Intrinsic Noise Sources

Thermal Noise

Thermal noise is generated by the random motion of electrons in resistors. The electron motion is caused by thermal energy. At zero Kelvin, the motion would cease and there would be no thermal noise.

The thermal noise generated by a resistor is the same for all types of resistors. It was discovered by Johnson in 1928 at Bell Laboratories. In 1929, Nyquist in Germany used a thermodynamic analysis of a transmission line to derive his famous Nyquist formula for the rms thermal noise generated in a band of frequencies. Let $B = f_2 - f_1$ be the bandwidth between f_1 and f_2, where $f_2 > f_1$. The rms noise in the band is given by

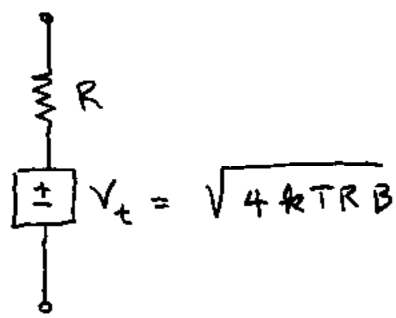
$$V_* = \sqrt{4kTRB} \qquad \text{volts rms}$$

where K = Boltzmann's constant
$= 1.38 \times 10^{-23}$ J/K
T = Kelvin temperature (K)
B = noise bandwidth (Hz)
R = resistance (Ω)

Noise calculations are often made for $T = 290$ K, which is room temperature. In this case, $4kT = 1.6 \times 10^{-20}$ W/Hz. Because the noise voltage varies as $\sqrt{T}$, normal changes in temperature have a small affect on the noise.

For example, the noise changes by only 16% if the temperature changes from 17 °C (63 °F) to 117 °C (243 °F).

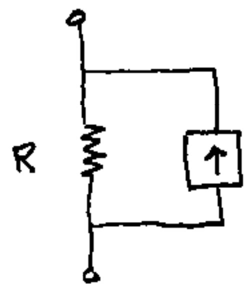
The Thévenin noise model of a resistor is



where we have used a square symbol for a noise source. The polarity of the source is arbitrary. The Thévenin circuit can be converted into a Norton circuit. The Norton current is given by

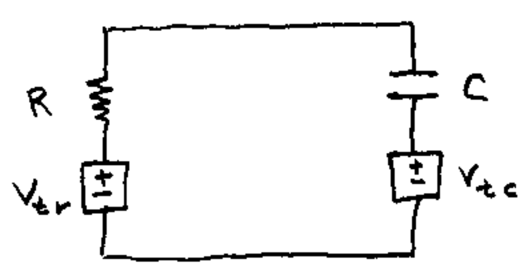
$$I_t = \frac{V_t}{R} = \sqrt{\frac{4kTB}{R}} \quad \text{amps rms}$$

The Norton circuit is



$$I_t = \sqrt{\frac{4kTB}{R}}$$

A resistor is the only passive element which can generate thermal noise. Capacitors and inductors do not generate thermal noise. We can prove this by contradiction. Assume a capacitor generates a noise V_{tc}. Connect it in parallel with a resistor which generates the noise V_{tr}. The circuit is as follows:



We assume R and C are in thermal equilibrium, i.e. at the same temperature.

The power delivered by the resistor to the capacitor is given by

$$P_c = Re\left(V_c I_c^*\right)$$

where

$$V_c = V_{tr} \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = V_{tr} \frac{1}{1+j\omega RC}$$

$$I_c = V_{tr} \frac{1}{R + \frac{1}{j\omega C}} = V_{tr} \frac{j\omega C}{1+j\omega RC}$$

$$\Rightarrow \quad P_c = Re\left(V_{tr} \frac{1}{1+j\omega RC} \times V_{tr}^* \frac{-j\omega C}{1-j\omega RC} \right)$$

