Chapter 1

First-Order Low-Pass Filtered Noise

Object

The object of this experiment is to become familiar with the characteristics of Gaussian noise. A spectrum analyzer, known as a Dynamic Signal Analyzer or DSA, is used to measure the noise bandwidth of a low-pass filter. A low-pass filter with a variable cutoff frequency is used to measure the spectral density of Gaussian noise.

Theory

The amplitude distribution of Gaussian noise is described by the Gaussian or normal probability density function. The standard deviation of a Gaussian noise voltage is the root-mean-square or rms value of the voltage. When Gaussian noise is band-limited with a filter, its Gaussian characteristics are not changed. However, its rms voltage is decreased.

White Gaussian noise has a spectral density that is independent of frequency. Thus the mean-square voltage in a frequency band Δf is directly proportional to Δf. When white Gaussian noise is applied to a filter, the mean-square noise voltage at the output of the filter is the spectral density at the input multiplied by the noise bandwidth B_n of the filter. For either a single-pole low-pass filter or a two-pole band-pass filter, the noise bandwidth is given by

$$B_n = \frac{\pi}{2} B_3$$

where B_3 is the filter $-3\,\mathrm{dB}$ bandwidth. For the single-pole low-pass filter, $B_3 = f_0$, where f_0 is the pole frequency. For the second-order band-pass filter, $B_3 = f_0/Q$, where f_0 is the center or resonance frequency and Q is the quality factor.

When using a voltmeter to measure the rms value of a voltage, the maximum peak voltage that can be applied to the meter without overloading the internal circuits divided by the full scale reading is called the crest factor of the meter. The peak to rms ratio for a sine wave is $\sqrt{2}$. Thus a meter that is designed to read correctly with a sine wave input must have a crest factor that is greater than $\sqrt{2}$. A commonly accepted crest factor for Gaussian noise is 4 times the rms value. Therefore, to accurately measure a Gaussian noise voltage, the rms voltage read by the meter should be less than the crest factor of the meter multiplied by the full scale voltage divided by 4.

The bandwidth of a measuring instrument plays an important role in any noise measurement. If the instrument bandwidth is less than the bandwidth of the signal being measured, then the measurement will be incorrect. Therefore, accurate measurements of noise require that the bandwidth of the measuring instrument be greater than that of the noise signal, preferably by a factor of 10. Often a filter is used to limit the bandwidth of noise to a bandwidth that is less than that of the instrument.

The type of meter used to measure noise voltages is an important consideration. A true rms responding meter is the best for such measurements. However, these meters are not universal and many are average responding meters. An average responding meter is normally calibrated to read the correct rms value of the input voltage only for a sine wave with a dc level of zero. If Gaussian white noise is measured on such a meter, the reading must be multiplied by a factor of 1.13 to obtain the correct rms value.

With white noise at its input, the rms noise voltage V_{rms} at the output of a filter with a noise bandwidth B_n is given by

$$V_{rms} = \sqrt{S_v B_n}$$

where S_v is the voltage spectral density of the white noise source. It follows that if a plot is made of $\log(V_{rms})$ versus $\log(B_n)$ the result should be a straight line with a slope of 1/2 and a y-intercept of $\log(\sqrt{S_v})$. This provides an experimental technique of determine the voltage spectral density of a noise source. Because the theoretical prediction is a straight line, well known linear regression techniques may be used to obtain an accurate estimate of S_v.

Laboratory Procedure

First Order Low-Pass Filter Design

Figure 1.1 shows two cascaded single-pole low-pass filters with op-amp buffers. It is given that $R_1 = 10\,\text{k}\Omega$, $R_2 = 1\,\text{k}\Omega$, $R_3 = 100\,\Omega$, and $C_1 = 100\,\text{pF}$.. The input filter has a $-3\,\text{dB}$ frequency of 1.6 MHz and prevents possible rf interference with the circuit. This frequency is significantly higher than the $-3\,\text{dB}$ frequency of the following filter which sets the bandwidth of the circuit. R_3 is included in the circuit to prevent possible oscillations due to capacitive loading on the second op amp.

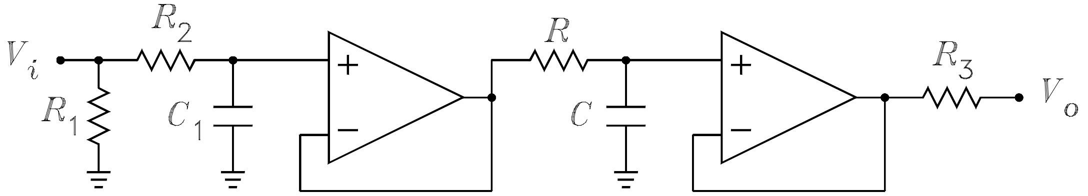

Figure 1.1: Low-pass filter circuit.

