# Impedance Compensation Networks for the Lossy Voice-Coil Inductance of Loudspeaker Drivers\*

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Two simple Zobel impedance compensation networks for the lossy voice-coil inductance of a loudspeaker driver are described. Design equations for the element values are given, and a numerical example is presented. The synthesis procedure can be extended to realize general *RC* networks which exhibit an impedance that decreases with frequency at a rate of -n dec/dec, where 0 < n < 1.

#### **0 INTRODUCTION**

A two-terminal network that is connected in series or in parallel with a circuit to cause its terminal impedance to be transformed into a desired impedance is commonly called a Zobel [1] network. In loudspeaker design a Zobel network consisting of a series resistor and capacitor connected in parallel with the voice coil of a driver has been described to compensate for the impedance rise at high frequencies caused by the voice-coil inductance [2]. If the inductance is lossless, the network can be designed so that the effective high-frequency impedance is resistive. By maintaining a resistive load on the crossover network, its performance is improved. However, the voice-coil inductance of the typical loudspeaker driver is not lossless. In this case a Zobel network consisting of one resistor and one capacitor can be used to obtain a resistive input impedance at only one frequency in the high-frequency range where the voice-coil inductance dominates.

In this engineering report two Zobel networks are described, one consisting of two resistors and two capacitors and the other consisting of three resistors and three capacitors. Each can be designed to compensate for the lossy voice-coil inductance of a driver. It is shown that the networks can be designed to approximate the desired impedance in an "equal-ripple" sense. Although the approximation can be improved with the use of more elements, it is shown by example that the simpler four element network can give excellent results with a typical driver. The effects of this network on the responses of second-order and thirdorder low-pass crossover networks for a specific driver are presented. At low frequencies the voice-coil impedance is dominated by its motional impedance. For infinite baffle systems the low-frequency impedance exhibits a peak at the fundamental resonance frequency of the driver. In [3] a modification of the circuit proposed in [2] is described which provides an additional compensation for this impedance peak. The circuit is also applicable to closed-box systems. Although the present report concerns impedance compensation at the high-frequencies where the voice-coil inductance dominates, the low-frequency compensation circuit proposed in [3] is reviewed. In addition, a modification of this circuit for vented-box systems is given.

It is assumed that the loudspeaker driver is operated in its small-signal range. Otherwise the voice-coil inductance becomes a time-varying nonlinear function, its value varying with diaphragm displacement. This would preclude a linear circuit analysis and make it impossible to derive the compensation networks.

The impedance approximation technique presented here has been used in the design of filters that convert white noise into pink noise. These circuits exhibit a gain slope of -3 dB per octave over the audio band. Example circuit diagrams of such filters can be found in [4], [5], and [6], but no design equations are given. In [5] the network is described as one in which "the zeros of one stage partially cancel the poles of the next stage." In [7] a similar network is described to realize an operational-amplifier circuit which exhibits a gain slope of +4.6 dB per octave over the audio band. The authors stated that the network component values were selected with the aid of a software optimization routine to match the desired slope. An analytical solution is given here for the design of such networks.

The general impedance compensation theorem described by Zobel can be succinctly summarized as follows. Given an impedance  $Z_1 = R_0 + Z_0$ , let an impedance  $Z'_1 =$ 

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 $R_0 + Z'_0$  be connected in parallel with  $Z_1$ . The condition that the parallel connection have a resistive impedance equal to  $R_0$  is that  $Z'_1 = R_0^2/Z_1$ . This is a general result that is not specific to loudspeakers. For completeness, its derivation is given in the following where the notation used is that for the voice-coil impedance.