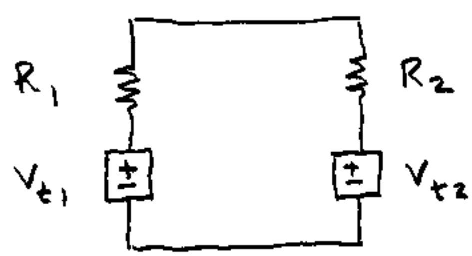
$$= Re\left(|V_{tr}|^2 \frac{-j\omega RC}{1+(\omega RC)^2} \right)$$

$$= 0$$

Thus the resistor delivers no power to the capacitor. This means that the resistor must be heating up if the capacitor delivers

delivers power to the resistor. But this is a contradiction if we assume the two are in thermal equilibrium. Thus the capacitor cannot generate a thermal noise. The same conclusion holds for an inductor. However, the winding resistance of an inductor generates thermal noise.

Suppose we have two unequal resistors connected in parallel. We assume they are in thermal equilibrium. The circuit is



The power delivered to R_2 by R_1 is

$$P_{12} = V_{R2} I_{R2} = V_{t1} \frac{R_2}{R_1+R_2} \times V_{t1} \frac{1}{R_1+R_2}$$

$$\Rightarrow \quad P_{12} = V_{t1}^2 \frac{R_2}{(R_1+R_2)^2} = \frac{4kTBR_1R_2}{(R_1+R_2)^2}$$

The power delivered by R_2 to R_1 can be obtained by interchanging the subscripts 1 and 2. Thus it follows that

$$P_{21} = \frac{4kTBR_2R_1}{(R_2+R_1)^2}$$

Thus we have $P_{21} = P_{12}$. Note that if $P_{21} \neq P_{12}$, one R would be heating up and the other would be cooling off. This would continue until they reach thermal equilibrium.

Available Noise Power

The available noise power P_n is defined as the maximum power one resistor can deliver to another

Let us consider R_1 to be fixed. The power it delivers to R_2 is

$$P_{12} = 4kTB \frac{R_1 R_2}{(R_1 + R_2)^2}$$

Note that $P_{12} = 0$ if $R_2 = 0$ or if $R_2 = \infty$. To maximize P_{12}, we set $dP_{12}/dR_2 = 0$.

$$\frac{dP_{12}}{dR_2} = 4kTB \frac{(R_1+R_2)^2 R_1 - 2R_1R_2(R_1+R_2)}{(R_1+R_2)^4}$$

This is zero if $R_2 = R_1$. Let $R_1 = R_2 = R$. Thus P_n is given by

$$P_n = 4kTB \frac{R^2}{(2R)^2}$$

$$= kTB \quad \text{watts}$$

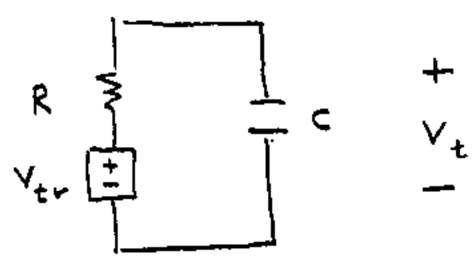
Note that this is independent of R.

Generalized Nyquist Formula

Let Z be a complex impedance that is a function of frequency. The thermal noise voltage generated over the band $B = f_2 - f_1$ is given by

$$V_t = \sqrt{4kTB\,\mathrm{Re}(Z)} \quad \text{volts rms}$$

Example



We can calculate V_t in two ways. First, we use voltage division.

$$V_t = V_{tr} \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = V_{tr} \frac{1}{1 + j\omega RC}$$

The rms value is given by $\sqrt{V_t V_t^*}$

$$V_{t(rms)}^2 = \frac{V_{tr}}{1+j\omega RC}\,\frac{V_{tr}^*}{1-j\omega RC} = \frac{|V_{tr}|^2}{1+(\omega RC)^2}$$

$$= \frac{4kTRB}{1+(\omega RC)^2}$$

$$\Rightarrow \quad V_{t(rms)} = \sqrt{\frac{4kTRB}{1+(\omega RC)^2}}$$

Because this varies with frequency, B must be very small. Otherwise you must integrate over the band.