For the second filter, the available capacitor values for C are 100 pF, 1000 pF, 0.01 μF, and 0.1 μF. For the resistors, standard 1% values are available. Determine suitable values of R and C to obtain $-3\,\text{dB}$ cutoff frequencies in the second filter of 100 Hz, 300 Hz, 1 kHz, 3 kHz, 10 kHz, 30 kHz, and 100 kHz. The $-3\,\text{dB}$ frequency is given by $f_3 = 1/(2\pi RC)$. Exact values are not necessary. Values within 5% of the specified ones are acceptable. Measure the exact values of the resistors and capacitors using the LCR meter and use these values to calculate the theoretical $-3\,\mathrm{dB}$ cutoff frequencies.

First-Order Low-Pass Filter Bandwidth Measurement

With reference to Figs. 1.2 and 1.3, assemble the filter circuit on a solderless breadboard. Use a $100\,\Omega$ resistor and $100\,\mu\mathrm{F}$ capacitor to decouple each power supply rail. Note that the $100\,\mu\mathrm{F}$ capacitors are electrolytic types which must be inserted with the correct polarity. Use the function generator and the oscilloscopes and/or DSA to measure the upper $-3\,\mathrm{dB}$ frequency for each filter. An input voltage of approximately 0.1 V peak should be used. Keep the resistors because each of these circuits will be reassembled. Record the measured data.

Noise Source Measurement

Observe the noise signal with both the oscilloscope and the DSA to observe the nature of the noise. Set the DSA to average the input signal 50 times. Program the DSA to measure the voltage over the frequency band $0 - 115\,\mathrm{kHz}$. Determine the spot noise spectral density ($\mathrm{V}/\sqrt{\mathrm{Hz}}$) of the noise source at a frequency of 10 kHz. This is given by noise voltage measured at 10 kHz divided by the square root of the measurement bandwidth of the signal analyzer. Obtain a screen dump of the display.

Filtered Noise

With reference to Figs. 1.2 and 1.3, assemble the 100 Hz filter elements on the solderless breadboard. Use the DMM to measure the dc voltage at the input and output of the circuit to assure that the dc voltage is in the millivolt range. If it is not, the input and output must be ac coupled for the next step in the procedure. Apply the noise signal from the DSA to the filter input. Connect oscilloscope to the output of the circuit. Observe the nature of the waveform on the oscilloscope and compare it to what is observed at the SOURCE output of the DSA.

Spectrum of Output Noise

With the noise signal from the SOURCE output of the DSA driving the circuit as in the previous procedure step, connect the circuit output to the input of the DSA. Program the analyzer to measure the total noise output and record the value. Also measure the output voltage with the HP3400 true rms voltmeter. Repeat for the other filters.

AC Line Corruption

Use the DSA to examine any ac line voltage corruption at the output of the 100 Hz filter. Set the cursor to 60 Hz and the frequency SPAN to 1 kHz and obtain a plot of the display.

Laboratory Report

Noise Bandwidths

Calculate the noise bandwidth of each of the filters using the measured half-power cutoff frequencies. Plot the measured output noise voltage as a function of the noise bandwidth of the filter. Use log-log scales for these plots. Use a linear regression analysis to obtain a value for the spectral density of the noise source. Compare this with the value measured directly at the output of the noise source.

Conclusions

Explain any discrepancy between the values obtained using the different approaches for determining the voltage spectral density of the noise source.

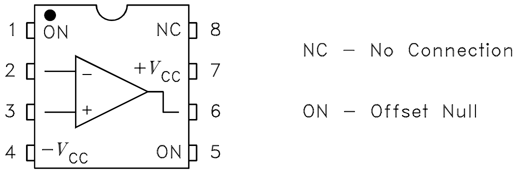

Figure 1.2: Pin Outs for 741 Type IC

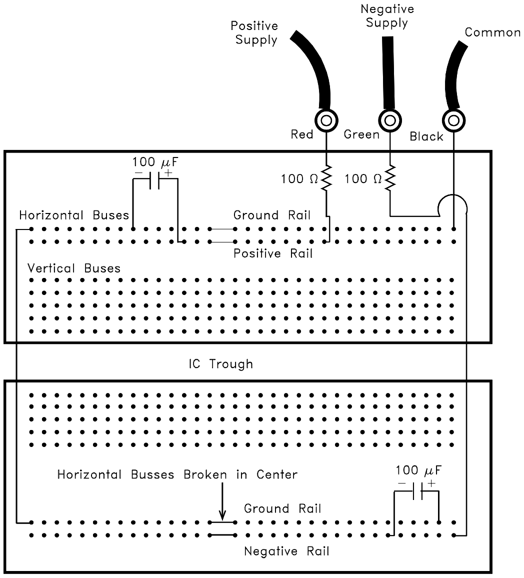

Figure 1.3: Solderless Breadboard Layout