On the Electroacoustic-Analogous Circuit for a Plane Wave Incident on the Diaphragm of a Free-Field Pressure Microphone*

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An analogous circuit for a plane wave incident on the diaphragm of a free-field pressure microphone is developed which models the average pressure increase at the diaphragm due to reflections.

0 INTRODUCTION

The impedance-analogous circuit for a plane wave incident on the diaphragm of a microphone is usually modeled by a pressure generator in series with the air load impedance on a piston in a long tube [1]. The pressure generator in the model represents the average pressure at the diaphragm when it is blocked, that is, when it is restrained so that its velocity is zero. A more useful model for the analysis of free-field microphones would place the acoustic pressure of the incident plane wave in the absence of the microphone in series with the air load impedance on the piston. The development of such a model is discussed here. It is shown that the incident plane wave pressure in the absence of the microphone must be multiplied by a transfer function which accounts for the average effects of reflections at the blocked diaphragm. The form of this transfer function is derived.

The inadequacy of the usual model for a plane wave incident on a microphone diaphragm has been discussed in Bauer [2]. Using a heuristic argument, he reasoned out an analogous circuit which he deemed "approximate" and cautioned the reader that "the circuit herein proposed should be used with prudence to avoid straining the equivalence beyond the point of validity." The writer found the ideas that Bauer expressed in his ingenious paper to be so intriguing that the analysis presented here was developed. It is shown that Bauer's "approximate" circuit can be obtained by using a more rigorous analysis, and the approximations in his circuit are identified.

1 THE ANALOGOUS CIRCUITS

Fig. 1 illustrates an acoustic plane wave of pressure amplitude p_{01} in free air that is incident on a long thin-wall tube of radius a with a lossless and massless piston in its end. The plane wave that is coupled into the tube confronts an acoustic resistance given by $R_A = \rho_0 c/S$, where ρ_0 is the air density, c is the velocity of sound,

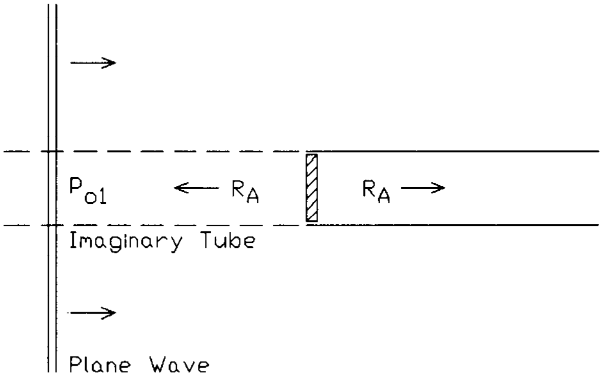

Fig. 1. Plane wave of pressure amplitude p_{01} incident on a lossless and massless piston in the end of a long tube.

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and $S = \pi a^2$ is the cross-sectional area of the tube. The same wave would be coupled into the tube if an imaginary tube of the same area were extended to the left of the piston, as shown by the dashed lines in the figure, in which a plane wave of the same pressure amplitude propogates as the plane wave in free air. The acoustic output impedance of this imaginary source tube is given also by $R_A = \rho_0 c/S$. It follows that the analogous circuit that predicts the pressure amplitude p_D at the piston is a source of pressure $2p_{01}$ in series with a source resistance R_A driving a load resistance R_A. The analogous circuit is shown in Fig. 2(a).

Suppose that the generator is now moved so that the plane wave propagates to the left from the load end of the original tube. The wave that is radiated into free air by the piston at the end of the tube is a spherical wave rather than a plane wave. Fig. 2(b) illustrates the analogous circuit for this case, where the pressure generator is denoted by p_{02}. For the pressure p_D at the piston to be correctly predicted by this circuit, the load impedance Z_A must be the impedance representing the air load on a piston in the end of a long tube. This impedance is given in Fig. 3, where the elements are given by [1] $M_{A1} = 0.6133\rho_0/\pi a$, $R_{A1} + R_{A2} = 1.505\rho_0 c/\pi a^2$, $R_{A2} = \rho_0 c/\pi a^2$ (the same as R_A), and $C_{A1} = 0.55\pi^2 a^3/\rho_0 c^2$. It follows therefore that the pressure source cannot be moved from one end of the circuit to the other without simultaneously changing the acoustic impedance for the free-air side of the circuit.