## **1 IMPEDANCE COMPENSATION CONDITION**

The voice coil of a loudspeaker driver exhibits both a series resistance and an inductance. In the following it is assumed that the resistance is separated and treated as a separate element, that is, not a part of the voice-coil inductance. Fig. 1 shows the voice-coil equivalent circuit of a driver in an infinite baffle [8]. The resistor  $R_E$  and the inductor  $L_E$  represent the voice-coil resistance and inductance. The elements  $R_{ES}$ ,  $L_{CES}$ , and  $C_{MES}$  model the motional impedance generated when the voice coil moves. These elements are related to the small-signal parameters of the driver by the equations [9]

$$R_{\rm ES} = \frac{Q_{\rm MS}}{Q_{\rm ES}} R_{\rm E} \tag{1}$$

$$L_{\rm CES} = \frac{R_{\rm E}}{2\pi f_{\rm S} Q_{\rm ES}} \tag{2}$$

$$C_{\rm MES} = \frac{Q_{\rm ES}}{2\pi f_{\rm S} R_{\rm E}} \tag{3}$$

where  $Q_{\rm MS}$  is the mechanical quality factor,  $Q_{\rm ES}$  is the electrical quality factor, and  $f_{\rm S}$  is the fundamental resonance frequency.

Above the fundamental resonance frequency, the capacitor  $C_{\text{MES}}$  becomes a short circuit and the voice-coil impedance can be approximated by  $R_{\text{E}}$  in series with  $L_{\text{E}}$ . The equivalent high-frequency circuit is shown in Fig. 2(a). A resistor  $R_1$  in series with a capacitor  $C_1$  is shown in parallel with the voice-coil impedance. At low frequencies the impedance of the circuit is  $R_{\text{E}}$ . If the inductor is lossless, the high-frequency impedance is  $R_1$ . If  $R_1 = R_{\text{E}}$  and  $R_1C_1 = L_{\text{E}}/R_{\text{E}}$ , it is straightforward to show that the



Fig. 1. Equivalent circuit of voice-coil impedance.



Fig. 2. (a) High-frequency voice-coil equivalent circuit with twoelement Zobel network. (b) Circuit used to derive Zobel network impedance transfer function.

circuit has an impedance equal to  $R_{\rm E}$  at all frequencies [2]. In this case  $R_1$  and  $C_1$  form a simple Zobel network, which cancels the lossless  $L_{\rm E}$  from the input impedance of the driver.

In [10] it is shown that a lossy voice-coil inductance has an impedance that can often be approximated by

$$Z_{\rm L}(j\omega) = L_{\rm e}(j\omega)^n = L_{\rm e}\left[\cos\left(\frac{n\pi}{2}\right) + j\sin\left(\frac{n\pi}{2}\right)\right]\omega^n \quad (4)$$

where  $L_{\rm e}$  and *n* are constants. Fig. 2(b) shows the circuit of Fig. 2(a) with  $L_{\rm E}$  replaced with  $Z_{\rm L}(j\omega)$  and  $C_{\rm 1}$  replaced with an impedance  $Z_{\rm 1}(j\omega)$ . Let  $Z_{\rm in}$  be the input impedance to the circuit. The source current  $I_{\rm s}$  can be written

$$I_{\rm s} = \frac{V_{\rm s}}{Z_{\rm in}} = \frac{V_{\rm s} - V_{\rm 1}}{R_{\rm 1}} + \frac{V_{\rm s} - V_{\rm L}}{R_{\rm E}} \,.$$
(5)

If  $Z_{in} = R_E$  and  $R_1 = R_E$ , this equation can be solved for  $V_s$  to obtain

$$V_{\rm s} = V_{\rm 1} + V_{\rm L} = V_{\rm s} \frac{Z_{\rm 1}}{R_{\rm E} + Z_{\rm 1}} + V_{\rm s} \frac{Z_{\rm L}}{R_{\rm E} + Z_{\rm L}}$$
(6)

where voltage division has been used to express  $V_1$  and  $V_L$  as functions of  $V_s$ . This equation can be solved for  $Z_1$  to obtain

$$Z_{1}(j\omega) = \frac{R_{\rm E}^{2}}{Z_{\rm L}(j\omega)} = \frac{R_{\rm E}^{2}}{L_{\rm e}\omega^{n}} \left[ \cos\left(\frac{n\pi}{2}\right) - j\sin\left(\frac{n\pi}{2}\right) \right].$$
(7)