For a second solution, we solve for $Re(Z)$.

$$Z = R \,\|\, \frac{1}{j\omega C} = \frac{R\,\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{R}{1+j\omega RC}$$

Multiply and divide by the conjugate of the denominator.

$$\Rightarrow \quad Z = \frac{R}{1+j\omega RC}\,\frac{1-j\omega RC}{1-j\omega RC} = R\,\frac{1-j\omega RC}{1+(\omega RC)^2}$$

$$\Rightarrow \quad Re(Z) = \frac{R}{1+(\omega RC)^2}$$

The thermal noise voltage is given by

$$V_{t(rms)} = \sqrt{4kTB\,Re(Z)} = \sqrt{\frac{4kTRB}{1+(\omega RC)^2}}$$

This is the same answer as from the first solution.

We solve for the total noise voltage from $f=0$ to $f=\infty$ as follows. First let $B = df$ and $\omega = 2\pi f$. Then integrate

$$V_{t(rms)}^2 = \int_0^\infty \frac{4kTR}{1+(2\pi fRC)^2}\, df$$

Let $x = 2\pi fRC$ and $df = dx/2\pi RC$

$$\Rightarrow \quad V_{t(rms)}^2 = \frac{2kT}{\pi C}\int_0^\infty \frac{dx}{1+x^2}$$

$$= \frac{2kT}{\pi C}\left[\tan^{-1}(x)\right]_0^\infty$$

$$= \frac{2kT}{\pi C} \times \frac{\pi}{2} = \frac{kT}{C}$$

$$\Rightarrow \quad V_{t(rms)} = \sqrt{\frac{kT}{C}} \quad \text{volts rms}$$

This is referred to as kT-over-C noise. Note that it is independent of R. The formula is often used to calculate the maximum thermal noise generated by any circuit shunted by a capacitor.

Example

It is estimated that the shunt capacitance between the two leads of a passive impedance is 2 pF. What is the maximum thermal noise voltage?

$$V_t = \sqrt{\frac{1.38\times10^{-23}\times 290}{2\times10^{-12}}} = 44.7 \ \mu V \ \text{rms}$$

The Spectral Density

The spectral density is defined as the mean-square noise voltage or current per unit bandwidth. The mean-square value is simply the square of the rms value. For thermal noise, the spectral density of the noise voltage is

$$S_v(f) = \frac{V_t^2}{B} = 4kTR \quad \frac{V^2}{Hz}$$

The spectral density of the noise current is

$$S_i(f) = \frac{I_t^2}{B} = \frac{4kT}{R} \quad \frac{A^2}{Hz}$$

To find the rms noise voltage or current in a band, you integrate the spectral density over the band and then take the square root.

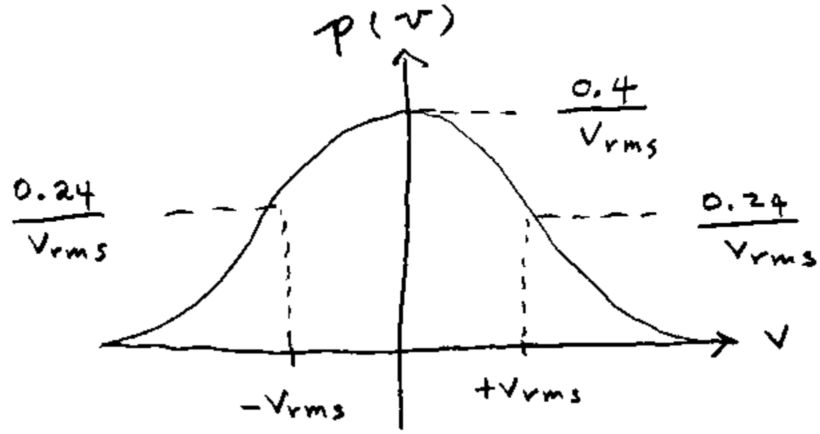
Characteristics of Thermal Noise

1. The frequency distribution is uniform, flat, or white. That is, it is independent of frequency. For example, the noise in the band from 100 Hz to 200 Hz is the same as it is in the band from 1,000 Hz to 1,100 Hz.

