ECE 6416 Quiz 1
October 7, 2009

Professor Leach Name______________________

Instructions. Print your name in the space above and at the top of all other pages in your quiz. Place a box around each answer. Express each numerical answer as a decimal number. Numerical values are $4kT_0 = 1.6 \times 10^{-20}$ J and $q = 1.6 \times 10^{-19}$ C. **Honor Code:** *I have neither given nor received help on this quiz.* Initials ________

1. A $10\,\text{k}\Omega$ resistor has a noise index of -15 dB. The dc current flowing through the resistor is $I_{DC} = 1$ mA. The frequency band of interest is the band from 20 Hz to 20 kHz.

   (a) Solve for the rms thermal noise voltage v_t in the band.

   $$v_t = \sqrt{4kTR\,(f_2 - f_1)} = 1.79\,\mu\text{V}$$

   (b) Solve for the rms excess noise voltage v_{ex} in the band.

   $$v_{ex} = \sqrt{10^{NI/10}\,(I_{DC}R)^2 \log(f_2/f_1)} = 3.08\,\mu\text{V}$$

   (c) Solve for the total rms noise voltage v_n in the band.

   $$v_n = \sqrt{v_t^2 + v_{ex}^2} = 3.56\,\mu\text{V}$$

2. A $0.01\,\mu$F capacitor is connected in parallel with a $10\,\text{k}\Omega$ resistor. Solve for the rms thermal noise voltage generated by the circuit in the band from 10 kHz to 100 kHz. You are given the following integral for reference:

$$\int_{x_1}^{x_2} \frac{dx}{a + bx^2} = \frac{1}{\sqrt{ab}}\left[\tan^{-1}\left(x_2\sqrt{\frac{b}{a}}\right) - \tan^{-1}\left(x_1\sqrt{\frac{b}{a}}\right)\right]$$

$$Z = R \| \frac{1}{j\omega C} = \frac{R}{1 + j\omega RC} \qquad \text{Re}\,(Z) = \text{Re}\left(\frac{R}{1 + j\omega RC}\frac{1 - j\omega RC}{1 - j\omega RC}\right) = \frac{R}{1 + (\omega RC)^2}$$

$$\begin{aligned} v_t^2 &= 4kTR\int_{10\,\text{kHz}}^{100\,\text{kHz}} \frac{df}{1 + (2\pi fRC)^2} \\ &= \frac{2kT}{\pi C}\left[\arctan(2\pi f_2 RC) - \arctan(2\pi f_1 RC)\right] = 3.61 \times 10^{-14}\,\text{V}^2 \end{aligned}$$

$$v_t = \sqrt{v_t^2} = 190\,\text{nV}$$

3. A non-inverting amplifier has the gain with feedback $A_v = 100$ and the input resistance $R_i = \infty$. With a source resistor $R_S = 100\,\text{k}\Omega$ connected from input to ground, the rms noise voltage at the output over the band $\Delta f = 100$ kHz is found to be $v_{no1} = 1.5$ mV. When a resistor $R_1 = 20$ kHz is connected in parallel with R_S the noise output voltage drops to $v_{no2} = 0.6$ mV. If the correlation between v_n and i_n is neglected, solve for the rms values of v_n and i_n over the band Δf at the amplifier input.

$$\begin{aligned} v_n^2 + i_n^2 R_S^2 &= \frac{v_{no1}^2}{A_v^2} - 4kTR_S\Delta f \Longrightarrow v_n^2 + 10^{10} i_n^2 = 6.5 \times 10^{-11} \\ v_n^2 + i_n^2 (R_S \| R_1)^2 &= \frac{v_{no2}^2}{A_v^2} - 4kTR_S \| R_1 \Delta f \Longrightarrow v_n^2 + 2.778 \times 10^8 i_n^2 = 9.333 \times 10^{-12} \end{aligned}$$

$$\Delta = \begin{vmatrix} 1 & 10^{10} \\ 1 & 2.778 \times 10^8 \end{vmatrix} = 2.778 \times 10^8 - 10^{10} = -9.722 \times 10^9$$

$$\begin{aligned} v_n^2 &= \frac{1}{\Delta} \begin{vmatrix} 6.5 \times 10^{-11} & 10^{10} \\ 9.333 \times 10^{-12} & 2.778 \times 10^8 \end{vmatrix} \\ &= \frac{6.5 \times 10^{-11} \times 2.778 \times 10^8 - 9.333 \times 10^{-12} \times 2.778 \times 10^8}{-9.722 \times 10^9} \\ &= 7.743 \times 10^{-12} \\ v_n &= 2.783\,\mu\mathrm{V} \end{aligned}$$

$$\begin{aligned} i_n^2 &= \frac{1}{\Delta} \begin{vmatrix} 1 & 6.5 \times 10^{-11} \\ 1 & 9.333 \times 10^{-12} \end{vmatrix} \\ &= \frac{9.333 \times 10^{-12} - 6.5 \times 10^{-11}}{-9.722 \times 10^9} \\ &= 5.726 \times 10^{-21} \\ i_n &= 75.67\,\mathrm{pA} \end{aligned}$$