It follows that  $Z_{in} = R_E$  if  $R_1 = R_E$  and  $Z_1(j\omega)$  is given by Eq. (7). In this case the high-frequency voice-coil impedance is resistive at all frequencies. Note that  $|Z_1(j\omega)| \propto \omega^{-n}$ so that a plot of  $|Z_1(j\omega)|$  versus  $\omega$  on log–log scales is a straight line with a slope of -n dec/dec. It should also be noted that  $Z_1(j\omega)$  is the dual of  $Z_L(j\omega)$  scaled by the factor  $R_E^2$ , which follows from the fundamental principle derived by Zobel.

#### **2 APPROXIMATING IMPEDANCE**

Fig. 3 shows the Bode magnitude plot of an impedance which exhibits a slope of -n dec/dec between the frequencies  $f_1$  and  $f_6$ . Also shown are the asymptotes of an approximating impedance which exhibit alternating slopes of -1 and 0. Four frequencies are labeled between  $f_1$  and  $f_6$  at which the slopes of the asymptotes change. In the general case, let there be N frequencies, where N is even and  $N \ge$ 4. In this case the number of asymptotes having a slope of 0 is (N - 2)/2. Let k be the ratio of the asymptotic approximating impedance to the desired impedance at  $f = f_1$ .



Fig. 3. Desired impedance and asymptotes of approximating impedance versus frequency.

given by

$$|Z_1| = L_e (2\pi f_1)^n.$$
(8)

The approximating impedance at  $f_1$  is labeled  $k|Z_1|$ .

With *n*,  $f_1$ , and  $f_N$  specified, the object is to specify *k* and  $f_2$  through  $f_{N-1}$  such that the ratios of each even subscripted frequency to the odd subscripted frequency to its left are equal and the intersection points (indicated by dots on the plot) occur at the geometric mean of the adjacent frequencies. In this case the lengths of the six dashed vertical lines in Fig. 3 are equal and the asymptotes of the approximating impedance approximate the desired impedance in an equal ripple sense between  $f_1$  and  $f_N$ .

It is straightforward to show that the following conditions must hold:

$$k = \left(\frac{f_2}{f_1}\right)^{\frac{1-n}{2}} = \left(\frac{f_4}{f_3}\right)^{\frac{1-n}{2}} = \dots = \left(\frac{f_N}{f_{N-1}}\right)^{\frac{1-n}{2}}$$
(9)

$$f_{2} = f_{1}^{1-n} f_{3}^{n}$$

$$f_{4} = f_{3}^{1-n} f_{5}^{n}$$
: (10)

$$f_{N-2} = f_{N-3}^{1-n} f_{N-1}^{n}$$

$$f_{3} = f_{2}^{n} f_{4}^{1-n}$$

$$f_{5} = f_{4}^{n} f_{6}^{1-n}$$

$$\vdots$$
(11)

 $f_{N-1} = f_{N-2}^n f_N^{1-n}$ 

Solutions to these equations are given next for the cases N = 4 and N = 6.

# 2.1 Case A: *N* = 4

Let  $f_1$  and  $f_4$  be specified. For N = 4, Eqs. (9)–(11) can be solved to obtain

$$k = \left(\frac{f_4}{f_1}\right)^{\frac{n(1-n)}{2(1+n)}}$$
(12)

$$f_2 = f_1^{\frac{1}{1+n}} f_4^{\frac{n}{1+n}}$$
(13)

$$f_3 = f_1^{\frac{n}{1+n}} f_4^{\frac{1}{1+n}}.$$

Let  $Z_1(f)$  be the approximating impedance function. It is given by

$$Z_{1}(f) = \frac{k |Z_{1}|}{j(f/f_{1})} \times \frac{1 + j(f/f_{2})}{1 + j(f/f_{3})}.$$
(15)

