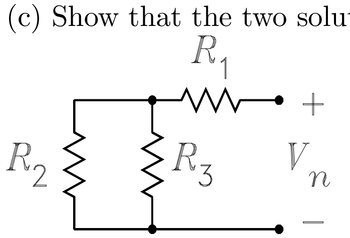
# ECE 6416 Quiz 1

October 6, 2010

Professor Leach Name______________________________

**Instructions.** Print your name in the space above and at the top of all other pages in your quiz. Clearly mark each answer. Express each numerical answer as a decimal number. Numerical values are $4kT_0 = 1.6 \times 10^{-20}$ J and $q = 1.6 \times 10^{-19}$ C. **Honor Code:** *I have neither given nor received help on this quiz.* Initials __________

1. The figure shows a resistive circuit.
   (a) Solve for the mean-square value of $V_n$ using the generalized Nyquist formula.
   (b) Solve for $V_n$ by replacing each resistor with its Thévenin model and using superposition. Convert the answer to the mean-square value.
   (c) Show that the two solutions are equivalent.




(a)

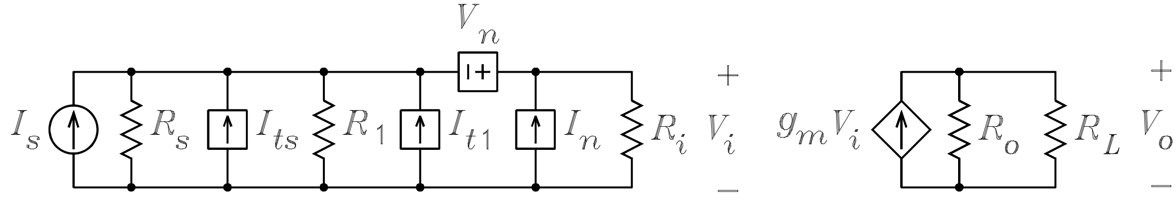
$$v_n^2 = 4kT\left(R_1 + R_2 \| R_3\right)\Delta f$$

(b)

$$\begin{aligned}
V_n &= V_{t1} + V_{t2}\frac{R_3}{R_2 + R_3} + V_{t3}\frac{R_2}{R_2 + R_3} \\
v_n^2 &= 4kTR_1\Delta f + 4kTR_2\Delta f\left(\frac{R_3}{R_2 + R_3}\right)^2 + 4kTR_3\Delta f\left(\frac{R_2}{R_2 + R_3}\right)^2 \\
&= 4kT\left(R_1 + \frac{R_2R_3^2 + R_2^2R_3}{\left(R_2 + R_3\right)^2}\right)\Delta f \\
&= 4kT\left(R_1 + R_2 \| R_3\right)\Delta f
\end{aligned}$$

(c) They are already equal to each other.

2. Shown is the noise model of an amplifier.
   (a) Solve for the noise current $I_{ni}$ in parallel with $I_s$ that generates the same noise as all noise sources in the circuit.
   (b) Set $I_{ni} = I_{ts} + V_n'/R_s + I_n'$ and solve for $V_n'$ and $I_n'$.
   (c) Solve for the mean-square values $v_n'^2 = \overline{V_n' V_n'^*}$ and $i_n'^2 = \overline{I_n' I_n'^*}$. In the expressions, let $\overline{V_n I_n^*} = v_n i_n \gamma$. Express all answers in terms of $v_n$ (or $v_n^2$), $i_n$ (or $i_n^2$), and $\gamma$.
   (d) Solve for $\overline{V_n' I_n'^*}$ in the expression for the correlation coefficient $\gamma' = \overline{V_n' I_n'^*}/v_n' i_n'$ between $V_n'$ and $I_n'$. You do not have to write out the equation for $\gamma'$.



(a)

$$\begin{aligned} I_{i(sc)} &= I_s + I_{ts} + I_{t1} + I_n + \frac{V_n}{R_s \| R_1} = I_s + I_{ni} \\ I_{ni} &= I_{ts} + I_{t1} + I_n + \frac{V_n}{R_s \| R_1} = I_{ts} + I_{t1} + I_n + \frac{V_n}{R_1} + \frac{V_n}{R_s} \end{aligned}$$

(b)

$$V_n' = V_n \qquad I_n' = I_{t1} + I_n + \frac{V_n}{R_1}$$

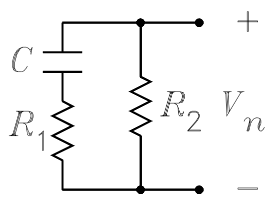
(c)

$$v_n'^2 = v_n^2 \qquad i_n'^2 = \frac{4kT\Delta f}{R_1} + i_n^2 + 2\gamma \frac{v_n i_n}{R_1} + \frac{v_n^2}{R_1^2}$$

(d)

$$\overline{V_n I_n'^*} = \overline{V_n \left( I_{t1} + I_n + \frac{V_n}{R_1} \right)^*} = \frac{\overline{V_n V_n^*}}{R_1} + \overline{V_n I_n^*} = \frac{v_n^2}{R_1} + \gamma v_n i_n$$

3. For $R_1 = 4\,\mathrm{k}\Omega$, $R_2 = 2\,\mathrm{k}\Omega$, and $C = 0.01\,\mu\mathrm{F}$ and $f = 10\,\mathrm{kHz}$,
   (a) Solve for the spot noise voltage in nV across the circuit due to $R_1$.
   (b) Solve for the spot noise voltage in nV across the circuit due to $R_2$.
   (c) Solve for the total spot noise voltage in nV across the circuit.




(a)

$$V_{n1} = V_{t1} \frac{R_2}{R_1 + R_2 + \frac{1}{j\omega C}} \qquad v_{n1}^2 = 4kTR_1 \left| \frac{R_2}{R_1 + R_2 + \frac{1}{j\omega C}} \right|^2 = 2.58\,\mathrm{nV}$$

(b)

$$V_{n2} = V_{t2} \frac{R_1 + \frac{1}{j\omega C}}{R_1 + R_2 + \frac{1}{j\omega C}} \qquad v_{n2}^2 = 4kTR_2 \left| \frac{R_1 + \frac{1}{j\omega C}}{R_1 + R_2 + \frac{1}{j\omega C}} \right|^2 = 3.92\,\mathrm{nV}$$

(c)

$$v_n^2 = \sqrt{v_{n1}^2 + v_{n2}^2} = 4.69\,\mathrm{nV}$$