An analogous circuit is sought which can be used to predict the average pressure at the piston without changing the impedance in the free-air side of the circuit when the pressure source is moved. The solution that is proposed here is given in Fig. 4. The analogous circuit in this figure shows two sources, one for the free-air part of the circuit and one for the original tube side. The pressure amplitude of the generator on the free-air side of the circuit is multiplied by $1 + Z_A/R_A$, which is a complex function of frequency. It follows from this circuit that the correct pressure at the piston is predicted when either source is activated.

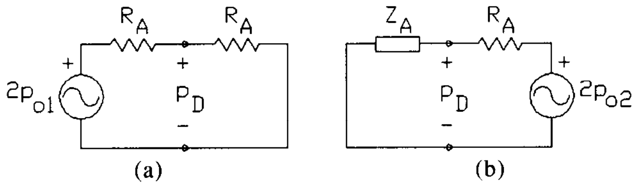

Fig. 2. Analogous circuit. (a) Predicting pressure p_D at piston when plane wave is incident from free-air side. (b) Predicting pressure p_D at piston when plane wave is incident from load end of tube.

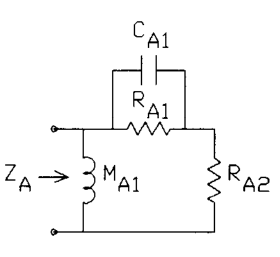

Fig. 3. Analogous circuit of air load impedance Z_A in Fig. 2.

If the quantity $1 + Z_A/R_A$ is denoted by the transfer function $T(s)$, where s is the complex frequency, it can be shown that $T(s)$ is given by

$$T(s) = \frac{1 + b_1 s + b_2 s^2}{1 + c_1 s + c_2 s^2} \qquad (1)$$

where the coefficients b_1, b_2, c_1, and c_2 are given by

$$b_1 = R_{A1} \| R_{A2} C_{A1} + \frac{M_{A1}}{(R_{A1} + R_{A2}) \| R_{A2}} \qquad (2)$$

$$b_2 = \frac{2 R_{A1} C_{A1} M_{A1}}{R_{A1} + R_{A2}} \qquad (3)$$

$$c_1 = R_{A1} \| R_{A2} C_{A1} + \frac{M_{A1}}{R_{A1} + R_{A2}} \qquad (4)$$

$$c_2 = \frac{R_{A1} C_{A1} M_{A1}}{R_{A1} + R_{A2}} \qquad (5)$$

The symbol $\|$ is used to denote the parallel combination of resistors. Since $T(s)p_{01}$ models the blocked pressure at the piston due to the incident free-air plane wave of pressure amplitude p_{01}, it follows that $T(s)$ represents the average effect of reflections on the pressure at the piston. Example plots of $20 \log |T(j2\pi f)|$ as functions of the frequency f for piston radii of ¼ and ½ in (6 and 12 mm) are given in Fig. 5. It can be seen from this figure that the increase in the average blocked pressure at the piston approaches 6 dB at the higher frequencies.

Several forms of the proposed model for a plane

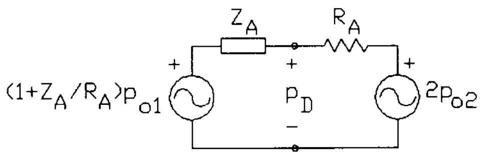

Fig. 4. Proposed analogous circuit predicting correct pressure p_D at piston when plane wave is incident from either free-air side or load end of tube.

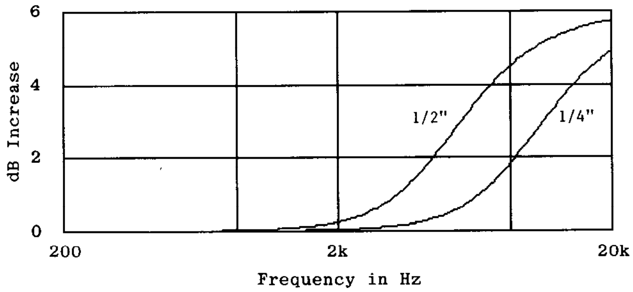

Fig. 5. Plots of $20 \log |T(j2\pi f)|$ as a function of frequency f for piston radii of ¼ and ½ in (6 and 12 mm).