#### 2.2 Case B: N = 6

Let  $f_1$  and  $f_6$  be specified. For N = 6, Eqs. (9)–(11) can be solved to obtain

$$k = \left(\frac{f_6}{f_1}\right)^{\frac{n(1-n)}{2(2+n)}}$$
(16)

$$f_2 = f_1^{\frac{2}{2+n}} f_6^{\frac{n}{2+n}}$$
(17)

$$f_3 = f_1^{\frac{1+n}{2+n}} f_6^{\frac{1}{2+n}} \tag{18}$$

$$f_4 = f_1^{\frac{1}{2+n}} f_6^{\frac{1+n}{2+n}}$$
(19)

$$f_5 = f_1^{\frac{n}{2+n}} f_6^{\frac{2}{2+n}}.$$
 (20)

The approximating impedance as a function of frequency for this case is given by

$$Z_{1}(f) = \frac{k|Z_{1}|}{j(f/f_{1})} \times \frac{1 + j(f/f_{2})}{1 + j(f/f_{3})} \times \frac{1 + j(f/f_{4})}{1 + j(f/f_{5})}.$$
 (21)

#### 2.3 Example Plots

To illustrate the accuracy of the approximating functions, let the impedance given by Eq. (7) be approximated over a three-decade band for the case n = 0.5. The smaller the value of n, the poorer the approximation. In the author's experience, the value of n for most loudspeaker drivers is in the range from 0.6 to 0.7. Thus the value n =0.5 results in an approximation that is worse than what can be expected with the typical driver.

Fig. 4 shows the calculated Bode magnitude plots. Curve a is the desired impedance. Curve b is the approximating impedance for N = 4. Curve c is the approximating impedance for N = 6. It can be seen that the approximating impedance functions ripple about the desired function over the band of interest with a maximum deviation occurring at the two frequency extremes. Between the two extremes, the maximum deviation is less than it is at the extremes because the design equations are derived from the asymptotes of the approximating function.



(14)

Fig. 4. Example plots of desired impedance (curve a) and approximating impedances (curves b, c) versus frequency for n = 0.5.

#### **3 THE COMPENSATING CIRCUITS**

## 3.1 Network A

Fig. 5(a) shows a circuit consisting of two capacitors and one resistor, which can be used to realize the impedance of Eq. (15). The impedance is given by

$$Z_1(s) = \frac{1}{s(C_1 + C_2)} \times \frac{1 + s/\omega_2}{1 + s/\omega_3}$$
(22)

where  $s = j\omega = j2\pi f$  and

$$\omega_2 = 2\pi f_2 = \frac{1}{R_2 C_2} \tag{23}$$

$$\omega_3 = 2\pi f_3 = \frac{C_1 + C_2}{R_2 C_1 C_2}.$$
(24)

The impedance of the circuit is equal to that of Eq. (15) if

$$C_1 = \frac{f_2}{2\pi f_1 f_3 k |Z_1|} \tag{25}$$

$$C_2 = \frac{f_3 - f_2}{f_2} C_1 \tag{26}$$

$$R_2 = \frac{1}{2\pi f_2 C_2} \,. \tag{27}$$

The circuit of Fig. 5(a) corresponds to Fincham's more general compensating network in [11].

## 3.2 Network B

Fig. 5(b) shows a circuit consisting of three capacitors and two resistors, which can be used to realize the impedance of Eq. (21). The impedance is given by

$$Z_{1}(s) = \frac{1}{s(C_{1} + C_{2} + C_{3})} \times \frac{(1 + s/\omega_{2})(1 + s/\omega_{4})}{s^{2}/(\omega_{3}\omega_{5}) + s(1/\omega_{3} + 1/\omega_{5}) + 1}$$
(28)

where

$$\omega_2 = 2\pi f_2 = \frac{1}{R_2 C_2} \tag{29}$$

$$\omega_4 = 2\pi f_4 = \frac{1}{R_3 C_3} \tag{30}$$

$$\omega_3 \omega_5 = 2\pi f_3 \times 2\pi f_5 = \frac{C_1 + C_2 + C_3}{R_2 R_3 C_1 C_2 C_3}$$
(31)

$$\frac{1}{\omega_3} + \frac{1}{\omega_5} = \frac{1}{2\pi f_3} + \frac{1}{2\pi f_5}$$
$$= \frac{R_2 C_2 (C_1 + C_3) + R_3 C_3 (C_1 + C_2)}{C_1 + C_2 + C_3}.$$
(32)

