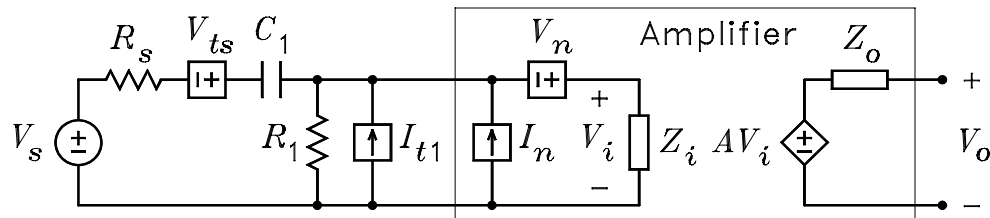


ECE 6416 Assignment 2

1. The figure shows the noise model of an amplifier. The frequency is $f = 1$ kHz. It is given that $R_s = 1$ k Ω , $R_1 = 2$ k Ω , $C_1 = 0.1$ μ F, $v_n/\sqrt{\Delta f} = 4$ nV/ $\sqrt{\text{Hz}}$, $i_n/\sqrt{\Delta f} = 8$ pA/ $\sqrt{\text{Hz}}$, $\gamma = 0$, $Z_i = 2800\angle -45^\circ$, $A = 20\angle 45^\circ$, $Z_o = 1200\angle 30^\circ$. The open-circuit input voltage with $Z_i = \infty$ can be written $V_{i(oc)} = KV_s + V'_{ni} = K(V_s + V'_{ni}/K)$, where $K = V_{i(oc)}/V_s$ when all noise sources are neglected and V'_{ni} is the open-circuit input voltage due to all noise sources. The latter is given by $V'_{ni} = V_{teq} + V_n + I_n Z_{eq}$, where V_{teq} is the thermal noise generated by the real part of the impedance $Z_{eq} = R_1 \parallel (R_s + 1/j\omega C_1)$. It follows that the equivalent noise voltage in series with V_s is $V_{ni} = V'_{ni}/K$.



- (a) Show that $|K| = 0.5889$ and $v'_{ni}/\sqrt{\Delta f} = 10$ nV.
- (b) Show that $v_{ni}/\sqrt{\Delta f} = 17.79$ nV.
- (c) If R_1 is replaced by an open circuit and C_1 is replaced by a short circuit, show that $v_{ni}/\sqrt{\Delta f} = 9.798$ nV. This calculation illustrates how the addition of series and parallel elements increase the noise.
- (d) Show that the addition of C_1 and R_1 increases the noise by 5.18 dB.
2. The figure shows the model of a two-stage amplifier. It is given that $R_S = 100$ Ω , $R_{i1} = R_{i2} = 1.2$ k Ω , $R_{o1} = R_{o2} = 5$ k Ω , $R_L = 2$ k Ω , $I_{o1} = g_{m1}V_{i1}$, $I_{o2} = g_{m2}V_{i2}$, $g_{m1} = g_{m2} = 13^{-1}$ S, $v_{n1}/\sqrt{\Delta f} = v_{n2}/\sqrt{\Delta f} = 2$ nV/ $\sqrt{\text{Hz}}$, and $i_{n1}/\sqrt{\Delta f} = i_{n2}/\sqrt{\Delta f} = 100$ pA/ $\sqrt{\text{Hz}}$. The noise sources can be assumed to be uncorrelated. V_{ni1} models the noise generated both by R_S and the first amplifier. It is given by $V_{ni1} = V_{ts} + V_{n1} + I_{n1}R_S$. V_{ni2} models the noise generated by the second stage and is given by $V_{ni2} = V_{n2} + I_{n2}R_{o1}$. The noise generated by R_L can be neglected. All other resistors are noiseless because their noise is contained in V_{ni1} and V_{ni2} .