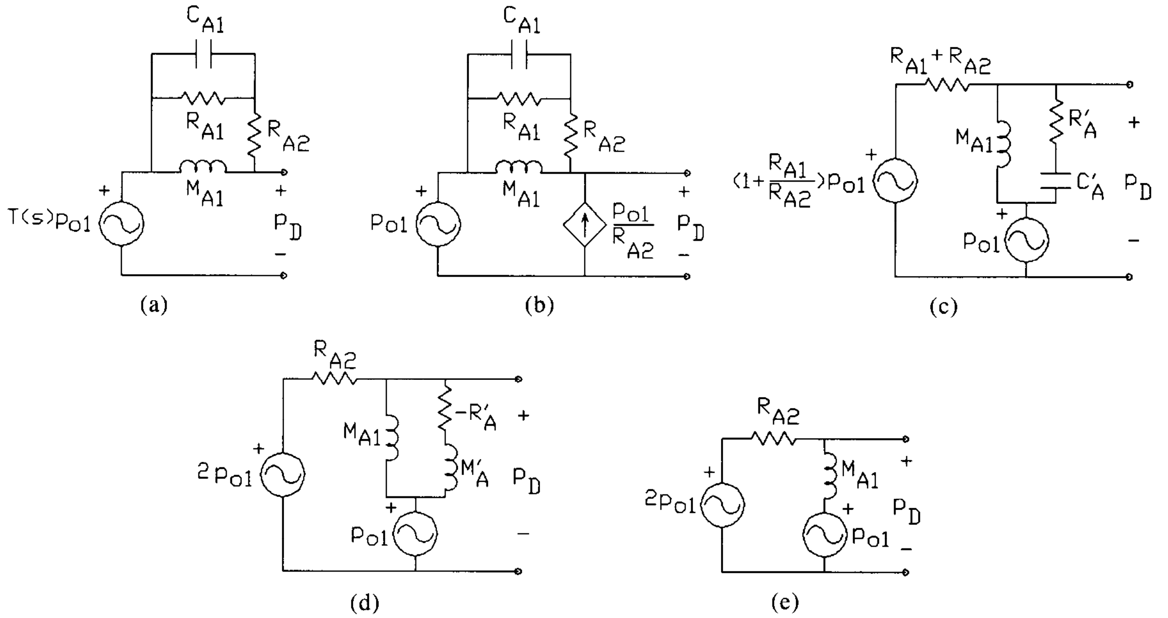

Fig. 6. (a) Summary of proposed analogous circuit. (b)–(d) Alternative forms of proposed circuit. (e) Approximate circuit proposed by Bauer.

wave incident on a piston in a long tube are given in Fig. 6. Fig. 6(a) summarizes the circuit developed here. Fig. 6(b), (c), and (d) give alternative models which follow by selectively making Thévenin and Norton equivalent circuits in the original circuit. In the circuits of Fig. 6(c) and (d) the primed circuit elements are given by

$$R'_{\mathrm{A}} = \frac{R_{\mathrm{A2}}(R_{\mathrm{A1}} + R_{\mathrm{A2}})}{R_{\mathrm{A1}}} \tag{6}$$

$$C'_{\mathrm{A}} = \frac{R^2_{\mathrm{A1}} C_{\mathrm{A1}}}{(R_{\mathrm{A1}} + R_{\mathrm{A2}})^2} \tag{7}$$

$$M'_{\mathrm{A}} = R^2_{\mathrm{A2}} C_{\mathrm{A1}} \ . \tag{8}$$

Note that the circuit of Fig. 6(d) contains a negative resistance. Fig. 6(e) gives the approximate circuit proposed by Bauer. His circuit follows from the one of Fig. 6(d) if the elements $-R'_{\mathrm{A}}$ and M'_{A} are omitted. It can be concluded that Bauer's circuit becomes exact only for the high-frequency limiting case.

2 CONCLUSIONS

A missing link in the electroacoustic modeling of a plane wave incident on the diaphragm of a free-field microphone is the effect of reflections at the diaphragm. It is well known that the effect is to boost the high-frequency response of the microphone. A method for incorporating the mechanism that causes this boost into the electroacoustic modeling of the microphone system has been proposed.

3 REFERENCES

[1] L. L. Beranek, *Acoustics* (McGraw-Hill, New York, 1954).

[2] B. B. Bauer, "On the Equivalent Circuit of a Plane Wave Confronting an Acoustical Device," *J. Acoust. Soc. Am. (Letter to the Editor)*, vol. 42, pp. 1095–1097 (1967 Nov.); reprinted in *J. Audio Eng. Soc.*, vol. 24, pp. 653–654 (1976 Oct.).

Dr. Leach's biography was published in the March issue.