The impedance of the circuit is equal to that of Eq. (21) if

$$C_1 = \frac{f_2 f_4}{2\pi f_1 f_3 f_5 k |Z_1|} \tag{33}$$

$$C_2 = \frac{f_2 - f_3 - f_5 + f_3 f_5 / f_2}{f_4 - f_2} C_1$$
(34)

$$R_2 = \frac{1}{2\pi f_2 C_2}$$
(35)

$$C_3 = \frac{f_3 - f_4 + f_5 - f_3 f_5 / f_4}{f_4 - f_2} C_1$$
(36)

$$R_3 = \frac{1}{2\pi f_4 C_3} \,. \tag{37}$$

# 4 COMPENSATING THE LOW-FREQUENCY DRIVER IMPEDANCE

The impedance  $Z_1(j\omega)$  in Fig. 2(b) is the dual of  $Z_L(j\omega)$ scaled by the factor  $R_E^2$ . Following [3], the impedance rise at resonance can be canceled by adding an impedance  $Z_2(j\omega)$  in parallel with  $Z_1(j\omega)$ , which is the dual of the motional impedance of the driver scaled by the factor  $R_E^2$ . This impedance consists of a series *RLC* circuit having the element values [3]

$$R_{\rm S} = \frac{Q_{\rm ES}}{Q_{\rm MS}} R_{\rm E} \tag{38}$$

$$L_{\rm S} = \frac{Q_{\rm ES}R_{\rm E}}{2\pi f_{\rm S}} \tag{39}$$

$$C_{\rm S} = \frac{1}{2\pi f_{\rm S} Q_{\rm ES} R_{\rm E}}.$$
(40)

The circuit for  $Z_2(j\omega)$  is shown in Fig. 6(a). The preceding equations apply to a driver in an infinite baffle. For a closed-box baffle the element values are given by

$$R_{\rm S} = \frac{Q_{\rm EC}}{Q_{\rm MC}} R_{\rm E} \tag{41}$$

$$L_{\rm S} = \frac{Q_{\rm EC}R_{\rm E}}{2\pi f_{\rm C}} \tag{42}$$

$$C_{\rm S} = \frac{1}{2\pi f_{\rm C} Q_{\rm EC} R_{\rm E}} \tag{43}$$

where  $Q_{\rm EC}$  is the closed-box electrical quality factor,  $Q_{\rm MC}$  is the closed-box mechanical quality factor, and  $f_{\rm C}$  is the closed-box resonance frequency [12].

The circuit for  $Z_2(j\omega)$  for a vented-box baffle is shown in Fig. 6(b). The element values for  $R_S$ ,  $L_S$ , and  $C_S$  are



Fig. 5. Circuits for approximating impedance  $Z_1(j\omega)$ .

calculated from Eqs. (38) through (40). It can be shown that the elements  $R_{\rm P}$ ,  $L_{\rm P}$ , and  $C_{\rm P}$  are given by

$$R_{\rm P} = \frac{\alpha Q_{\rm ES} Q_{\rm L} R_{\rm E}}{h} \tag{44}$$

$$L_{\rm P} = \frac{\alpha Q_{\rm ES} R_{\rm E}}{2\pi f_{\rm S} h 2} \tag{45}$$

$$C_{\rm P} = \frac{1}{2\pi f_{\rm S} \alpha Q_{\rm ES} R_{\rm E}} \tag{46}$$

where  $\alpha = V_{AS}/V_B$  is the system compliance ratio,  $Q_L$  is the enclosure quality factor at the Helmholtz resonance frequency, and  $h = f_B/f_S$  is the system tuning ratio [13].