2. When viewed on an oscilloscope, the noise has no definable peak value or frequency.

3. The amplitude distribution follows a Gaussian or normal distribution. It is given by

$$p(v) = \frac{1}{V_{rms}\sqrt{2\pi}} \exp\left(-\frac{v^2}{2V_{rms}^2}\right)$$



The probability that $|v| < V_1$ is given by the integral

$$P(v < V_1) = \int_{-V_1}^{V_1} p(v)\,dv$$

The probability that $|v| > V_1$ is given by

$$P(|v| > V_1) = 1 - P(|v| < V_1)$$

The crest factor is defined as the ratio of the peak value to the rms value. But the peak value cannot be defined. Thus a statistical definition must be used by specifying the percent of time a peak is exceeded. Let kV_{rms} be the peak. The percent of time that $|V|$ exceeds kV_{rms} is given by

$$\% = 100 \times \left[1 - P(|V| < kV_{rms}) \right]$$

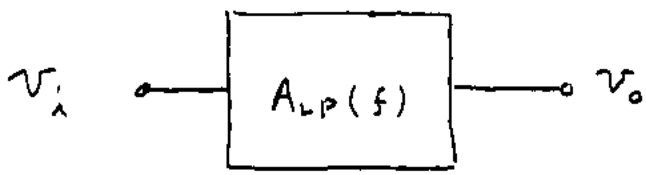
The following table gives several values.

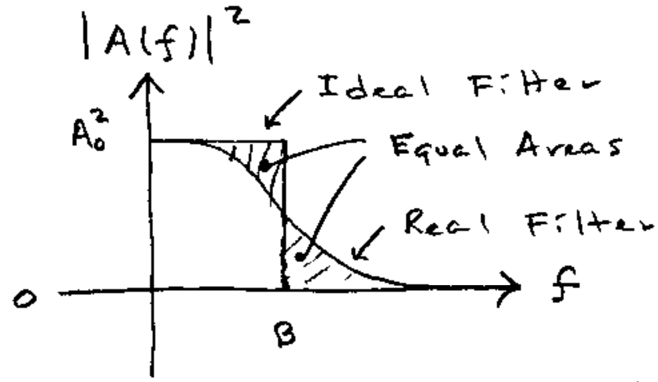
% of Time that $\|V\| > kV_{rms}$	k
1%	2.6
0.1%	3.3
0.01%	3.9
0.001%	4.4
0.0001%	4.9

Equivalent Noise Bandwidth

The equivalent noise bandwidth of a filter is the bandwidth of an ideal filter which passes the same rms noise as the filter, where the input noise is white.

Low Pass Filter

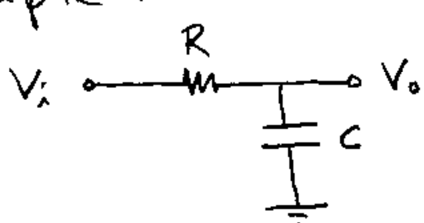




$$A_0^2 B = \int_0^\infty |A(f)|^2 \, df$$

$$\Rightarrow B = \frac{1}{A_0^2} \int_0^\infty |A(f)|^2 \, df$$

Example:



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$$\frac{V_o}{V_i} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

Let $\omega = 2\pi f$ and $f_o = \frac{1}{2\pi RC}$

$$A(f) = \frac{1}{1 + j\frac{f}{f_o}} \qquad |A(f)|^2 = \frac{1}{1 + \left(\frac{f}{f_o}\right)^2}$$

$$A_o = A(0) = 1$$

$$B = \int_0^\infty \frac{df}{1 + \left(\frac{f}{f_o}\right)^2} = f_o \int_0^\infty \frac{dx}{1 + x^2}$$