- (a) Show that the voltage gain can be written as follows:

$$\begin{aligned} A &= \frac{V_{i1(oc)}}{V_s} \times \frac{I_{o1(sc)}}{V_{i1(oc)}} \times \frac{V_{i2(oc)}}{I_{o1(sc)}} \times \frac{I_{o2(sc)}}{V_{i2(oc)}} \times \frac{V_o}{I_{o2(sc)}} \\ &= 1 \times G_{m1} \times R_{o1} \times G_{m2} \times (R_{o2} \parallel R_L) \end{aligned}$$

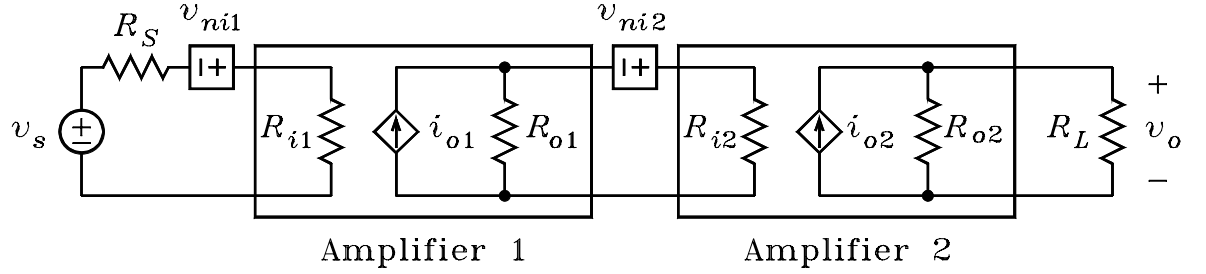
where

$$G_{m1} = \frac{I_{o1(sc)}}{V_{i1(oc)}} = \frac{V_{i1}}{V_{i1(oc)}} \frac{I_{o1}}{V_{i1}} = \frac{R_{i1}}{R_S + R_{i1}} g_{m1} = 14.08^{-1} \text{ S}$$

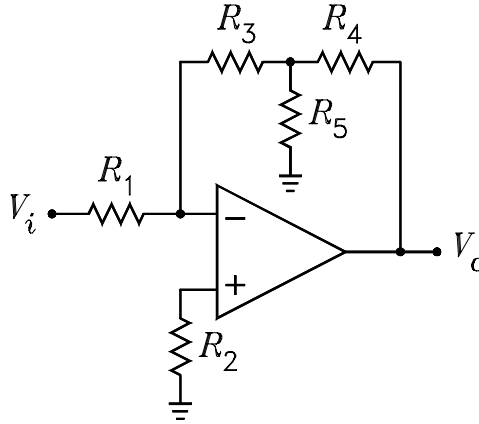
$$G_{m2} = \frac{I_{o2(sc)}}{V_{i2(oc)}} = \frac{V_{i2}}{V_{i2(oc)}} \frac{I_{o2}}{V_{i2}} = \frac{R_{i2}}{R_{o1} + R_{i2}} g_{m2} = 67.17^{-1} \text{ S}$$

to give

$$A = 7551 \text{ (77.6 dB)}$$



- (b) Show that components of $v_{ni}/\sqrt{\Delta f}$ due to v_{ts} , v_{n1} , i_{n1} , v_{n2} , and i_{n2} , respectively, are $1.265 \text{ nV}/\sqrt{\Delta f}$, $2 \text{ nV}/\sqrt{\Delta f}$, $10 \text{ nV}/\sqrt{\Delta f}$, $5.633 \text{ pV}/\sqrt{\Delta f}$, and $1.408 \text{ nV}/\sqrt{\Delta f}$.
- (c) Show that $v_{ni}/\sqrt{\Delta f} = 10.37 \text{ nV}$.
3. The figure shows an op amp circuit. Except for noise, the op amp is ideal. It is given that $R_1 = 1 \text{ k}\Omega$, $R_2 = 500 \Omega$, $R_3 = 901 \text{ k}\Omega$, $R_4 = 9.9 \text{ k}\Omega$, and $R_5 = 100 \Omega$. The op amp noise is modeled by a noise voltage source V_n in series with the non-inverting input and two noise current sources I_{n1} and I_{n2} from each input to ground. It is given that $v_n/\sqrt{\Delta f} = 10 \text{ nV}/\sqrt{\text{Hz}}$ and $i_{n1}/\sqrt{\Delta f} = i_{n2}/\sqrt{\Delta f} = 1 \text{ pA}/\sqrt{\text{Hz}}$. All correlation effects can be neglected.