#### **5 NUMERICAL EXAMPLE**

One sample of the JBL model 2241H 18-in (0.457-m) professional woofer was selected to illustrate the networks

and their application. The dc voice-coil resistance was found to be  $R_{\rm E} = 5.1 \ \Omega$ . The voice-coil impedance was measured at 62 frequencies between 14.8 Hz and 20 kHz with an MLSSA analyzer. The data in the range from 1.8 to 20 kHz were used to calculate the lossy voice-coil inductance parameters. Calculations on the MLSSA data yielded the parameters  $R_{\rm ES} = 26.9 \ \Omega$ ,  $L_{\rm CES} = 38.1 \ mH$ ,  $C_{\rm MES} = 424 \ \mu\text{F}$ , n = 0.764, and  $L_{\rm e} = 0.0150$ . Fig. 7 shows the measured magnitude and phase of the impedance as circles and the impedance calculated from the equation

$$Z_{\rm VC}(j\omega) = R_{\rm E} + L_{\rm e}(j\omega)^n + \left(\frac{1}{R_{\rm ES}} + \frac{1}{j\omega L_{\rm CES}} + j\omega C_{\rm MES}\right)^{-1}$$
(47)

shown as a solid line, where  $\omega = 2\pi f$ . The figure shows excellent agreement between the measured and calculated

$$Z_{2}(s) \xrightarrow{R_{S}} L_{S} C_{S}$$

$$Z_{2}(s) \xrightarrow{(a)} Z_{2}(s) \xrightarrow{(b)} Z_{2}(s) \xrightarrow{(b)} Z_{2}(s)$$

Fig. 6. Compensation circuits for low-frequency impedance rise. (a) Infinite-baffle and closed-box drivers. (b) Vented-box drivers.



Fig. 7. Impedance measured and calculated from Eq. (47) (------) for JBL driver. (a) Magnitude. (b) Phase.

data, thus verifying the calculated values of the driver parameters.

The element values for the Zobel networks were calculated to compensate for the voice-coil inductance over the

Table 1. Summary intermediate calculations.				
	Network A	Network B		
Ν	4	6		
k	1.24	1.15		
$ Z_1 $	$4.77 \ \Omega$	$4.77 \ \Omega$		
$f_1$	300 Hz	300 Hz		
$f_2$	1.85 kHz	958 Hz		
$\bar{f_3}$	3.24 kHz	1.37 kHz		
$f_4$	20 kHz	4.38 kHz		
$f_5$		6.26 kHz		
$f_6$		20 kHz		

Table 1. Summary intermediate calculations

Fable	2.	Element	values.
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	Network A	Network B
$\overline{R_1}$	5.1 Ω	5.1 Ω
$\dot{C_1}$	51.2 μF	47.4 μF
$R_2$	2.23 Ω	5.25 Ω
$\overline{C_2}$	38.5 µF	31.6 µF
$R_3$		2.03 Ω
$C_3$		17.9 μF

frequency band from  $f_1 = 300$  Hz to  $f_N = 20$  kHz. Table 1 summarizes the intermediate calculations for the two networks. Table 2 gives the calculated element values. Network A is the network of Fig. 5(a). Network B is that of Fig. 5(b). The element values for the optional network to compensate for the impedance rise at resonance have the values  $R_{\rm S} = 0.858 \ \Omega$ ,  $L_{\rm S} = 11 \ {\rm mH}$ , and  $C_{\rm S} = 1460$ µF. It is quite obvious that these values would be impractical in a passive crossover network. Indeed, an 11-mH air-core inductor would in all probability have a series resistance greater than 0.858  $\Omega$ . For these reasons, the impedance  $Z_2(j\omega)$  has been omitted in the following. However, it would be expected that the element values would fall in a more practical range for midrange and tweeter drivers which have a much higher resonance frequency than the driver considered here.