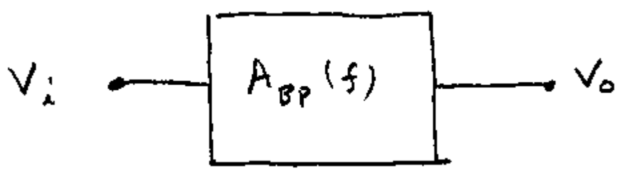
$$= \frac{\pi}{2} f_o$$

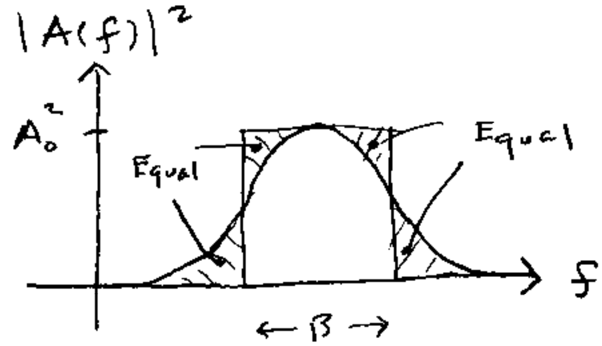
For the case $A(f) = \frac{1}{\left(1 + j\frac{f}{f_o}\right)^n}$

n	B
1	$1.57 f_3$
2	$1.22 f_3$
3	$1.15 f_3$
4	$1.13 f_3$
5	$1.11 f_3$

As n increases, B approaches f_3, which is the $-3\,dB$ bandwidth of the entire filter.

Band Pass Filter





$$B = \frac{1}{A_o^2} \int_0^{\infty} |A(f)|^2 \, df$$

Example : Second - order BPF

$$\frac{V_o}{V_i} = K \frac{\frac{1}{Q}\frac{s}{\omega_o}}{\left(\frac{s}{\omega_o}\right)^2 + \frac{1}{Q}\left(\frac{s}{\omega_o}\right) + 1}$$

Let $s = j2\pi f \qquad \omega_0 = 2\pi f_0$

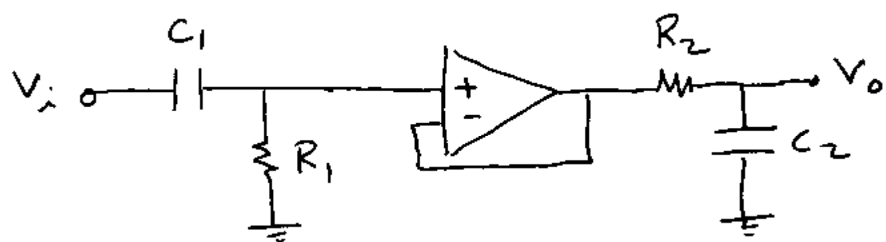
$$A(f) = K \frac{j \frac{1}{Q} \frac{f}{f_0}}{1 - \left(\frac{f}{f_0}\right)^2 + j \frac{1}{Q} \frac{f}{f_0}}$$

$$|A(f)|^2 = K^2 \frac{\left(\frac{1}{Q} \frac{f}{f_0}\right)^2}{\left[1 - \left(\frac{f}{f_0}\right)^2\right]^2 + \left(\frac{1}{Q} \frac{f}{f_0}\right)^2}$$

It can be shown that the -3 dB bandwidth is $B_3 = f_0/Q$ and the noise bandwidth is

$$B = \frac{\pi}{2} \frac{f_0}{Q}$$

Example:



$$\frac{V_o}{V_i} = \frac{R_1 C_1 s}{1 + R_1 C_1 s} \times \frac{1}{1 + R_2 C_2 s}$$

Let $R_1C_1 = 1/\omega_1$ and $R_2C_2 = 1/\omega_2$

$$\Rightarrow \quad \frac{V_o}{V_i} = \frac{A/\omega_1}{1 + A/\omega_1} \times \frac{1}{1 + A/\omega_2}$$

$$= \frac{A/\omega_1}{\frac{A^2}{\omega_1\omega_2} + \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)A + 1}$$

$$= \frac{\frac{1}{\omega_1}}{\frac{1}{\omega_1} + \frac{1}{\omega_2}} \quad \frac{\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)A}{\frac{A^2}{\omega_1\omega_2} + \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)A + 1}$$

$$= \frac{\omega_2}{\omega_1 + \omega_2} \quad \frac{\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)A}{\frac{A^2}{\omega_1\omega_2} + \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)A + 1}$$

$$\Rightarrow \quad K = \frac{\omega_2}{\omega_1 + \omega_2} \qquad \omega_0 = \sqrt{\omega_1\omega_2}$$

$$\frac{1}{Q\omega_0} = \frac{1}{\omega_1} + \frac{1}{\omega_2} \quad \text{or} \quad Q = \frac{1}{\omega_0\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)}$$