- (a) Using Norton equivalent circuits, show that the voltage at the non-inverting input is given by

$$V_+ = \left(I_{n1} + \frac{V_n}{R_2} + I_{t2} \right) R_2$$

- (b) Using Norton equivalent circuits, show that the voltage at the inverting input is given by

$$V_- = \left(\frac{V_i}{R_1} + \frac{V_o}{R_4 + R_3 \parallel R_5} \frac{R_5}{R_3 + R_5} + I_{n2} + I_{teq} \right) [R_1 \parallel (R_3 + R_4 \parallel R_5)]$$

where I_{teq} is the noise current generated by the resistance $R_{eq} = R_1 \parallel (R_3 + R_4 \parallel R_5)$.

- (c) Set $V_+ = V_-$ and show that the voltage gain has the value $A = V_o/V_i = -90.1 \times 10^3$.
- (d) Use the equation to show that the spot noise output voltage due to the thermal noise of all resistors has the value $v_{no}/\sqrt{\Delta f} = 442 \mu\text{V}/\sqrt{\text{Hz}}$.
- (e) Use the equation to show that the spot noise output voltage due to the op-amp noise sources has the value $v_{no}/\sqrt{\Delta f} = 908 \mu\text{V}/\sqrt{\text{Hz}}$.
- (f) Use rms addition of the thermal noise and op-amp noise to show that the total spot noise output voltage is $v_{no}/\sqrt{\Delta f} = 1.01 \text{ mV}/\sqrt{\text{Hz}}$.
- (g) Divide $v_{no}/\sqrt{\Delta f}$ by A to show that the equivalent spot noise voltage in series with v_i is $v_{ni}/\sqrt{\Delta f} = 11.2 \text{ nV}/\sqrt{\text{Hz}}$.
4. An amplifier has a voltage gain $V_o/V_i = 10$ (20 dB) and an input resistance $R_i = 10 \text{ k}\Omega$. It is driven from a voltage source which has an open-circuit rms output voltage $V_s = 1 \text{ V}$ and an output resistance $R_s = 1 \text{ k}\Omega$. The rms noise at the amplifier output in the band from 100 Hz to 100 kHz is $v_{no} = 28.4 \mu\text{V}$. When a $9 \text{ k}\Omega$ resistor is added in series with R_s , the rms output noise increases to $v'_{no} = 67.5 \mu\text{V}$. If the V_n and I_n of the amplifier are not correlated, show that $v_n = 2.56 \mu\text{V}$ and $i_n = 1.26 \text{ nA}$.
5. N identical amplifiers having $v_n/\sqrt{\Delta f} = 1.09 \text{ nV}/\sqrt{\text{Hz}}$ and $i_n/\sqrt{\Delta f} = 11.4 \text{ pA}/\sqrt{\text{Hz}}$ are driven from a source having the output resistance $R_s = 10 \Omega$. Show that it would take 10 amplifiers in parallel to minimize the noise. For $N = 10$, show that $v_{ni}/\sqrt{\Delta f} = 639 \text{ pV}/\sqrt{\text{Hz}}$.
6. An input transformer is to be used to connect the source of problem 5 to the amplifier. If the winding resistances of the transformer can be neglected, show that the noise is a minimum if the transformer has the turns ratio $n = 3.1$. Show that $v_{ni}/\sqrt{\Delta f} = 639 \text{ pV}/\sqrt{\text{Hz}}$.