Fig. 8 shows the magnitude and phase of the voice-coil impedance with and without Zobel network A. The plots are calculated from the measured voice-coil data and not those predicted by Eq. (47). The plots for network B are not shown because, for all practical purposes, they are not distinguishable from those of network A. However, this may not be the case with drivers that have a lower value of n.

To evaluate the effect of the Zobel networks on the performance of passive crossover networks, the voice-coil



Fig. 8. Impedance of JBL driver with and without Zobel network A. (a) Magnitude. (b) Phase.

voltage of the JBL driver was calculated for a source voltage of 1 V rms with second- and third-order low-pass crossover networks. The crossover frequency was chosen to be  $f_c = 800$  Hz, which might be a typical value when this driver is used with a midrange horn. The circuit diagrams are shown in Fig. 9. Second-order networks are usually designed for critical damping. The crossover frequency is the -6-dB frequency of the network. The element values for the second-order network in Fig. 9(a) are given by

$$L_1 = \frac{R_{\rm E}}{\pi f_{\rm c}} = 2.03 \text{ mH}$$
(48)

$$C_1 = \frac{1}{4\pi f_c R_E} = 8.43 \ \mu F. \tag{49}$$

$$\begin{array}{c}
\begin{array}{c}
L_1 \\
L_2 \\
L_1 \\
L_2 \\$$

Fig. 9. (a) Second-order crossover network. (b) Third-order crossover network.

Third-order networks are usually designed for a Butterworth response. The crossover frequency is the -3-dB frequency of the network. The element values for the third-order network in Fig. 9(b) are given by

$$L_1 = \frac{3R_{\rm E}}{4\pi f_{\rm c}} = 1.52 \text{ mH}$$
(50)

$$C_1 = \frac{2}{3\pi f_{\rm c} R_{\rm E}} = 52 \ \mu \text{F} \tag{51}$$

$$L_2 = \frac{R_{\rm E}}{4\pi f_{\rm c}} = 0.507 \text{ mH.}$$
(52)

Figure 10(a) shows the calculated voice-coil voltage for the second-order crossover network with and without Zobel network A. With the network, the response follows what would be expected of a second-order crossover network. Without the network, the voltage exhibits a peak at 1.8 kHz that is 16.7 dB greater than the response with the network. Fig. 10(b) shows the calculated voice-coil voltage for the third-order crossover network with and without Zobel network A. With the network, the response follows what would be expected of a third-order crossover network. Without the network, the voltage exhibits a peak at



Fig. 10. Voice-coil voltage of JBL driver with (curve a) and without (curve b) Zobel network A. (a) Second-order crossover network. (b) Third-order crossover network.

630 Hz that is 10 dB greater than the response with the network. Above 1.6 kHz the response without the network lies above the response with the network and exhibits a slope of approximately -43 dB/dec. The slope with network A approaches -60 dB/dec, which is the correct slope for a third-order network. Crossover simulations with Zobel network B have been omitted because the results were almost identical. However, this may not be the case with drivers having a lower value of *n*. The plots in Fig. 10 were calculated using the measured voice-coil data and not that predicted by Eq. (47). The plots show some evidence of the rise in impedance at the fundamental resonance frequency of the driver. This could be eliminated by the addition of the circuit in Fig. 6(a) in parallel with the Zobel network.

# **6 CONCLUSION**

The high-frequency rise in the voice-coil impedance of a loudspeaker driver caused by a lossy voice-coil inductance can be approximately canceled in the audio band by an *RC* Zobel network connected in parallel with the voice coil. The simplest network consists of two resistors and two capacitors. More complicated networks have three or more resistors and three or more capacitors. For a typical driver, the simplest network can yield excellent results. Because the lossy voice-coil inductance of crossover networks, the parameters *n* and  $L_e$  should be included in the list of specifications for drivers as an aid in the design of Zobel compensation networks.

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