Let $\omega_1 = 2\pi f_1$, $\omega_2 = 2\pi f_2$, $\omega_0 = 2\pi f_0$

$$\Rightarrow \quad B = \frac{\pi}{2}\frac{f_0}{Q} = \frac{\pi}{2} f_0^2\left(\frac{1}{f_1} + \frac{1}{f_2}\right) = \frac{\pi}{2}(f_1 + f_2).$$

Shot Noise

Shot noise is associated with current flow across a potential barrier such as a pn junction. It is due to the flow of discrete charges which make up the total or average current. The number of discrete charges per unit time fluctuates about the average value. The fluctuation generates a current noise called shot noise.

Shot noise was discovered in vacuum tubes. In 1918, Schottky derived his famous formula for the rms shot noise current. It is given by

$$I_{sh} = \sqrt{2 q I_{dc} B} \quad \text{amps rms}$$

where q = electron charge
$= 1.6 \times 10^{-19}$ C

I_{dc} = dc current flow (amps)
B = noise bandwidth (Hz)

The spectral density is given by

$$S_i(f) = \frac{I_{sh}^2}{B} = 2qI_{dc} \quad \frac{A^2}{Hz}$$

Shot noise is generated in junction diodes and in BJTs. FETs do not generate shot noise because there is no potential barrier over which the current carriers flow.

Avalanche Noise

Avalanche noise is similar to shot noise. It occurs in zener diodes when they are biased in their breakdown region. However, avalanche noise is much greater than shot noise for the same I_{dc}.

Contact Noise

Contact noise is caused by a fluctuating resistance due to imperfect contact between two materials. It occurs in switch contacts, relay contacts, resistors, diodes, BJTs, and FETs. Other names for contact noise are flicker noise, 1/f noise, and low frequency noise.

The spectral density of contact noise is given by

$$S_i(f) = \frac{K_f I_{dc}^m}{f^n}$$

where K_f = flicker noise coefficient
m = flicker noise exponent
$n \cong 1$

For excess noise in resistors, $m = 2$.
For excess noise in diodes, BJTs, and FETs, $m = 1$.

Popcorn Noise

Also called burst noise. If amplified and fed to a loudspeaker, it sounds like corn popping. Thermal and shot noise sound more like a background frying noise. Popcorn noise is caused by manufacturing defects. It occurs in bursts, the width of which vary from microseconds to seconds. The rate is random and varies from several hundred pulses per second to less than one pulse per minute. The pulse amplitudes are typically 2-100 times the thermal noise. The spectral density varies as $1/f^n$, where $n \cong 2$. It is a current noise, so it is worse in high Z circuits.

Addition of Noise Voltages

Let $v_a(t)$ and $v_b(t)$ be two noise voltages.

$$v_{sum}(t) = v_a(t) + v_b(t)$$

Let $v_a(t)$ have the rms value V_a and $v_b(t)$ have the rms value V_b. What is the rms value of the sum?

$$V_{sum}^2 = \overline{[v_a(t) + v_b(t)]^2}$$

$$= \overline{v_a^2(t)} + 2\,\overline{v_a(t)\,v_b(t)} + \overline{v_b^2(t)}$$

$$= V_a^2 + 2\,\overline{v_a(t)\,v_b(t)} + V_b^2$$

If $v_a(t)$ and $v_b(t)$ <u>are</u> statistically independent, then $\overline{v_a(t)\,v_b(t)} = 0$. In this case

$$V_{sum} = \sqrt{V_a^2 + V_b^2}$$

In some cases, the voltages are correlated. In this case, we define the correlation coefficient γ as follows:

$$\gamma = \frac{\overline{v_a(t)\, v_b(t)}}{V_a V_b}$$

$$\Rightarrow \quad \overline{v_a(t)\, v_b(t)} = \gamma V_a V_b$$

In this case V_{sum} is given by

$$V_{sum} = \sqrt{V_a^2 + 2\gamma V_a V_b + V_b^2}$$

It can be shown that $-1 \le \gamma \le +1$.

