(r_s + R_{ts}\right) = i'_d\left(r_s + R_{ts}\right) \Longrightarrow i'_d = \frac{v_{tg}}{r_s + R_{ts}}$$

(c) Solve for v_o.

$$v_o = -i'_d r_{id} \| R_D \| R_L = -\frac{v_{tg}}{r_s + R_{ts}} r_{id} \| R_D \| R_L = v_i \frac{R_1 \| R_2}{R_i + R_1 \| R_2} \frac{-r_{id} \| R_D \| R_L}{r_s + R_{ts}}$$

(d) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_i} = \frac{R_1 \| R_2}{R_i + R_1 \| R_2} \frac{1}{r_s + R_{ts}} (-r_{id} \| R_D \| R_L)$$

(e) Solve for r_{in}.

$$r_{in} = R_1 \| R_2$$

(f) Solve for r_{out}.

$$r_{out} = r_{id} \| R_D$$

Example 4 *Use the T-model solutions to calculate the values of A_v, r_{in}, and r_{out} for Example 1.*

$$A_v = 0.994 \times \left(2.667 \times 10^{-3}\right) \times \left(-6.019 \times 10^{3}\right) = -15.957$$

$$r_{in} = 833.3\,\mathrm{k\Omega} \qquad r_{id} = 61.95\,\mathrm{k\Omega} \qquad r_{out} = 8.61\,\mathrm{k\Omega}$$