Examples:

$$v_b(t) = k\, v_a(t) \quad \Rightarrow \quad \gamma = 1$$

$$v_b(t) = -k\, v_a(t) \quad \Rightarrow \quad \gamma = -1$$

$$v_a(t) = V_1 \cos\omega t \qquad v_b(t) = V_2 \sin\omega t$$

$$v_a(t)\, v_b(t) = V_1 V_2 \cos\omega t \sin\omega t$$

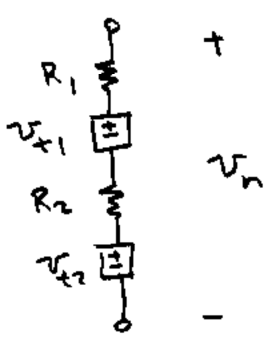
$$= \frac{V_1 V_2}{2} \sin 2\omega t$$

$$\Rightarrow \overline{v_a(t)\, v_b(t)} = 0 \quad \Rightarrow \quad \gamma = 0$$

In general, thermal noise and shot noise generated by different devices are independent.

Example -

Series Resistors



$$\overline{v_n^2} = \overline{(v_{n1} + v_{n2})^2}$$

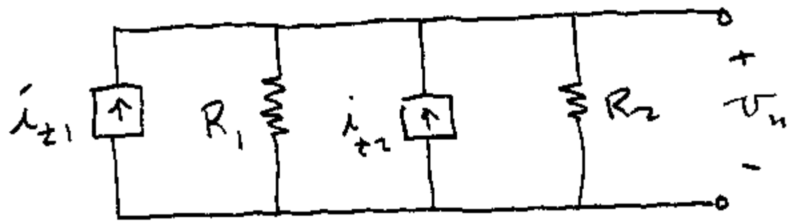
$$= \overline{v_{n1}^2} + \overline{v_{n2}^2}$$

$$= 4kTR_1B + 4kTR_2B$$

$$= 4kT(R_1 + R_2)B$$

$$\Rightarrow \quad V_n = \sqrt{4kT(R_1+R_2)B}$$

Parallel Resistors



$$v_n = (i_{t1} + i_{t2})\, R_1 \| R_2$$

$$\overline{v_n^2} = \overline{(i_{t1}+i_{t2})^2}\,(R_1\|R_2)^2$$

$$= \left(\overline{i_{t1}^2} + \overline{i_{t2}^2}\right)(R_1\|R_2)^2$$

$$= \left(\frac{4kTB}{R_1} + \frac{4kTB}{R_2}\right)(R_1\|R_2)^2$$

$$= \frac{4kTB}{R_1\|R_2}\,(R_1\|R_2)^2$$

$$= 4kT\,(R_1\|R_2)\,B$$

$$\Rightarrow \quad V_n = \sqrt{4kT(R_1+R_2)B}$$

Noise Measurements

Noise is measured at the output of an amplifier. It can be referred to the input by dividing by the amplifier gain.

Three requirements for a noise measuring voltmeter:

1. It should be a true rms meter
2. It should have a crest factor of 4 or greater
3. Its bandwidth should be at least 10 times the noise bandwidth of the circuit being measured.

Some voltmeters respond to the average value of the rectified voltage rather than the rms valve. The meter is calibrated to read the correct valve for sine waves only. When measuring Gaussian noise, the meter reading should be multiplied by 1.13. Peak reading meters should not be used.

When using an oscilloscope, the peak-to-peak noise voltage is divided by 8 to obtain the rms value.